Resolvable spaces

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joint work with

István Juhász and Zoltán Szentmiklóssy

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Let $\kappa > 1$ be a cardinal. A topological space X is **κ-resolvable** iff X contains κ **disjoint dense subsets.**

- **resolvable** iff it is **2-resolvable**
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If D is **dense** and U is a non-empty **open** set then $U \cap D \neq \emptyset$.

Fact

If X is κ-resolvable then $\kappa \leq \Delta(X) = \min\{|U| : U \in \tau_X \setminus \{\emptyset\}\}$.

$\Delta(X)$ is the **dispersion character** of X.

Definition (Ceder, Pearson, 1967)

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Theorem

*A topological space X is **maximally resolvable** provided*

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(2) *ordered,*

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- A topological space $X = \langle X, \tau \rangle$ is **maximal** iff it is **crowded** and no **strictly stronger** topology on X is crowded.
- A crowded space is **submaximal** iff the dense subsets are **open**.
- A space X is **NODEC** if all nowhere dense subsets of X are closed discrete.

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Theorem (Hewitt, 1943)

*There are **maximal**, and so **strongly irresolvable**, T_2 spaces.*

Problem (Ceder, Pearson 1967)

Does ω-**resolvable** imply **maximally resolvable**?

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Does ω-**resolvable** imply **maximally resolvable**?

Natural idea: $X_0 = \langle \kappa, \tau_0 \rangle$, $\Delta(X_0) > \omega$, $\{D_n : n < \omega\}$ are pairwise disjoint dense sets.

Let $\tau \supset \tau_0$ be a topology on κ s.t.

(a) $X = \langle \kappa, \tau \rangle$ is crowded

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- X is ω-resolvable.
- $\Delta(X) > \omega$? Yes, if **every** $A \in [\kappa]^\omega$ **is discrete in** τ_0
- Is X ω_1-irresolvable?
- **Plan:**
  (1) D_n is irresolvable subspace in X for each $n < \omega$
  (2) **If a space is countable union of irresolvable spaces then it is NOT ω_1-resolvable.**
- (1) is clear. **(2) is false**

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Does ω-**resolvable** imply **maximally resolvable**?

Natural idea: $X_0 = \langle \kappa, \tau_0 \rangle$, $\Delta(X_0) > \omega$, $\{D_n : n < \omega\}$ are pairwise disjoint dense sets.
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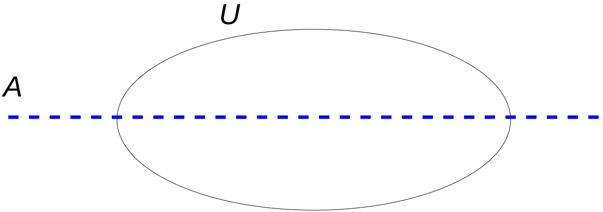

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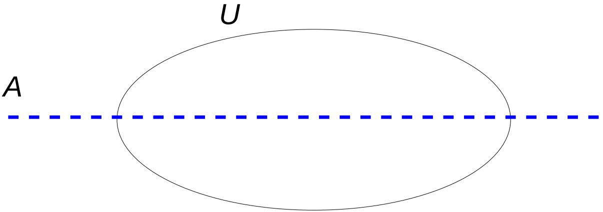

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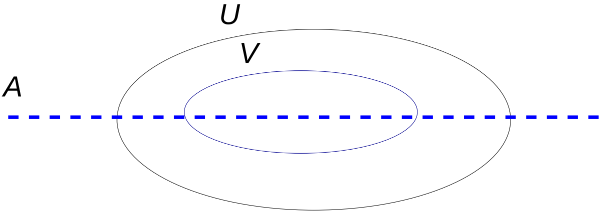

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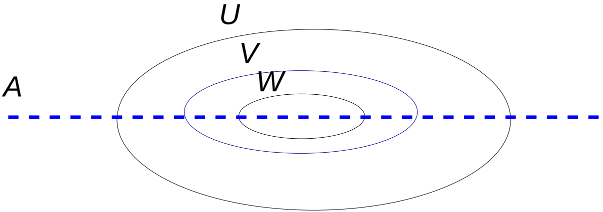

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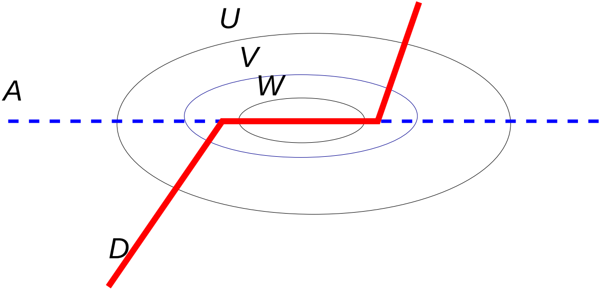

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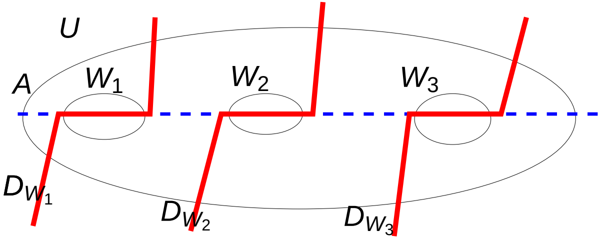

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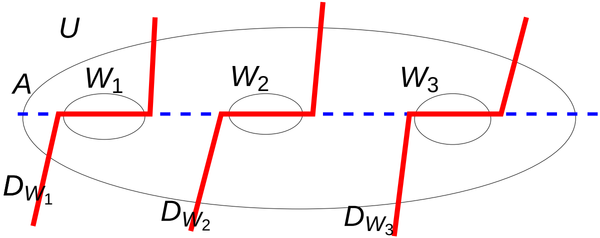

Fact

*If $\mathcal{D} \Vdash_\tau A$ **is dense in** U then A **is dense in** U.*

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Lemma

If $\mathcal{D}$ is ***disjoint family*** *of dense sets, and τ is* ***$\mathcal{D}$-forced*** *then every $D \in \mathcal{D}$ is* ***strongly irresolvable****, ($D \cap U$ is irresolvable for each $U \in \tau$).*

Proof: Assume $S, T \subset D \cap U$ are dense. We show $S \cap T \neq \emptyset$.
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Existence of $\mathcal{D}$-forced spaces

$\mathcal{D}$-forced spaces: **dense subspaces of the Cantor cube $D(2)^\lambda$**

Natural one-to-one correspondence between

- dense subspaces of the Cantor cube $D(2)^\lambda$ of size κ
- independent families of 2-partitions of κ indexed by λ.

Let $X = \{f_\alpha : \alpha < \kappa\} \subset D(2)^\lambda$ be dense.

For $\xi < \lambda$ and $i < 2$ let $B^i_\xi = \{\alpha < \kappa : f_\alpha(\xi) = i\}$.

$\langle B^0_\xi, B^1_\xi \rangle$ is a partition of κ.

$\mathcal{B} = \{\langle B^0_\xi, B^1_\xi \rangle : \xi < \lambda\}$ is a family of partition which is

- **independent** ($\forall \varepsilon \in Fin(\lambda, 2)$ $\mathcal{B}[\varepsilon] = \cap\{B^{\varepsilon(\xi)}_\xi : \xi \in dom(\varepsilon)\} \neq \emptyset$.)
- **separating** ($\forall \alpha \neq \beta \in \kappa\, \exists \xi < \lambda\, |\{\alpha, \beta\} \cap B^0_\xi| = 1$)

Let $X_\mathcal{B} = \langle \kappa, \tau_\mathcal{B} \rangle$, where $\{\mathcal{B}[\varepsilon] : \varepsilon \in Fin(\lambda, 2)\}$ is a base for $\tau_\mathcal{B}$

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For $\xi < \lambda$ and $i < 2$ let $B^i_\xi = \{\alpha < \kappa : f_\alpha(\xi) = i\}$.

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X and $X_\mathcal{B}$ are homeomorphic

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X' is dense, and X' and $X_\mathcal{B}$ are homeomorphic.

Existence of $\mathcal{D}$-forced spaces

$\mathcal{D}$-forced spaces: **dense subspaces of the Cantor cube $D(2)^\lambda$**

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Main theorem

Let

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(1) $\forall D \in \mathcal{D}$ is κ-**dense** in $\tau_{\mathcal{C}}$,

(2) $\tau_{\mathcal{C}}$ is $\mathcal{D}$-**forced**,

(3) $\tau_{\mathcal{C}}$ is **NODEC**, (i.e. every $\tau_{\mathcal{C}}$-nowhere dense set is closed discrete)

(4) Every $A \in \left[\kappa\right]^{<\kappa}$ is closed discrete.

Moreover, if $I \subset 2^\kappa$ with $|I| = 2^\kappa$ then we can assume that

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Corollary 1.

For each $\kappa \geq \omega$ there is a **submaximal** space X of cardinality κ which is dense subspace of $D(2)^{2^\kappa}$.

Proof: Let $\mathcal{B} = \{\langle B^0_\xi, B^1_\xi \rangle : \xi \in 2^\kappa\}$ be a independent, separating family of 2-partitions of κ, let $\mathcal{D} = \{\kappa\}$.
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Theorem (Illanes)

If $X = \cup\{X_i : i < n\}$, $n < \omega$, and X_i are ***hereditarily irresolvable*** *subspaces (**=every crowded subspace is irresolvable**) then X is not $n+1$-resolvable.*

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If X is not n-resolvable for some $n < \omega$ then there is a hereditarily irresolvable open subspace U in X.

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For each $\omega \leq \lambda = \operatorname{cf}(\lambda) \leq \kappa$ there is a 0-dimensional T_2 space $X = \langle \kappa, \tau \rangle$, s.t. X is c.c.c., $\Delta(X) = \kappa$, X is **not λ-resolvable**, but **hereditarily μ-resolvable** for each $\mu < \lambda$.

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Compactness

Positive results

Theorem (Illanes)

*If X is k-**resolvable** for each $k < \omega$ then X is ω-**resolvable**.*

Theorem (Bashkara-Rao)

*If $cf(\lambda) = \omega$ and X is μ-**resolvable** for each $\mu < \lambda$ then X is λ-**resolvable**.*

Problem

What happens if $\omega < cf(\lambda) < \lambda$?

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*If $cf(\lambda) = \omega$ and X is **μ-resolvable** for each $\mu < \lambda$ then X is **λ-resolvable**.*

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What happens if $\omega < cf(\lambda) < \lambda$?

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Compactness

A weak positive result

Theorem

If $\hat{c}(X) \leq cf(\lambda) < \lambda \leq \Delta(X)$, and

$(\star)$ *for each dense subspace Y if $\Delta(Y) \geq \lambda$ then Y is μ-**resolvable** for each $\mu < \lambda$,*

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Extraresolvability

Definition (Malykhin, 1998)

X is **extraresolvable** iff there are dense sets $\{D_\alpha : \alpha < \Delta(X)^+\}$ such that $D_\alpha \cap D_\beta$ is **nowhere dense** for $\alpha \neq \beta$.

Problem (Comfort – Hu)

X is countable and maximally resolvable $\stackrel{?}{\Longrightarrow}$ extraresolvable?

M. Hrušak: $\mathfrak{i} = \mathfrak{c} \Longrightarrow$ **NO**

Corollary 5

For each $\kappa \geq \omega$ there is 0-dim. T_2, ccc $X = \langle \kappa, \tau \rangle$ s.t. $\Delta(X) = \kappa$, X is **hereditarily maximally resolvable**, but **not extraresolvable**.

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A space X is **almost κ-resolvable** there are dense sets $\{D_\alpha : \alpha < \kappa\}$ such that $D_\alpha \cap D_\beta$ is **nowhere dense** for $\alpha \neq \beta$.

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If $\{D_n : n < \omega\}$ dense, $D_n \cap D_m$ is nowhere dense, then D_0, $D_1 \setminus D_0$, $D_2 \setminus D_0 \cup D_1, \ldots$ are pairwise disjoint dense sets.

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*Let X be an **extraresolvable** space with $\Delta(X) \geq \omega_1$. Is X then **ω_1-resolvable**?*

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A recent application of the Main Theorem

Let X be an **extraresolvable** space with $\Delta(X) \geq \omega_1$. Is X then ω_1**- resolvable**?

Theorem (Comfort, Hu)

*If κ is an infinite cardinal such that GCH **first fails at** κ then there is a 0-dimensional T_2 space X with $|X| = \Delta(X) = \kappa^+$ such that X is κ**-resolvable**, **extraresolvable** but **not** κ^+**-resolvable**.*

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Theorem (Pavlov, 2000)

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*If $\kappa = cf(\kappa)$, $\Delta(X) = \kappa \geq \hat{s}(X)$ then X is **maximally resolvable**.*

Problem

What if $\Delta(X) = \hat{s}(X)$ is singular?

Some notions and notations

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Monotonically normal spaces

For any topological space X we write

$$\mathcal{M}(X) = \{\langle x, U\rangle \in X \times \tau(X) : x \in U\}.$$

The elements of $\mathcal{M}(X)$ are the **marked open sets.**
The space X is **monotonically normal** iff it is T_1 and it admits a **monotone normality operator**, that is a function $H : \mathcal{M}(X) \to \tau(X)$ such that

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Theorem

If X is crowded, ***monotonically normal*** *then*

(a) X *is* ω***-resolvable***,

(b) X *is* ***almost*** $\min(2^\omega, \omega_2)$***-resolvable***.

Problem

X *crowded, monotonically normal* $\stackrel{?}{\Longrightarrow}$ X ***maximally resolvable*** *?*

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*If κ is a **measurable cardinal**, then there is a monotonically normal space X with $|X| = \Delta(X) = \kappa$ which is **not ω_1-resolvable**.*

Theorem

A crowded monotonically normal space X is maximally resolvable provided $|X| < \aleph_\omega$.

Theorem

*It is consistent (modulo a **supercompact cardinals**) that there is a monotonically normal space X with $|X| = \Delta(X) = \aleph_\omega$ which is **not ω_2-resolvable**.*

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The pdf file of my talk can be downloaded from my homepage.