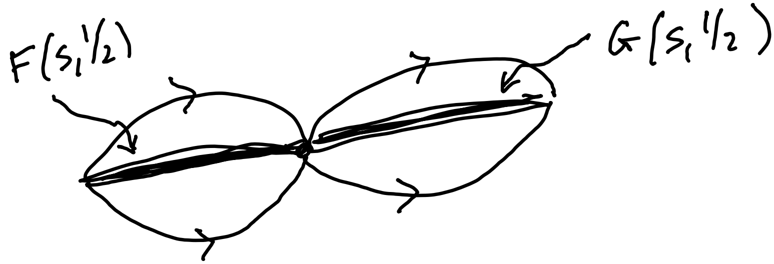
$$H(s, 1/2) = \begin{cases} F(2s, 1/2), & s \in [0, 1/2] \\ G(2s-1, 1/2), & s \in [1/2, 1] \end{cases}$$

is the concatenation of 2 "inbetween" paths, namely, of

$\gamma(s) = F(s, 1/2)$ and $\delta(s) = G(s, 1/2)$



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c.) H is continuous by the Pasting Lemma.

Consider $D = [0,1] \times [0,1]$

$A = [0, 1/2] \times [0,1]$ closed in D

and

$B = [1/2, 1] \times [0,1]$ closed there too

Note that $D = A \cup B$.

Also D is the domain of the homotopy H and in fact

$$H(s,t) = \begin{cases} F(2s,t) & \text{if } (s,t) \in A \\ G(2s-1,t) & \text{if } (s,t) \in B \end{cases}$$

is an alternative description of H.

Since F, G are continuous on A, resp B

we only need to check if

$F = G$ on $A \cap B$. Then H is continuous by the Pasting Lemma.

Now, $A \cap B = \{ s = 1/2, \; t \in [0,1] \}$

and there

$$F(2s,t) = F(1,t) = b$$

$$G(2s-1,t) = G(0,t) = b$$

So they are equal on $A \cap B$ indeed.