Resolvable spaces

Lajos Soukup

joint work with

lstván Juhász and Zoltán Szentmiklóssy

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences; Eötvos University, Budapest

Advances in Set-Theoretic Topology, Erice 2008

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Definition

Let $\kappa > 1$ be a cardinal. A topological space X is κ -resolvable iff X contains κ disjoint dense subsets.

- resolvable iff it is 2-resolvable
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If *D* is **dense** and *U* is a non-empty **open** set then $U \cap D \neq \emptyset$.

Fact

If X is κ -resolvable then $\kappa \leq \Delta(X) = \min\{|U| : U \in \tau_X \setminus \{\emptyset\}\}.$

 $\Delta(X)$ is the **dispersion character** of *X*.

Definition (Ceder, Pearson, 1967)

X is **maximally resolvable** iff it is $\Delta(X)$ -resolvable.

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- A crowded space is **submaximal** iff the dense subsets are **open**.
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There are maximal, and so strongly irresolvable, T₂ spaces.

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Natural idea: $X_0 = \langle \kappa, \tau_0 \rangle$, $\Delta(X_0) > \omega$, $\{D_n : n < \omega\}$ are pairwise disjoint dense sets.

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Counterexamples

El'kin, Malykhin, Eckertson, Hu: either not T₂ or not in ZFC.

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Definition

$\langle X, \tau \rangle$ top. spaces, \mathcal{D} family of dense sets, $A \subset X$ and $U \in \tau$.

 $\mathcal{D} \Vdash_{\tau} A$ is dense in U

iff $\forall V \subset U$ open $\exists W \subset V$ open and $D \in D$ such that $D \cap W \subset A$.

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$\mathcal{D} \Vdash_{\tau} A$ is dense in U

iff $\forall V \subset U$ open $\exists W \subset V$ open and $D \in \mathcal{D}$ such that $D \cap W \subset A$.



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Fact

If $\mathcal{D} \Vdash_{\tau} A$ is dense in U then A is dense in U.

Soukup, L (Rényi Institute)

Resolvable spaces

- $\mathcal{D} \Vdash_{\tau} A$ is dense in U
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Image: A matrix and a matrix

An easy application

 τ is \mathcal{D} -forced iff *A* is dense in *U* implies $\mathcal{D} \Vdash_{\tau} A$ is dense in *U*, (i.e. $\forall V \subset U$ open $\exists W \subset V$ open $\exists D \in \mathcal{D} D \cap W \subset A$).

Lemma

If \mathcal{D} is **disjoint family** of dense sets, and τ is \mathcal{D} -forced then every $D \in \mathcal{D}$ is strongly irresolvable, $(D \cap U \text{ is irresolvable for each } U \in \tau)$.

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Proof: Assume S, T \subset D \cap U are dense. We show S \cap T \neq \emptyset.

S is dense in U and \tau is \mathcal{D}-forced \Longrightarrow \exists V \subset U open \exists D_S \in \mathcal{D} s. t.

V \cap D_S \subset S \subset D.

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Existence of \mathcal{D} -forced spaces

 \mathcal{D} -forced spaces: **dense subspaces of the Cantor cube** $D(2)^{\lambda}$ Natural one-to-one correspondence between

- dense subspaces of the Cantor cube $D(2)^{\lambda}$ of size κ
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Let $X = {f_{\alpha} : \alpha < \kappa} \subset D(2)^{\lambda}$ be dense.

For $\xi < \lambda$ and i < 2 let $m{B}^i_{\xi} = \{ lpha < \kappa : f_{lpha}(\xi) = i \}.$

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- dense subspaces of the Cantor cube $D(2)^{\lambda}$ of size κ
- independent families of 2-partitions of κ indexed by λ .

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(5) $\forall \xi \notin I C^i_{\xi} = B^i_{\xi}$,

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Existence of \mathcal{D} -forced spaces

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Is it provable in ZFC that the Cantor cube $D(2)^{c}$ has a dense countable submaximal subspace?

Corollary 1.

For each $\kappa \ge \omega$ there is a **submaximal** space *X* of cardinality κ which is dense subspace of $D(2)^{2^{\kappa}}$.

Proof: Let $\mathcal{B} = \{ \langle B_{\xi}^0, B_{\xi}^1 \rangle : \xi \in 2^{\kappa} \}$ be a independent, separating family of 2-partitions of κ , let $\mathcal{D} = \{\kappa\}$.

By main theorem we obtain C s.t.

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By Lemma, $\langle \kappa, \tau_{\kappa} \rangle$ is strongly irresolvable.

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If $X = \bigcup \{X_i : i < n\}$, $n < \omega$, and X_i are hereditarily irresolvable subspaces (=every crowded subspace is irresolvable) then X is not n + 1-resolvable.

Corollary 3

For $\omega \leq \mu < \lambda < \kappa$ there is a 0-dimensional T_2 space $X = \langle \kappa, \tau \rangle$ such that $\Delta(X) = \kappa$,

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If $\omega \leq \mu < \sigma < \lambda$, $c(X) \leq \sigma$, and $\{D_{\alpha} : \alpha < \mu\}$ and $\{E_{\beta} : \beta < \lambda\}$ are partitions into hereditarily irresolvable sets, then there is a partition $\{F_{\gamma} : \gamma < \sigma\}$ into hereditarily irresolvable sets.

Soukup, L (Rényi Institute)

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Resolvable spaces

If $X = \bigcup \{X_i : i < n\}$, $n < \omega$, and X_i are **hereditarily irresolvable** subspaces (**=every crowded subspace is irresolvable**) then X is not n + 1-resolvable.

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Theorem

If X is not n-resolvable for some $n < \omega$ then there is a hereditarily irresolvable open subspace U in X.

Corollary 4.

For each $\omega \leq \lambda = cf(\lambda) \leq \kappa$ there is a 0-dimensional T_2 space $X = \langle \kappa, \tau \rangle$, s.t. X is c.c.c., $\Delta(X) = \kappa$, X is not λ -resolvable, but hereditarily μ -resolvable for each $\mu < \lambda$.

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If X is k-resolvable for each $k < \omega$ then X is ω -resolvable.

Theorem (Bashkara-Rao)

If $cf(\lambda) = \omega$ and X is μ -resolvable for each $\mu < \lambda$ then X is λ -resolvable.

Problem

What happens if $\omega < cf(\lambda) < \lambda$?

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What happens if $\omega < cf(\lambda) < \lambda$?

If $\hat{c}(X) \leq cf(\lambda) < \lambda \leq \Delta(X)$, and

(*) for each dense subspace Y if $\Delta(Y) \ge \lambda$ then Y is μ -resolvable for each $\mu < \lambda$,

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If $\hat{c}(X) \leq cf(\lambda) < \lambda \leq \Delta(X)$, and

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Image: A matrix and a matrix

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Problem (Comfort – Hu)

X is countable and maximally resolvable $\stackrel{?}{\Longrightarrow}$ extraresolvable?

M. Hrušak: $i = c \Longrightarrow NO$

Corollary 5

For each $\kappa \ge \omega$ there is 0-dim. T_2 , ccc $X = \langle \kappa, \tau \rangle$ s.t. $\Delta(X) = \kappa$, X is hereditarily maximally resolvable, but not extraresolvable.

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Soukup, L (Rényi Institute)

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If κ is an infinite cardinal such that GCH **first fails at** κ then there is a 0-dimensional T₂ space X with $|X| = \Delta(X) = \kappa^+$ such that X is κ -resolvable, extraresolvable but not κ^+ -resolvable.

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For every cardinal κ there is a 0-dimensional T_2 space X with $\Delta(X) = \kappa$ that is **almost** $2^{\Delta(X)}$ -resolvable (so extraresolvable) but not ω_1 -resolvable.

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If $\Delta(X) > s(X)^+$ then X is **maximally resolvable**.

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What if $\Delta(X) = \hat{s}(X)$ is singular?

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$$\mathcal{M}(X) = \{ \langle x, U \rangle \in X \times \tau(X) : x \in U \}.$$

The elements of $\mathcal{M}(X)$ are the **marked open sets.** The space X is **monotonically normal** iff it is T_1 and it admits a **monotone normality operator**, that is a function $H : \mathcal{M}(X) \to \tau(X)$ such that

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- If X is crowded, monotonically normal then
- (a) X is ω -resolvable,
- (b) X is almost $min(2^{\omega}, \omega_2)$ -resolvable.

Problem

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If κ is a **measurable cardinal**, then there is a monotonically normal space *X* with $|X| = \Delta(X) = \kappa$ which is **not** ω_1 -**resolvable**.

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A crowded monotonically normal space X is maximally resolvable provided $|X| < \aleph_{\omega}$.

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The pdf file of my talk can be downloaded from my homepage.

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