Extremal problems for geometric hypergraphs

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Abstract

In this talk, we discuss a variety of problems and results for ordered, geometric and convex geometric hypergraphs. We give a short proof of the following version of the Erdős-Ko-Rado Theorem for geometric hypergraphs: the maximum size of a family of triangles from a set P_n of n points in the plane, no three collinear and not containing two triangles which geometrically share at least one point, is

$$\triangle(n) := \begin{cases} \dfrac{n(n-1)(n+1)}{24} & \text{if } n \geq 3 \text{ is odd} \\ \dfrac{n(n-2)(n+2)}{24} & \text{if } n \geq 4 \text{ is even.} \end{cases}$$

For convex geometric hypergraphs – here the vertex set is the set of vertices of a regular n-gon – $\triangle(n)$ is roughly the number of triangles containing the centroid of the n-gon. In this setting, we determine the extremal function for six of the eight possible pairs of triangles in convex position, and give bounds on the remaining two. These results extend earlier theorems of Aronov, Dujmović, Morin, Ooms and da Silveira and answer some problems posed by Frankl and Kupavskii. We discuss some extensions to r-uniform hypergraphs.

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