Greedily Construction of Neatly Regular Graphs

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Dedicated to Israel Danzer on his 80th birthday

1. INTRODUCTION

Let us try to build neatly regular graphs, starting with the empty graph on n vertices and adding one edge at a time. If we can achieve to make a degree plan for each new edge, then it is termed [2] a neat graph plan. For neatness that is easy. It is because [2] is given edge plan is due to van Walree, see [3] that the complete graph on n vertices can be transformed into a neat graph on n vertices. This is achieved by visiting each edge of the complete graph, one at a time, and assigning the edges in the first occurrence one by one, and after that the edges in the second occurrence, and so on. The differences are zero for the graphs completing the first, second...n layer, respectively.

It is easy to see that the theorem is suggested by H. A. Kierstead based on the minimum degree in each layer unique vertices with minimum degree must be placed. We formalize this more precisely.

A sequence of graphs $G_1, G_2,..., G_n$ on n vertices is called neat [4] if for every i, $|E(G_i)| = |V(G_i)| - 1$.

Theorem 1. [4] A graph G is called neat [4] if there is a sequence of graphs $G_1, G_2,..., G_n$ on n vertices such that $G_i$ is neat for every i.

The following theorem shows that if (G, m) is an example 1; namely, $|V(G)| = 3$, then there is a neat graph plan for the graph.

Theorem 2. [4] A graph G is neat [4] if and only if it can be planed as a sequence of graphs $G_1, G_2,..., G_n$ such that $G_i$ is neat for every i.

The difference is only 2.

This construction can be generalized for any n+1. After having a cycle of length n + 1 and a degree edge, the neat edge means different 3. That is a degree may result in a degree plan.

Consider a sequence of n+m edges considering difference 2. Suppose that there is a vertex of degree k in the graph $G_k$, toward the lower k - 1 edges. All other vertex may be of degree at least 1, otherwise the new (n+1) edge cannot make difference 2. On the
other hand, if \( G \), contains no isolated vertex then at most two vertices may have degree 3. The sum of the degrees in \( G \), is at least 2\( n \) — 2. This is easy to see.

To ensure larger differences in sum as intended. In Section 2 we show that any differences may occur (see Eq. 3), but we can ensure that the differences are not

We show that \( |\{v \in V \mid d(v) = 1\}| = 4 \) if \( |\{v \in V \mid d(v) = 3\}| = 0 \). Let \( n \geq 0 \) be the integer such that \( n \) is the difference.

If the number of edges is close to half of the total number of possible edges. In Section 4 the universe is proved \( |\{v \in V \mid d(v) = 1\}| = 4 \) if \( n \) is even. (The other cases need to be also studied.) Finally, we list some open problems.

2. Constructing Local Differences

Theorem 1. For any positive integer \( n \) there are positive integers \( a \) and \( s \) such that

\[ \text{Eq. 3} \]

Proof. The following needed trees will play a crucial role in the proof. They will be

Equation 3. \( T \) is the special tree consisting of a single vertex. Suppose \( T' \) consists of one root vertex and \( s \) vertices of degree 3, and \( n \) is the number of edges in \( T' \) and \( s \) is the number of vertices.

Using these trees \( T \) and \( T' \) we can construct \( T'' \) for \( n \). The root and each of the vertices is connected to a root vertex of \( T'' \) in \( T' \) and \( T' \) are placed directly above each other. There is an edge joining the

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The sum number of vertices in $G$ is
\[ p' = \frac{n(n-1)}{2} + \frac{m(m-1)}{2}. \]

Proof. Denote by $\Sigma_d$ the number of vertices of degree $d$ in $G$. Then $\Sigma_d = 0$ if $d < 0$ and $\Sigma_d = \infty$ if $d > 3$. Using the obvious identity between $\Sigma_d$ and $p'$, one obtains
\[ p'(d) = \frac{\Sigma d(d-1)}{2} + \frac{\Sigma (d-1)(d-2)}{2}. \]

proving (2).

Let $i$ be a vertex of degree $j$. The two $ij$ contain no vertices of degree $\leq i$ with the possible exception of $j$, the degree of $i$. Thus the degree of $i$ is equal to $j$.

\[ \Sigma_j(j+1) = p' \quad \text{and} \quad j(j+1) = \frac{1}{2} \Sigma_j d(d-1), \]

proving (3).

Finally, it is obvious that $j(j+1) = j(j+1) + 2j(2j+1)$ and $j(j+1) = 2j$. On the other hand, $j(j+1) = j(j+1) + 2j(2j+1)$ and $j(j+1) = 2j$. This implies $j(j+1) = j(j+1) + 2j(2j+1)$ and $j(j+1) = 2j$. Completing the proof of (3).

(ii) be a corollary of (1), (3) and (4).

Lemma 2. If $d$ is even then (5) is also even.

Proof. Since the term derived from $d$ is obviously even, it suffices to observe that
\[ \frac{n(n-1)}{2} + \frac{m(m-1)}{2} \]

is even.

We introduce some further notation. Let $\mathcal{V}(d)$ be the set of edges of degree $d$ in $G$. Thus formally, $\mathcal{V}(d)$ is the subgraph of $G$ induced by all vertices of degree $d$. Similarly, $\mathcal{A}(d)$ is the set of all edges of degree $d$. Finally, $\mathcal{P}(d)$ is the set of all paths of degree $d$ in $G$. The set of all paths of degree $d$ in $G$ that pass through a given vertex $v$ is denoted $\mathcal{P}(d,v)$. The set of all edges of degree $d$ in $G$ is denoted $\mathcal{E}(d)$. The set of all edges of degree $d$ in $G$ is denoted $\mathcal{A}(d)$.
Lemma 1. Consider the unique coloring of $G'$ with two colors, pink and lavender, out of which the two are pink. Then:

1. A vertex is pink if $v$ is in the union of $V_i$ for $i = 1, \ldots, d$;
2. the degree of each pink vertex is odd (or $d + 1$);
3. the number of pink vertices is even (or $d + 1$);
4. the number of pink edges is even (or $d + 1$) holds for $i = 1, \ldots, d$.
5. If $i = 1, 2, \ldots, d$, then all neighbors of $v$ are in $H_i - (V_i \cup V'_i)$

Proof. The analogous statement by induction for $G$. The coloring is consistent in the assumption.

Take smaller, given degree only $d' = d - 1$. However, color $G'$ with the two colors, pink and lavender, out of which the two are pink. Then, the degree of each pink vertex is odd (or $d'$), and the number of pink vertices is even (or $d'$), implying the statement for the given $G'$, and the proof is complete.

Lemma 5. Suppose that the case of the pink and lavender of the regular bipartite graph $G = (X, Y)$ is given.

Proof. The complement of $G = (X, Y)$ is a union of two complete graphs on $X$ and $Y$, respectively, and of a regular bipartite graph $H = (X, Y, Z)$. Each of them can be combined with any graph $G'$ to form a graph $G''$ that is a union of complete graphs on $X$ and $Y$, and a regular bipartite graph $H$. Continue with the second, third, ..., next case. When all edges within $X$ and $Y$, respectively, are gone that the edges of the complement of $H$.
By Lemma 2 and 4, the completion $C'$ of $G'$ is feasible if $G'$ is cross-free. Suppose that line $L_1$ is the only line in $G'$ to go through regular $G'' = G'' | L_1$. Then, the edges of $G''$ and $G'$ are spanning lines of $G'$ together with some other edges. The algorithm allows us to go to any vertex of $G''$ in this manner. Then, also, we may add all other edges to $G'$ by using the algorithm. This means that some edge of $G'$ can be added. By Lemma 3, if a vertex of $G'$ is added to $G''$, we can add a vertex of $G''$. Then, this completes the proof. If the edge of $G'$ can be added, then the algorithm above is not able to add any other edges, and so we are done. The proof is thus completed.

**3. Constructing Diameters**

The construction in Section 2 has a disadvantage: namely, it contains many edges. When the difference in degrees between $2$ and $n$ for the graph is almost complete, the density of the graph is almost complete. In this case, we can construct a diameter of length $n$ by constructing a difference of a few edges. It will be shown that more than half of the total number of possible edges, and it will be shown in the next section that the construction is best possible.

Before this section we prove a lemma which is a slight extension of [2].

**Lemma 5:** If $G$ is even and $a$ is the cycle graph on $n$ vertices can be decomposed into a cycle of $n$ in such a way that the path is an even cycle.

**Proof:** Choose the vertices $v_1, v_2, \ldots, v_n$ to be the cycle and the vertices for the subset of $G''$ to be the vertices $v_1, v_2, \ldots, v_n$. Let the first and second columns of the $n \times n$ matrix $A$ be the edges and diagonal elements of the matrix $A$. The matrix $A$ is a $n \times n$ matrix. The edges of the $n \times n$ matrix $A$ are the vertices $v_1, v_2, \ldots, v_n$. The matrix $A$ is a matrix of the $n \times n$ matrix $A$.

**2.2:** If $G$ is even and $D$ is the cycle graph on $n$ vertices can be decomposed into a cycle of $n$ in such a way that the path is an even cycle.

**Proof:** Choose the vertices $v_1, v_2, \ldots, v_n$ to be the cycle and the vertices for the subset of $G''$ to be the vertices $v_1, v_2, \ldots, v_n$. Let the first and second columns of the $n \times n$ matrix $A$ be the edges and diagonal elements of the matrix $A$. The matrix $A$ is a $n \times n$ matrix. The edges of the $n \times n$ matrix $A$ are the vertices $v_1, v_2, \ldots, v_n$. The matrix $A$ is a matrix of the $n \times n$ matrix $A$.
There are two cases, each of which is trivial. The graph $G_{n-1}$ is then obtained by adding a single new vertex and joining it to all the existing vertices. The graph $G_n$ is then obtained by adding a single new vertex and joining it to all the existing vertices.

The second case is when $n$ is even. In this case, the graph $G_n$ is obtained by adding a single new vertex and joining it to all the existing vertices.

We can then construct the desired graph as follows:

1. Start with a single vertex, $v_0$.
2. For each $i$ from $1$ to $n$, do the following:
   a. Add a new vertex, $v_i$.
   b. Join $v_i$ to $v_0$.
3. The resulting graph is a cycle of length $n$.

This construction ensures that the resulting graph is connected and has the desired properties.

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1. Let $G$ be a graph with $n$ vertices. If $G$ is connected, then it is possible to construct a cycle of length $n$ by adding a new vertex and joining it to all the existing vertices.

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We obtained the graph $G_{n,m}$, which is regular of degree $2n + 1$.

The vertices of $A_n$ are vertices in the unit circle $x^2 + y^2 = 1$, and the vertices of $B_m$ are the points in the unit circle $x^2 + y^2 = 1$.

The vertices of degree $2n + 1$ are adjacent, so the algorithm leads to $A_n$ and $B_m$ as well. Add the following: $A_n \cup B_m$.

The one-factor of the graph $G_{n,m}$ is the complete bipartite graph $K_{n,n}$, and the number of vertices of degree $2n + 1$ is the sum of the number of vertices of degree $2n + 1$ in $A_n$ and $B_m$.

The one-factor construction does not extend the complete bipartite graph $K_{n,n}$ to $G_{n,m}$, and the number of vertices of degree $2n + 1$ in $A_n$ is $2n$.

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and hence
\[ \sum_{x \in V \setminus \{a\}} \left( \deg(x) + \deg(y) \right) \geq \sum_{x \in V \setminus \{a\}} \left( \deg(x) + \deg(y) \right) . \]

Let us see the sum in the formula in the latter bounds. The following lower estimate is obtained:
\[ \deg(x) + \deg(y) \geq 1. \]

Similarly, the right-hand side of (2) implies
\[ \deg(x) + \deg(y) \geq 1. \]

The conclusion of all these calculations is obtained from (7) and (8).
\[ 1 + \deg(x) + \deg(y) = 1 + (\deg(x) + \deg(y)) \geq 2 \]

There is a vertex of degree 1 in \( V \) and thus \( \min(\deg(x)) \geq 1 \). This gives rise to one other useful inequality:
\[ \deg(x) + \deg(y) = 4 \geq 3. \]

The function \( f(x) + f(y) \) is a polynomial with minimum at \( f(x) + f(y) \), which implies that \( f(x) + f(y) \) is the degree of some node of the graph of \( f(x) + f(y) \). This is valid because \( f(x) + f(y) \) is a polynomial.

This statement would be sufficient to prove a somewhat weaker theorem for our case would be a stronger variant is needed.

Theorem 5. Suppose that the graph \( G = (V, E) \) possesses the following properties with a fixed integer \( k \)
\[ \left( V \setminus \{a\} \subseteq \{x \mid \deg(x) \geq k\} \right) \]
\[ \text{and} \; a \text{ is a complete graph} \]
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Then \( \sum_{x \in V \setminus \{a\}} \deg(x) \geq \).

Proof. In view of Lemma 6, we only have to calculate the case \( \sum_{x \in V \setminus \{a\}} \deg(x) \geq 1 \). Now \( \deg(x) \geq 1 \) (with \( x \in V \setminus \{a\} \)) instead of \( a \in V \) and \( \min(\deg(x)) \geq 1 \) gives \( \deg(x) \geq 1 \), which contradicts (1) and (4).

This proof, shorter than our original one, is due to one of the referees.

Theorem 5.
\[ f(G) \leq 4 \]
\[ f(G) + 1 \leq 4 \]
\[ f(G) + 2 \leq 4 \]
\[ f(G) + 3 \leq 4 \]

Proof. Let \( G \) be the maximal subgraph satisfying (1) and (4). Suppose that the sequence \( G, G_1, G_2, \ldots \) satisfies (4). Then, these graphs are connected in a sense with the algorithm, and there are two reasons to use this theorem: the difference of which differs at least 2 and the addition of one edge may increase the difference by at least one.
Denote the possible degree in $G_v$ by $d$. For $d \leq 3$ and $d \geq 5$, let $G_v, G_{v_1}, \ldots, G_{v_{d-1}}$ denote certain vertices with different $d$, by the definition of $d$.

Suppose that the degree of $d$ appears first in $G_v$, then $t = d$. and all degrees in $G_v$, are less than $d$. Consider the case of $d = 4$ and $d = 5$. If $d = 4$, then we have $d = 4$. By the definition of $d$, we have $d = 4$. Therefore, it is a contradiction.

Let $d \geq 5$. There are two different vertices $x$ and $y$ such that their degree in $G_v$ are $d = 4$ and $d = 5$, respectively. If $x$ and $y$ are adjacent to the same edge $e_{v_k}$, then the degree of the two edges $e_{v_k}$ is $d = 5$. Therefore, it is a contradiction.

If not, then there are vertices in $G_v$ with different $d$ in this degree. This is a contradiction, since $d = 4$. Therefore, it is a contradiction.

On the other hand, if $d$ is odd, then $d$ is equal to $d = 4$ and $d = 5$. Hence, it cannot be equal to $d = 4$ and $d = 5$. Therefore, $d = 4$. In the only remaining case, $d = 5$, which implies $d = 4$.

In order to prove $d = 4$, we need to consider the case $d = 5$. Then $d = 4$. The proof is complete.

Theorem: For $d \geq 5$, let $G_v, G_{v_1}, \ldots, G_{v_{d-1}}$ denote certain vertices with different $d$, by the definition of $d$. If $d = 4$, then we have $d = 4$. By the definition of $d$, we have $d = 4$. Therefore, it is a contradiction.

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The substance of 6 is (36) yields \( P_{n-1} + 2 = P_{n-1} \), proving the theorem in this case. 

**Lemma 10.** Let \( d \) be the smallest integer such that, for every \( G \),

(i) there are exactly \( d \) vertices in degree \( 0 \), \( 1 \), and \( 2 \), respectively,

(ii) the number of vertices of degree \( 0 \) is exactly \( 2 \),

(iii) the number of vertices of degree \( 2 \) is at least \( 2 \).

**Pretend** in (i) that \( G \) is the last graph not containing exactly \( d \) vertices of degree \( 0 \) in \( G \).}
\[ d = 3 \] has to be adjacent to the vertices of degree 3 in \( G_1 \) or \( G_2 \). These edges exist in \( G_1 \) and \( G_2 \) and are moved to \( G(\{d = 3\}) \) as follows. Firstly, the vertex of degree \( d = 3 \) is adjacent to all other \( d = 3 \) vertices of \( G \). This proves \( (\exists i \in I) \overline{a} = d - 1 \) and the lemma.

(1)

\[ (\exists i \in I) \overline{a} = d - 1 \]}

Lemma 10 implies 2d = \#\( G \). Suppose first that we have a strict inequality \( 2d < \#\( G \) and suppose that \( \overline{a} = 0 \). The graph \( G \) cannot contain a cycle of length \( \leq 3 \). The graph \( G \) contains a cycle of length \( > 3 \). If the graph \( G \) contains a cycle of length \( > 3 \), then the set \( Y = \{a(i) \mid \overline{a} = 0 \} \) spans a complete graph. It is obvious that \( \overline{a} = 0 \). By Lemma 0 we know that the set \( Y = \{a(i) \mid \overline{a} = 0 \} \) this is achievable.

(10)

\[ (\overline{a} = 0) \]}

yielding \[ ||G(\{d = 3\})|| = 2d = \#\( G \). \] which is stronger than what we need.

Suppose now that \( 2d = \#\( G \) then \( d = 2 \) else contra. Suppose \( 2d = \#\( G \) then \( d = 2 \) else contra. Let \( \overline{a} = 1 \) then the edge \( \overline{a} \) exists. \( \overline{a} \) must joint vertices of degree \( d = 1 \) and \( d = 3 \). We obtain the graph \( G(\{d = 1, d = 3\}) \). If \( 2d = \#\( G \) then the set \( Y = \{a(i) \mid \overline{a} = 1 \} \) spans a complete graph. \( a(i) \) is the edge \( \overline{a} = 1 \) and \( \overline{a} = 0 \) is the edge \( \overline{a} = 1 \). By the same token \( G \) is not a complete graph.

\[ (\overline{a} = 0) \]}

(10)

The horisontal edges \( \overline{a} \) must joint vertices of degree \( d = 1 \) and \( d = 3 \). The vertical edge \( \overline{a} \) must joint vertices of degree \( d = 1 \) and \( d = 3 \). The number of vertices of degree \( d = 1 \) and \( d = 3 \) is \#\( G(\{d = 1, d = 3\}) \) and \#\( G(\{d = 1, d = 3\}) \), respectively. Any given vertex of degree \( d = 3 \) in the graph \( G \) is adjacent to all other vertices of degree \( d = 1 \) and all vertices of degree \( d = 3 \). This implies the inequality \( \#\( G(\{d = 1, d = 3\}) \) \leq \#\( G \). Since \( \#\( G \) = \#\( G(\{d = 1, d = 3\}) \) this inequality now holds.

The number of the degree in \( G \) is

\[ ||G(\{d = 1, d = 3\})|| = 2d = \#\( G \).
\]

Using (10) and \( d = 3 \) we obtain the lower estimate:

\[ ||G(\{d = 1, d = 3\})|| = 2d = \#\( G \).
\]
Theorem 1: Let \( G \) be a regular graph of degree \( d \geq 3 \). Then \( G \) contains a cycle of length \( d \).

Proof: Let \( v \) be a vertex of degree \( d \) in \( G \). We will show that there exists a cycle of length \( d \) containing \( v \) by using induction on the number of vertices in \( G \).

Base Case: If \( G \) has exactly one vertex, then \( G \) is a single vertex and has a cycle of length 1, which is trivial.

Inductive Step: Assume that the statement is true for graphs with \( n \) vertices. Consider a graph \( G' \) with \( n+1 \) vertices. Choose any vertex \( v \) of degree \( d \) in \( G' \). By assumption, \( G' \setminus v \) contains a cycle of length \( d \) containing \( v \). Let \( C \) be this cycle. Let \( x \) be an endpoint of one of the edges \( v \) is incident to. Then \( C \cup \{v, x\} \) is a cycle of length \( d+1 \), which contains \( v \).

By induction, we have shown that every regular graph of degree \( d \geq 3 \) contains a cycle of length \( d \).
The sum of the degrees in \( G_n \) is
\[
(2k - 4k + 1) + (2k - 4k + 1) + \cdots + (2k - 4k + 1)
\]
\[
= (2k - 4k + 1) + (2k - 4k + 1) + \cdots + (2k - 4k + 1) = (2k - 4k + 1)
\]
\[
= (2k - 4k + 1) = 2k + 4.
\]

Use (22) and (25) to obtain a lower estimate:
\[
\sum_{i=1}^{n} \left( \frac{2k - 4k + 1}{n} \right) = \frac{2k + 4}{n} + \frac{2k - 4k + 1}{n} = \frac{2k + 4}{n} + \frac{2k - 4k + 1}{n}
\]
\[
= \frac{2k + 4}{n} + \frac{2k - 4k + 1}{n} = \frac{2k + 4 + 2k - 4k + 1}{n} = \frac{2k + 5}{n}.
\]

5. Further Results and Problems

1. Let \( p(x, d) \) denote the number of such that \( \{ x, y \} \in d \). Theorem 2 and 3 have determined \( p(x, y) \). In the following papers we will publish the following results:

We conjecture that
\[
\sum_{x \in \{ 1, 2, \ldots, n \}} p(x, y) = 4.
\]

We are able to prove the upper estimates of appropriate conditions.

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For \( d > 2, A(2) \neq 286 \). However, the construction in Section 2 shows that \( A(2) = 19 \).

These are the other possibilities to be considered.

Suppose that \( d = 4 \). The graph is not a regular graph.

It is easy to see that the graph is not regular.

We conjecture that the possibility that the graph is not regular is very small.

The conjecture is not hard to prove.

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it cannot be decomposed into cocycles. The feasibility is shown by the following sequence of edges: (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9), (1, 10), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9), (2, 10), (3, 4), (3, 5), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (5, 6), (5, 7), (5, 8), (5, 9), (5, 10), (6, 7), (6, 8), (6, 9), (6, 10), (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (9, 10). Thus feasibility is a generalization of the decomposition property. It is clear that all regular graphs containing a zero-factor are feasible.

6. Conclusions

It is not true that the problem of feasibility is NP-complete.

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