Final Exam

1. (a) What is the norm of a gaussian integer? **(2 points)**

   (b) Prove that any gaussian integer divides its norm (among the gaussian integers). **(4 points)**

2. (a) State Chebyshev's theorem about the number of primes up to a certain positive $x \geq 2$. **(2 points)**

   (b) Assume $p > q > 0$ are prime numbers such that $p + q$ and $p - q$ are also prime numbers. Give all the possibilities for the pair p, q. **(4 points)**

3. (a) State the Euler-Fermat theorem. **(2 points)**
   (b) Prove that there exist integers $100 < k < n$ such that $2^n - 2^k$ is divisible by 2017. **(4 points)**

4. (a) Which integers can be represented as the sum of four squares? **(2 points)**

   (b) Prove that if a gaussian integer can be represented as the sum of some gaussian squares (squares of gaussian integers), then it can be represented as the sum of eight gaussian squares. **(4 points)**