You say you've got a real solution
Well, you know
We'd all love to see the plan
You ask me for a contribution
Well, you know
We are doing what we can
(John Lennon)

Preface

The forerunner of this book had a modest beginning in July 1977 at the Discrete Geometry Week (organized by H.S.M. Coxeter) in Oberwolfach, Germany. There, William Moser distributed a list of 14 problems that he called RPDG (Research Problems in Discrete Geometry). The problems had first appeared in a 1963 mimeographed collection of 50 problems proposed by Leo Moser (1921–1970) with the title "Poorly formulated unsolved problems in combinatorial geometry." Five new editions of RPDG appeared between 1977 and 1981, with hundreds of copies mailed to interested geometers; reviews of RPDG appeared in Mathematics Magazine 53 (1980) p. 189; American Mathematical Monthly 87 (1980) p. 236; Zentralblatt für Mathematik Zbl 528.52001 and Mathematical Reviews MR 84c:51003, MR 85h:52002. The 1986 edition of RPDG reported on the solution of several outstanding problems in earlier editions and was prepared with the collaboration of János Pach; the 1993 edition appeared as DIMACS Technical Report 93-32, 131 pp. We had hoped to publish a book soon thereafter. Indeed, Paul Erdős, the great problem proposer and collector, wrote a preface for that book in the expectation that it would soon be published. However, the book-writing project languished until 2000, when Peter Brass joined the project; his hard and careful work was instrumental in bringing the project to a conclusion. The book finally exists.

Many problems had to be left out, for in a subject with an active research community and a tradition of problem proposing it is natural that the number of open problems explodes over time. Our selection of problems is subjective, and many areas, such as art gallery problems, Helly-type questions, stochastic geometry, and problems about convex polytopes, are completely missing. We decided not to delay further, since a published incomplete book is more useful than an unpublished book (which would also be incomplete). Perhaps later in this century we will expand the collection in a second edition and report then that many current problems have been solved. Meanwhile, we invite the readers to submit their comments, corrections, and new problems to the site http://www.math.nyu.edu/~pach/. Whenever it was possible, we tried to give proper credit to the original problem proposers and problem solvers, but we have surely made many mistakes. We apologize for them, and we urge our readers to point out any

II Preface

error of this kind that they may discover.

Our aim all along has been to achieve a collection of research problems in discrete geometry containing a statement of each problem, an account of progress, and an up-to-date bibliography. It was meant to be a resource for everyone, but particularly for students and for young mathematicians, to help them in finding an interesting problem for research. Apart from the important open problems in the field, we have included a large number of less well known but beautiful questions whose solutions may not require deep methods. We wish the reader good luck in finding solutions.

We sincerely thank all those who helped us with encouragement, information, and corrections. These include Boris Aronov, Vojtech Bálint, Imre Bárány, András Bezdek, Károly Bezdek, Károly Böröczky Jr., Helmut Brass, Erik Demaine, Adrian Dumitrescu, Herbert Edelsbrunner, György Elekes, Christian Elsholtz, Gábor Fejes Tóth, Eli Goodman, Ronald Graham, Branko Grünbaum, Heiko Harborth, Martin Henk, Aladár Heppes, Ferran Hurtado, Dan Ismailescu, Gyula Károlyi, Arnfried Kemnitz, Wlodzimierz Kuperberg, Endre Makai, Rados Radoičić, Andrej Raĭgorodskĭi, Imre Z. Ruzsa, Micha Sharir, Alexander Soifer, József Solymosi, Konrad Swanepoel, Gábor Tardos, Csaba D. Tóth, Géza Tóth, Pavel Valtr, Katalin Vesztergombi, Jörg Wills, Chuanming Zong, and two students, Zheng Zhang and Mehrbod Sharifi. We apologize to those whose names have inadvertently been left out. We thank Marion Blake, David Kramer, Ina Lindemann, Paula Moser, and Mark Spencer for valuable editorial assistance, and Danielle Spencer for her help in preparing the cover design. We thank the mathematics libraries at the Free University Berlin, the Technical University Braunschweig, the Mathematische Forschungsinstitut Oberwolfach and at Courant Institute, New York University; our work would not have been possible without access to these excellent libraries. We also thank all our friends who obtained literature for us that we could not get ourselves.

This book is dedicated to Gisela and Helmut Brass and to Heiko Harborth (respectively parents and advisor of Peter Brass); to Beryl Moser and Leo Moser (respectively wife and brother of William Moser); to Klára and Zsigmond Pál Pach (parents of János Pach).

City College New York McGill University City College New York, NYU, and Rényi Institute Peter Brass William Moser János Pach