

Inverse group theory

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Abstract

Inverse group theory considers questions of the following form: Let \mathcal{F} be a group-theoretic construction which produces a group $\mathcal{F}(G)$ from any given group G . The inverse problem is: Given a group G , is there a group H such that $G = \mathcal{F}(H)$? The best studied version is where $\mathcal{F}(G)$ is the derived group of G , in which case a solution to the inverse problem is called an *integral* of G .

Although some basic questions remains unanswered – for example, we do not know whether the problem “Given a finite group G , is it integrable?” is decidable – quite a lot is known, for example about finite and profinite groups, abelian and nilpotent groups, and varieties of groups.

A beautiful theorem of Eick gives a precise answer to the inverse Frattini problem for finite groups. Apart from this, not too much is known about other inverse problems; I will summarise some of the results which are known, concentrating mostly on finite groups.