**Eralier Set Theory Midterm tests**
**BME TTK**

1. Prove that there do not exist sets $A, B$ such that

$$\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B).$$

2. Let $\langle A_n : n \in \omega \rangle$ be a system of sets. Prove that

$$\bigcup_{k\in\omega} \bigcap_{n>k} A_n \quad \subseteq \quad \bigcap_{k\in\omega} \bigcup_{n>k} A_n.$$

3. Prove that for any system of sets $\langle A_i : i \in I \rangle$ and set $B$ the following holds:

$$\Big(\bigcup_{i\in I} A_i\Big) - B \;=\; \bigcup_{i\in I} \Big(A_i - B\Big).$$

4. For each $n \in \omega$ let $A_n \subseteq \mathbb{R}^2$ be a convex subset of the plane. Suppose $A_n \subseteq A_{n+1}$ holds for all $n \in \omega$. Prove that

$$\bigcup_{n\in\omega} A_n$$

is a convex set. (Reminder: a subset $B \subseteq \mathbb{R}^2$ of the plane is convex iff for all $P, Q \in B$ we have $\overline{PQ} \subseteq B$ where $\overline{PQ}$ is the line segment joining $P$ and $Q$.)

5. Prove that if $A$ is a transitive set, then $\mathcal{P}(A)$ is also a transitive set.

6. Give infinitely many, pairwise non-isomorphic linear orders on the underlying set $\omega$.

7. Let $\mathcal{A} = \langle A, \leq^{\mathcal{A}} \rangle$ partially ordered set that satisfies the conditions of Zorn's lemma and let $\mathcal{B} = \langle B, \leq^{\mathcal{B}} \rangle$ be a substructure of $\mathcal{A}$ (that is, $B \subseteq A$ and for all $x, y \in B$, $x \leq^{\mathcal{B}} y$ iff $x \leq^{\mathcal{A}} y$). Is it true, that $\mathcal{B}$ necessarily satisfies the conditions of Zorn's Lemma?

8. Prove that for any sets $A, B, C$

$$A - (B \cup C) \subseteq \overline{(A \cap B) \cup (A \cap C) \cup (B \cap C)}.$$

9. Prove that for any set $A$ we have $\cup\mathcal{P}(A) = A$.

10. Let $\langle A, \leq \rangle$ be a partially ordered set. Applying Zorn's Lemma (or with any other way) prove that there exists $X \subseteq A$ such that $\langle X, \leq \rangle$ is linearly ordered and

$$(\forall a \in A - X)(\exists x \in X)(a \text{ and } x \text{ are incomparable }).$$

11. Prove that, if $A \sim B$, then $\mathcal{P}(A) \sim \mathcal{P}(B)$.

12. Prove that if $A$ is a set, then the class $\{E : E$ is an equivalence relation on $A\ \}$ is also a set.

13. Let $\emptyset < \alpha$ be an ordinal. Prove that $\alpha$ is a successor ordinal if and only if $\cup\alpha \in \alpha$.

14. If $\langle A, < \rangle$ is an ordered set, then on $A^P$ the lexicographic order $<^{lex}$ is defined as follows: for any $\langle a_0, a_1 \rangle, \langle b_0, b_1 \rangle \in A^P$, $\langle a_0, a_1 \rangle <^{lex} \langle b_0, b_1 \rangle$ iff ($a_0 < b_0$ or $a_0 = b_0$ and $a_1 < b_1$). Prove that if $\langle A, < \rangle$ is a well ordering, then $\langle A^P, <^{lex} \rangle$ is also a well-ordering.

15. Prove that for any sets $A, B, C$ we have

$$(A - (B - (A - B))) \cup C \subseteq (A \cup C).$$

16. For a set $A$ let $T(A) = \{x \subseteq A : x \text{ is a transitive set }\}$. Prove, that if $\cup T(A) = A$, then $A$ is a transitive set.

17. Prove that if $\kappa$ is an infinite ordinal, then there exists a limit ordinal $\alpha$ such that $\kappa \sim \alpha$ and $\kappa \in \alpha$.

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Always give reasons!