11/12th Emléktábla Workshop

Combinatorics and geometry / Matroid optimization 07.04 - 07.08.2022.

Preliminary Schedule for 12^{th} group:

Monday

Arrival in the hotel from 14:00.
15:00-16:30 Short introduction/presentation of problems - Part I
16:30-18:00 Discussions
18:00 Dinner
19:30 Short introduction/presentation of problems - Part II (if needed)

Tuesday-Thursday

9:30-18:00 Working in groups of 3-6
12:30 Lunch (except Wednesday)
18:00 Dinner
19:30 Presentations of daily progress

Friday

Check-out at 10:00.

List of Participants

Combinatorics and geometry

Eyal Ackerman, University of Haifa at Oranim
Péter Ágoston, ELTE
Gergely Ambrus, Rényi Institute
Martin Balko, Charles Univerity, Prague
János Barát, University of Pannónia
Zoltán Blázsik, Rényi Institute
Gábor Damásdi, ELTE
Nóra Frankl, Rényi Institute
Rado Fulek, UCSD
Dániel Gerbner, Rényi Institute
Eric Gottlieb, Rhodes College, Memphis
Anna Halfpap, University of Montana
Attila Jung, ELTE
Balázs Keszegh, Rényi Institute
Nathan Lemons, Los Alamos
Viola Mészáros, Berlin
Dániel Nagy, Rényi Institute
Zoltán Lóránt Nagy, ELTE
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Russ Woodroofe, University of Primorska

Matroid optimization

Erika Bérczi-Kovács, ELTE
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Lydia Mirabel Mendoza Cadena, ELTE-MTA
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András Sebő, Grenoble
Eszter Szabó, ELTE
Lilla Tóthmérész, ELTE
Kitti Varga, Rényi Institute
Yutaro Yamaguchi, Osaka University
Yu Yokoi, NII
Ran Ziv, Tel-Hai College

Covering the circuits of block matroids by paths

by Kristóf Bérczi and Tamás Schwarcz

A matroid M is **strongly base orderable** if for any two bases A, B there exists a bijection $\varphi : A \to B$ such that $A - X + \varphi(X)$ is a basis for every $X \subseteq A$; note that this implies $B - \varphi(X) + X$ being a basis as well. Strongly base orderable matroids generalizes fundamental matroid classes that appear in combinatorial optimization problems, such as gammoids (and so partition, laminar, and transversal matroids). However, they do not include paving or graphic matroids, so the results for them are not applicable, for example, to graph theoretic problems.

A possible interpretation of strongly base orderability is as follows: for any pair A, B of disjoint bases of a strongly base orderable matroid M, there exists a graph G consisting of a matching between the elements of A and B such that G covers every circuit of M that lie in $A \cup B$. Here **covering** means that every circuit of M spans at least one edge of G. As a relaxation of this property, we conjecture that an analogous statement holds for arbitrary matroids where G is a path instead of a matching.

Conjecture 1 (Bérczi, Schwarcz)**.** *For disjoint bases A, B of a matroid M, there exists a graph G consisting of an alternating path between A and B such that G covers every circuit of M that lie in $A \cup B$.*

Note that the elements of A and B have to appear alternatingly along the path. One might wonder: why a path is considered instead of a cycle? The path version seems to be stronger, but the two variants are in fact equivalent. Indeed, a path can simply be closed to get a cycle. To see the reverse implication, take the direct sum of the matroid with a uniform matroid of rank 1 on the set $\{a, b\}$, that is, $A' := A + a$ and $B' := B + b$ are bases of the new matroid. Now arrange the elements of A' and B' around a cycle such that the elements of A' and B' receive odd and even numbers, respectively, and every circuit of the matroid contains two consecutive elements. As a and b are parallel in the extended matroid, they must follow each other in the cyclic ordering. Hence the path obtained after breaking up the cycle by deleting a and b satisfies the requirements of the conjecture.

A weaker version of the conjecture is also of interest. A matroid M is **base orderable** if for any two bases A, B there exists a bijection $\varphi : A \to B$ such that $A - a + \varphi(a)$ and $B + a - \varphi(a)$ are bases for every $a \in A$. In terms of covering circuits, this property is equivalent to the existence of a graph G consisting of a matching between the elements of A and B such that G covers the fundamental circuits of the elements of A with respect to B, and the fundamental circuits of the elements of B with respect to A. A relaxation of this property would be the following.

Conjecture 2 (Bérczi, Schwarcz)**.** *For disjoint bases A, B of a matroid M, there exists a graph G consisting of an alternating path between A and B such that G covers $C(a, B)$ for $a \in A$ and $C(b, A)$ for $b \in B$.*

The conjectures hold for paving matroids, graphic matroids, and matroids on at most 8 elements. However, they are wide open in general.

All-ones vector in a base-polyhedron

by András Frank

Let b be a non-negative, integer-valued, non-decreasing, submodular function on ground-set S for which $b(S) = |S|$ and $b(s) = 2$ for each $s \in S$. Consider the base-polyhedron $B := B(b) := \{x : \widetilde{x}(Z) \leq b(Z)$ for every $Z \subset S$ and $\widetilde{x}(S) = b(S)\}$, where $\widetilde{x}(Z) := \sum[x(s) : s \in Z]$, and suppose that the identically 1 vector $m_1 := (1, 1, \ldots, 1)$ is in B, that is, $|Z| \leq b(Z)$ for every $Z \subseteq S$.

Problem 1. *Does B always have a vertex z for which $m_2 := 2m_1 - z$ is in B? In other words, is m_1 the arthmetic mean of a vertex z of B and another element of B?*

The existence of such a vertex z would provide a simple (iterative) certificate for the property that m_1 is in B. Indeed, since $m_1 = (z + m_2)/2$, for certifying that m_1 belongs to B it is enough to certify that $m_2 \in B$. But m_2 is a $(0, 1, 2)$-valued vector and hence checking whether m_2 is in B is equivalent to the original problem on a smaller ground-set (obtained by deleting those components s where $m_2(s) = 0$ and "contracting" those components s where $m_2(s) = 2$).

The only known certificate for m_1 being a member of B is that we express m_1 as a convex combination of vertices of B. But here the coefficients may be wild. In this light, the conjecture can be viewed as a purely combinatorial certificate for $m_1 \in B$. Such a certificate may give rise to constructing a simple algorithm for deciding whether $m_1 \in B$. The only known algorithm for this problem relies on a general-purpose submodular function minimizer subroutine.

Note that in the special case when $b = 2r$ for a rank-function r of a matroid M with $2r(S) = |S|$, then (by Edmonds and Fulkerson) $m_1 \in B$ if and only if S can be partitioned into two bases of M.

For a certificate or algorithm, it would be enough to prove the conjectere only for its special case when $b(X) > |X|$ holds for every non-empty proper subset X of S.

Balanced submodular flows

by Alpár Jüttner

In balanced optimization problems, the aim is to find a most equitable distribution of resources. Several problems have been analysed in the literature such as the balanced spanning tree problem studied by Camerini [2] and by Longshu Wu [6]. Another example is the balanced assignment problem by Martello[4]. Ahuja proposed a parametric simplex method for the general balanced linear programming problem [1]. Punnen et al. introduced a strongly NP-hard problem, which is called the quadratic balanced optimization problem, and showed some algorithms in a special case. Scutella studied the balanced network flow problem and presented a strongly polynomial algorithm to solve it [5],[3]. We would like to focus on extending these results to submodular flows, and also on finding min-max characterizations of the optimal solutions.

References

[1] R. K. Ahuja. The balanced linear programming problem. *European Journal of Operational Research*, 101(1):29–38, 1997.

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[4] S. Martello, W. Pulleyblank, P. Toth, and D. de Werra. Balanced optimization problems. *Operations Research Letters*, 3(5):275–278, 1984.

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Optimal Pricing for Two Matroids

by Naonori Kakimura

Conjecture 1 (Dütting–Végh (personal communication 2017))**.** *Let $M_1 = (S, \mathcal{B}_1)$ and $M_2 = (S, \mathcal{B}_2)$ be matroids with a common ground set S such that there exist disjoint bases $B_1 \in \mathcal{B}_1$ and $B_2 \in \mathcal{B}_2$ with $B_1 \cup B_2 = S$. Then, there exists a function $p : S \to \mathbb{R}$ (called a* price vector*) satisfying the following conditions.*

1. *If B_1 is a minimum-cost base in $\mathcal{B}_1$ with respect to p, then $S \setminus B_1 \in \mathcal{B}_2$.*

2. *If B_2 is a minimum-cost base in $\mathcal{B}_2$ with respect to p, then $S \setminus B_2 \in \mathcal{B}_1$.*

The conjecture arises in the context of pricing items in combinatorial markets [1]. Suppose that there are two buyers and each buyer $i \in \{1, 2\}$ wants to buy a set of items that forms a basis in $\mathcal{B}_i$. If buyer i comes to a shop first, then she chooses a cheapest set B_i in $\mathcal{B}_i$ with an arbitrary tie-breaking rule. The requirements mean that, regardless of the choice of B_i, the remaining set $S \setminus B_i$ is an item set the other buyer wants. Thus, whoever comes first, both of the buyers can get desired item sets.

The conjecture is known to be true if matroids are either partition matroids or strongly-base orderable matroids, and, in these cases, a desired price vector can be found in polynomial time [2].

References

[1] Vincent Cohen-Addad, Alon Eden, Michal Feldman, and Amos Fiat, The invisible hand of dynamic market pricing, *Proceedings of the 2016 ACM Conference on Economics and Computation*, 383–400, 2016.

[2] Kristóf Bérczi, Naonori Kakimura, Yusuke Kobayashi, Market Pricing for Matroid Rank Valuations, *SIAM J. Discret. Math.*, 35(4), 2662–2678, 2021.

Matroid basis with bounded intersections

by Tamás Király

Let $M = (V, r)$ be a matroid, let $H = (V, \mathcal{E})$ be a hypergraph of maximum degree Δ, and let $f_e \leq g_e$ ($e \in \mathcal{E}$) be lower and upper bounds on the hyperedges of H. We consider the following linear program (LP) for a cost vector $c \in \mathbb{R}^V_+$:

$$\begin{aligned} \min \ & \sum_{v\in} c_v x_v & \\ \text{subject to } & \sum_{v\in V} x_v = r(V) & \\ & \sum_{v\in U} x_v \leq r(U) & \forall U \subseteq V \\ & 0 \leq x_v \leq 1 & \forall v \in V \\ & f_e \leq \sum_{v\in E} x_v \leq g_e & \forall e \in \mathcal{E}. \end{aligned}$$

Clearly, integer feasible solutions are characteristic vectors of bases B that satisfy the additional constraints $f_e \leq \sum_{v\in E} |B \cap e| \leq g_e$ for every $e \in \mathcal{E}$.

Problem 1. *Is it true that if (LP) has a feasible solution, then there is a basis B such that $f_e - \Delta + 1 \leq \sum_{v\in E} |B \cap e| \leq g_e + \Delta - 1$ for every $e \in \mathcal{E}$?*

Problem 2. *Let OPT denote the optimum value of (LP). Is it true that there is a basis B such that $\sum_{v\in B} c_v \leq OPT$ and $f_e - \Delta + 1 \leq \sum_{v\in E} |B \cap e| \leq g_e + \Delta - 1$ for every $e \in \mathcal{E}$?*

By a result of Singh and Lau [1], the answer to the second question is positive for graphic matroids. We also know that the answer is positive if only upper bounds (or only lower bounds) are present [2]. The following may be an easier question: is the statement true if each hyperedge has either a lower bound or an upper bound, but not both?

References

[1] M. Singh, L.C. Lau, Approximating minimum bounded degree spanning trees to within one of optimal, Journal of the ACM 62 (2015), 1–19

[2] T. Király, L.C. Lau, M. Singh, Degree bounded matroids and submodular flows, Combinatorica 32 (2012), 703–720

Matroid Intersection Reconfiguration

by Yusuke Kobayashi

Let $M_1 = (S, \mathcal{I}_1)$ and $M_2 = (S, \mathcal{I}_2)$ be matroids with a common ground set S. We say that two common independent sets $I, I' \in \mathcal{I}_1 \cap \mathcal{I}_2$ are *adjacent* if $|I \setminus I'| = |I' \setminus I| = 1$. For $I_{\text{ini}}, I_{\text{tar}} \in \mathcal{I}_1 \cap \mathcal{I}_2$, *a reconfiguration sequence between* I_{ini} *and* I_{tar} is a sequence $\langle I_1, I_2, \ldots, I_\ell \rangle$ such that $I_1 = I_{\text{ini}}$, $I_\ell = I_{\text{tar}}$, $I_i \in \mathcal{I}_1 \cap \mathcal{I}_2$ for $i = 1, \ldots, \ell$, and I_i and I_{i+1} are adjacent for $i = 1, \ldots, \ell - 1$.

Given matroids M_1, M_2, and their common independent sets $I_{\text{ini}}, I_{\text{tar}} \in \mathcal{I}_1 \cap \mathcal{I}_2$, Matroid Intersection Reconfiguration asks whether there exists a reconfiguration sequence between I_{ini} and I_{tar}.

Problem 1. *Is* Matroid Intersection Reconfiguration *solvable in polynomial time?*

A few special cases can be solved in polynomial time: Reconfiguration of (bipartite) matchings [1] and Reconfiguration of directed trees [2].

References

[1] T. Ito, E.D. Demaine, N.J.A. Harvey, C.H. Papadimitriou, M. Sideri, R. Uehara, Y. Uno, On the complexity of reconfiguration problems, *Theoretical Computer Science*, 412, 1054–1065, 2011.

[2] T. Ito, Y. Iwamasa, Y. Kobayashi, Y. Nakahata, Y. Otachi, K. Wasa, Reconfiguring directed trees in a digraph, *Proceedings of COCOON 2021*, 343–354, 2021.

Matroid Intersection

by Dani Kotlar and Ran Ziv

Let M_1 and M_2 be two matroids defined on the same ground set S.

Problem 1. *Assuming in each of the two matroids S can be partitioned into three bases, can S be partitioned into two subsets S_1 and S_2 such that each S_i ($i = 1, 2$) spans S in both M_1 and M_2? Some insight can be gained by formulating an equivalent question on the duals of M_1 and M_2.*

We call a set $A \subseteq S$ a *matroidal matching* if $A \in M_1 \cap M_2$. A partition of S into two subsets is called *equitable* if the subsets' sizes differ by at most 1.

Problem 2. *Assuming S has a partition into two matroidal matchings, does S have a partition into two equitable matroidal matchings?*

Hatami-Shor for matroids

by Dani Kotlar and Ran Ziv

A partial transversal of size k in a Latin square is a set of k distinct entries no two of which lie in the same row or in the same column. A known conjecture of Ryser and Brualdi states that any odd order Latin square contains a partial transversal of size n (in which case the transversal is not partial) and any even order Latin square contains a partial transversal of size $n-1$.

The best result towards proving this conjecture is due to Hatami and Shor [2]:

Theorem 1. *Every Latin square has a partial transversal of length at least* $n - 11.053 \log^2 n$

We can generalize the notion of Latin square by taking the entries from a ground set of a matroid instead of the set $\{1, 2, \ldots, n\}$, and requiring the rows and columns to be bases or independent sets. We call this a *matroidal Latin square* (abbrv. MLS) (see [3]) . Note that a Latin square is a matroidal Latin square over a partition matroid. A *partial independent transversal* (abbrv. PIT) is, an independent set of entries no two of which lie in the same row or in the same column. A PIT in an MLS cannot have in general size larger than $n-1$, regardless of whether n is even or odd ([3]). So far we know that in any MLS there is a PIT of size at least $n - \sqrt{n}$. This follows from the more general result in [1] stating that any n pairwise disjoint sets of size n in the intersection of two matroids have a rainbow set of size at least $n - \sqrt{n}$ in that intersection.

Problem 1. *Can the method of proof in [2] be applied to matroidal Latin squares to obtain the bound* $n - O(\log^2 n)$*?*

Using basic properties of circuits, most of the proof in [2] can be adapted to MLSs, but there is one argument that does not seem to work. I would like to find a way to circumvent this argument. Details will be provided.

References

[1] R. Aharoni, D. Kotlar, and R. Ziv. Rainbow sets in the intersection of two matroids. *Journal of Combinatorial Theory, Series B*, 118:129–136, 2016.

[2] P. Hatami and P. W. Shor. A lower bound for the length of a partial transversal in a latin square. *Journal of Combinatorial Theory, Series A*, 115(7):1103–1113, 2008.

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Simultaneous combinatorial problems

by Péter Madarasi

Let $G = (V, E)$ be an undirected graph, and let $E_1, \ldots, E_k \subseteq E$ be such that $E = \bigcup_{i=1}^{k} E_i$. Find a maximum size subset M of the edges such that $M \cap E_i$ is a b_i-matching for all $i = 1, \ldots, k$. This problem is known to be NP-hard even for bipartite graphs when $k = 2$ and $b_i \equiv 1$. However, when sets E_i $(i = 1, \ldots, k)$ restricted to the edges incident to v form a laminar family for each node v, we get back the *Laminar matchoid problem* [1], which is polynomial-time solvable. In fact, the latter case remains tractable even if we have an additional laminar family $\mathcal{L}$ on V and prescribe an upper bound on the degree-sum in each set in $\mathcal{L}$ [2] — which gives back the so-called *Hierarchical b-matching problem* [3] in the special case $k = 0$. To prove this, one can formulate the problem as an integer linear program of the form

$$\max\{cx : x \in \mathbb{Z}^n, d \leq x \leq c, a \leq Mx \leq b\}, \tag{0.1}$$

where $M \in \mathbb{Z}^{m \times n}$ and $\sum_{i=1}^{m} |M_{ij}| \leq 2$ for all j, which is polynomial-time solvable [4]. The following question remains open.

Problem 1. *Is the problem solvable when we have additional parity constraints on the degree-sum in each set in $\mathcal{L}$? More generally, can we solve (0.1) when we pose parity constraints on the coordinates of x?*

Instead of b-matchings, one can require that $M \cap E_i$ has some other structure. For example, the case of trees reduces to the matroid intersection problem when $k = 2$, whereas the case of directed $s - t$ paths is open:

Problem 2. *Let $D = (V, A)$ be a directed graph, and let $A_1, A_2 \subseteq A$ be such that $A = A_1 \cup A_2$. Find a (minimum size) subset P of the arcs such that both $P \cap A_1$ and $P \cap A_2$ are $s - t$ paths.*

References

[1] T. A. Jenkyns, *Matchoids : a Generalization of Matchings and Matroids*, Thesis (Ph.D.)–University of Waterloo (1974).

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Locality Gap of the Spanning Tree Neighborhood for Expanding Search

by Jannik Matuschke

In the Expanding Search problem, a searcher wants to find an item which is hidden at a random node of an undirected graph $G = (V, E)$ according to a known distribution $p \in \Delta(V)$. The searcher starts at a given vertex $s \in V$ and can move to adjacent vertices along the edges of the graph. Traversing an edge $e \in E$ for the first time incurs a cost of $c(e) \in \mathbb{R}_+$, but any additional traversal of the same edge (in any direction) is free—in other words, the searcher can move within the set of previously visited nodes at no cost. The item is found once the searcher reaches the node at which it is located.

The searcher wants to devise a strategy for finding the item at minimum cost. Such a strategy can be encoded by a spanning tree T of G together with a bijection $\sigma : \{1, \ldots, |V| - 1\} \to T$, specifying the order in which the edges are traversed for the first time. The expected cost of the search strategy then is

$$C(T, \sigma) := \sum_{v \in V} \sum_{i=1}^{k_\sigma(v)} p(v) c(\sigma(i)),$$

where $k_\sigma(v) := \min\{k : \{\sigma(1), \ldots, \sigma(k)\} \text{ contains an } s\text{-}v\text{-path}\}$.

It is known that finding an optimal search strategy is NP-hard [2]. However, given a spanning tree T, a bijection σ_T minimizing $C(T, \sigma_T)$ can be computed efficiently via dynamic programming [1]. This suggests the following local-search procedure using the edge-swap neighborhood of the graphic matroid $\mathcal{T}$ of G to find a good strategy for the searcher.

1. Let $T \in \mathcal{T}$.
2. While there is $e \in T$ and $e' \in E \setminus T$ with $T' := T \cup \{e\} \setminus \{e'\} \in \mathcal{T}$ and $C(T', \sigma_{T'}) < C(T, \sigma_T)$, replace T by T'.
3. Return T.

A tree $T \in \mathcal{T}$ is locally optimal if there is no $e \in T$ and $e' \in E \setminus T$ with $T' := T \cup \{e\} \setminus \{e'\} \in \mathcal{T}$ and $C(T', \sigma_{T'}) < C(T, \sigma_T)$. Let $\mathcal{T}'$ denote the set of all locally optimal spanning trees of G. We are interested in bounding the locality gap $\dfrac{\max_{T \in \mathcal{T}'} C(T, \sigma_T)}{\min_{T \in \mathcal{T}} C(T, \sigma_T)}$.

Problem 1. *Can the locality gap of the above local search procedure be bounded by a constant?*

References

[1] S. Alpern and T. Lidbetter. Mining coal or finding terrorists: The expanding search paradigm, *Operations Research* 61:265–279, 2013.

[2] I. Averbakh and J. Pereira. The flowtime network construction problem, *IIE Transactions* 44:681–694, 2012.

Partition into common bases in base orderable matroids

by Tamás Schwarcz and Kristóf Bérczi

A matroid $M = (E, \mathcal{B})$ on ground set E with family of bases $\mathcal{B}$ is called **base orderable** (resp. **strongly base orderable**) if for every two bases $A, B \in \mathcal{B}$ there exists a bijection $\varphi \colon A \to B$ satisfying the following property (BO) ((SBO), respectively):

$$A - x + \varphi(x) \in \mathcal{B} \text{ and } B + x - \varphi(x) \in \mathcal{B} \text{ for every } x \in A, \tag{BO}$$

$$A - X + \varphi(X) \in \mathcal{B} \text{ and } B + X - \varphi(X) \in \mathcal{B} \text{ for every } X \subseteq A. \tag{SBO}$$

The following property of strongly base orderable matroids was shown by Davies and McDiarmid.

Theorem 1 ([1])**.** *Let $M_1 = (E, \mathcal{B}_1)$ and $M_2 = (E, \mathcal{B}_2)$ be strongly base orderable matroids such that E can be partitioned into k disjoint bases of M_i for $i = 1, 2$. Then E can be partitioned into k common bases of M_1 and M_2.*

It is natural to ask how one can relax strongly base orderability in Theorem 1. It is open whether the analogous statement holds for base orderable matroids.

Question 1. *Can we replace strongly base orderability by base orderability in Theorem 1?*

An even weaker assumption on the matroids would be that neither of them has a minor isomorphic to the graphic matroid of K_4. However, an example on 8 elements shows that this condition does not guarantee the existence of k disjoint common bases (not even if $k = 2$ and one of the matroids is a partition matroid).

In particular, we are interested in the special case $k = 2$, when the matroids are **block matroids**, that is, their ground set can be partitioned into two disjoint bases. In this special case, one can slightly relax the strongly base orderability condition in Theorem 1. Let us call a block matroid **locally strongly base orderable** if its ground set can be partitioned into bases B_1 and B_2 such that there exists a bijection $\varphi : B_1 \to B_2$ satisfying (SBO).

Proposition 2. *If $M_1 = (E, \mathcal{B}_1)$ and $M_2 = (E, \mathcal{B}_2)$ are locally strongly base orderable block matroids, then E can be partitioned into two common bases of M_1 and M_2.*

By this observation, an affirmative answer to the following question would provide an affirmative answer to Question 1.

Question 2. *Is every base orderable block matroid locally strongly base orderable?*

References

[1] J. Davies and C. McDiarmid. Disjoint common transversals and exchange structures. *Journal of the London Mathematical Society*, 2(1):55–62, 1976.

Packing and Hitting Rectangles

by András Sebő

The *packing number*, ν, of a given family of sets the maximum number of pairwise disjoint sets in the family, while their *hitting number* is the minimum number, τ, of points meeting all of them (with a non-empty intersection). Obviously, $\tau \geq \nu$. A more than half century old conjecture of Wegner (1965, [2]) asks whether $\tau \leq 2\nu - 1$ for rectangles in the plane. Replacing "2" by any larger constant the validity of the conjecture is also not known.

We studied the simplest special cases of this conjecture with Marco Caoduro [1]: by taking small values of some parameters, or by studying squares. I state here two of the most frustrating questions that we could not answer with Marco. The first concerns axis-parallel rectangles where none of the points is covered more than twice, the second concerns squares.

Problem 1. *Given a set of axis-parallel rectangles such that every point of the plane contains at most two of them, is it true that there are always* $\lfloor \frac{n}{2} \rfloor$ *disjoint ones among them ?*

Trying to prove Wegner's conjecture for the case when each point is covered at most twice, an easy induction shows its equivalence with this statement for sets of axis-parallel rectangles with a factor-critical intersection-graph.

The packing and hitting problems are NP-hard also for axis-parallel unit squares. Wegner's conjecture is easy for them, but for sets of axis-parallel squares of arbitrary size it is not known. There are no better examples for axis-parallel squares than those giving the 3/2 ratio.

For not necessarily axis-parallel unit squares τ can be as large as 3 and is always at most 4. But can it be 4 ? (For unit disks in the plane the exact bound of 3 is known.) The target of the following problem is to understand the difference between maximum clique of the interesection graph and the maximum number of sets containing a given point. An example would improve the lower bound for arbitrary large ν by taking disjoint copies of the example.

Problem 2. *We ask the following questions both for (not necessarily axis-parallel) squares, and unit squares. How large can be the minimum size of a hitting set of pairwise intersecting squares ? If no point is contained in more than two squares what is the maximum number of pairwise intersecting squares ?*

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Integer Caratheodory for Common Bases of Matroids

by András Sebő

The complexity status of covering the common ground-set of two matroids by common independent sets of the two, had been a long-standing open problem until Kristóf [2] has proved it to be NP-hard. This makes the polynomially solvable special cases, or the question of solvability with an error of 1 even more precious. Both of these induce a range of interesting open problems, *I think tied with the "IRUP" (integer round up) and "MIRUP" (modified integer round up) properties respectively.*

A nice summary of the results on packing common independent sets of two matroids, and of the related integer decomposition property – an equivalent reformulation of the above mentiond IRUP, and also implicitly containing MIRUP with the matroid example of Aharoni and Berger [1]) – can be found on the following "Egres Open" pages:

`http://lemon.cs.elte.hu/egres/open/Packing_common_bases`,

`http://lemon.cs.elte.hu/egres/open/Integer_polyhedra` .

We restrict ourselves here to quick definitions sufficient to explain the specific challenge we state: sets are represented by their incident vectors indexed by the ground-set. We abuse terminology by using the same term for a set and its incident vector. In particular, the incidence vector of a matroid basis will also be said to be a matroid basis and the incidence vector of an arborescence (as an edge-set) will also be said to be an arborescence. We denote the common rank of M_1 and M_2 by ρ.

Let us say that the pair (M_1, M_2) of connected matroids on the same ground-set S is *IRUP* if they have at least one common basis, and for any $w : S \to \mathbb{N}$, *if* w/k *with* $k = \sum_{s\in S} w(s)/\rho$ *is in the convex hull of common bases, then* w *is the sum of common bases.*

Does then the "Integer Caratheodory Property" hold" ? Before defining the problem precisely we state it in the special case of arborescences in digraphs rooted in a vertex r, called *r-arobrescences.*

Problem 1. *If w is the sum of r-arborescences, is it also a non-negative integer combination of r-arborescenses ?*

This is one of the simplest open special cases of the *Integer Carathedory problem* in (cf. in Schrijver white or yellow book, in more details in [3], later developments in [6], [4], with pointers to weaker statements involving the union of two sets of linearly independent bases).

Problem 2. *Let M_1 and M_2 be an IRUP pair of matroids. If w is the sum of common bases, is it also the non-negative integer combination of a linearly independent set of common bases ?*

The origins of the problem lie in the $M_1 = M_2$ case raised by Cunningham in the eighties, and which had also been open for a long time, until Gijswijt and Regts [4] proved that indeed, independent sets of matroids have the *Integer Caratheodory Property*, opening new perspectives. I am not sure anybody has considered the problem ever since.

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Maximizing the number of edge-disjoint directed cuts in orientations

by Lilla Tóthmérész

Suppose that $G = (S, T, E)$ is an undirected bipartite graph, and we want to find the orientation that maximizes the number of disjoint directed cuts. Then

Proposition 1. *The orientation in which each edge is oriented towards S maximizes the number of disjoint directed cuts.*

Proof. The maximal number of (undirected) cuts of G is obviously an upper bound. Let $\overrightarrow{G}$ be the orientation on which each edge is oriented towards S. By the result of Frank [1, Theorem 9.6.12], the maximal number of disjoint cuts in G is equal to the maximal number of disjoint dicuts in $\overrightarrow{G}$. Hence the upper bound is attainable, and $\overrightarrow{G}$ is a maximizer. $\square$

Problem 1. *Can we characterize the orientations attaining the maximum?*

Problem 2. *What can be said about non-bipartite graphs?*

We can generalize the problem to non-bipartite graphs in a different way: Let us call an assignment $\ell : V \to \mathbb{Z}$ a good layering for G, if $|\ell(u) - \ell(v)| \leq 1$ for each $uv \in E(G)$ and $\{uv \in E(G) : |\ell(u) - \ell(v)| = 1\}$ is a spanning subgraph.

Then we can define an oriented spanning subgraph $\overrightarrow{G_\ell}$ by $E(\overrightarrow{G_\ell}) = \{\overrightarrow{uv} : \ell(v) = \ell(u) + 1\}$.

For a bipartite graph, the subgraphs corresponding to good layerings are all complete orientations. Also, $\overrightarrow{G}$ corresponds to the layering with $\ell(v) = 1$ for $v \in S$ and $\ell(v) = 0$ for $v \in T$.

Now we can ask:

Problem 3. *Which good layerings ℓ maximize the number of disjoint dicuts for $\overrightarrow{G_\ell}$?*

The motivation behind studying $\overrightarrow{G_\ell}$ for good layerings is that these oriented subgraps correspond to facets of the symmetric edge polytope, which is a polytope associated to G. Problem 3 would give us some information on which facets of this polytope are "small" in a certain sense.

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Tractability/Intractability of Parity Constraints

by Yutaro Yamaugchi

Throughout, let $G = (V, E)$ be a connected undirected graph. A *path* means a simple path, i.e., each vertex can appear at most once. A path is said to be *odd* (or *even*) if the number of traversed edges is odd (or even, resp.).

1 Shortest Odd Path in Conservative Undirected Graphs

Let $\ell \colon E \to \mathbb{R}$ be edge length, and let $s, t \in V$. We say that ℓ is *conservative* if G contains no cycle C with $\ell(C) = \sum_{e \in C} \ell(e) < 0$.

If ℓ is nonnegative, a shortest s–t path can be found in polynomial time by Dijkstra's method as with the directed setting. In addition, as two different applications of the weighted matching problem in general graphs, it is well-known that

- if ℓ is conservative, a shortest s–t path can be found in polynomial time (note that a reduction to the directed setting fails because duplicating a negative edge yields a negative cycle);
- if ℓ is nonnegative, a shortest odd s–t path can be found in polynomial time.

Thus, the following question naturally arises.

Problem 1 (cf. [1, Section 29.11e])**.** *Is there a polynomial-time algorithm for finding a shortest odd s–t path if ℓ is conservative?*

Note that the problem is NP-hard if ℓ is not conservative because it includes the Hamiltonian path problem. It is also known that, in the directed setting, feasibility test is already NP-complete.

2 Lexicographically Smallest Odd Path in Undirected Graphs

Suppose that the vertices in V are indexed by $1, 2, \dots, n = |V|$, and let $s, t \in V$. A path is regarded as a sequence of vertices, and then two paths can be compared in the lexicographic order. By a naive depth first search, one can find a lexicographically smallest s–t path in linear time.

One can design a rather simple $\mathrm{O}(nm)$-time algorithm for finding a lexicographically smallest odd s–t path, where $m = |E|$. The key property is as follows.

Fact 1. *In any 2-connected graph that is not bipartite, for any pair of vertices u and w, there exist both odd and even u–w paths.*

By this fact, all s–t paths have the same parity if and only if all the 2-connected components between s and t (i.e., any s–t path traverses inside) are bipartite. Thus, for fixed t, by checking bipartiteness of each 2-connected component, the possible parities of u–t paths for all $u \in V$ are computed in linear time. Based on this feasibility check, one can greedily proceed from s to the neighbor u of s having the minimum index subject to an even u–t path remains in $G - s$, and recurse by replacing G, s, t with $G - s, u, t$ (also "odd" with "even"). The recursion depth is trivially bounded by n, and the total computational complexity is bounded by $\mathrm{O}(nm)$.

Problem 2. *Is there an $\mathrm{o}(nm)$-time algorithm for finding a lexicographically smallest odd s–t path in undirected graphs?*

3 Exact Parity Matching in Red-Blue Edge-Colored Graphs

Suppose that each edge in E is colored by one of two fixed colors, say red and blue. It is known that there exists a polynomial-time randomized algorithm for finding a perfect matching with exactly k red edges, where k is also included in the input. In contrast, any polynomial-time deterministic algorithm is not known even for the parity constraint case, i.e., for finding a perfect matching with an odd number of red edges in red-blue edge-colored graphs.

Problem 3 (cf. [2])**.** *Is there a polynomial-time deterministic algorithm for finding a perfect matching with an odd number of red edges in red-blue edge-colored graphs?*

If we restricted ourselves to bipartite graphs, the problem can be solved by finding an arbitrary perfect matching in the input graph and, if it is not desired parity, by finding a directed cycle with an odd number of red edges in the residual graph (the latter part is not so trivial but still elementary). This problem is naturally extended to matroid intersection, including other special cases such as arborescences. Note that these are special cases of three matroid intersection.

Problem 4. *Is there a polynomial-time algorithm for finding a common base with an odd number of red elements in red-blue element-colored matroid intersection? What kinds of matroids are tractable? Is there any characterization?*

Note that the main issue is as follows: in the general matroid intersection case, even if one can find an "odd" cycle in the exchange bipartite graph, the cycle itself may not be exchangeable.

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Matroid Generalizations of Some Game Theoretic Problems

by Yu Yokoi

We present some game theoretic problems, which have been (partially) solved for base-orderable matroids but are open for general matroids.

A matroid $(E, \mathcal{I})$ is called *base-orderable* (or *weakly base-orderable*) if, for any two bases B_1, B_2, there exists a bijection $\varphi\colon B_1 \to B_2$ such that, for every $e \in B_1$, both $B_1 - e + \varphi(e)$ and $B_2 + e - \varphi(e)$ are bases. A class of base-orderable matroids includes gammoids, which include laminar matroids and transversal matroids. Base-orderability is known to be minor-closed.

1 Stable matching with ties under matroid constraints

Stable matching is a well-studied combinatorial structure proposed by Gale and Shapley [1]. Its matroid generalization, called *matroid-kernel*, is proposed by Fleiner [2, 3], who showed that a matroid-kernel always exists and can be found efficiently. Another generalization is the model with ties (or indifferences) in the preference lists. In the model with ties, unlike the original model, stable matchings have different cardinalities (when the underlying bipartite graph is incomplete). The problem of finding a maximum stable matching in this setting, called MAX-SMTI, is known to be NP-hard and the current best approximation ratio is 1.5 [4, 5, 6].

We can naturally define a common generalization of the above two. Let $(E, \mathcal{I}_1, \succsim_1)$, $(E, \mathcal{I}_2, \succsim_2)$ be two *weakly ordered matroids* on E. That is, for each $k = 1, 2$, the pair $(E, \mathcal{I}_k)$ is a matroid on E and $\succsim_k$ is a weak preference relation on E. We use the notation "$\succsim_k$: e_1 (e_2 e_3) e_4 e_5 (e_6 e_7 e_8)" to mean that $\succsim_k$ is defined as $e_1 \succ_k e_2 \sim_k e_3 \succ_k e_4 \succ_k e_5 \succ_k e_6 \sim_k e_7 \sim_k e_8$.

We call $X \subseteq E$ a *matroid-kernel* if $X \in \mathcal{I}_1 \cap \mathcal{I}_2$ and every $e \in E \setminus X$ satisfies the following condition for some $k \in \{1, 2\}$: $X + e \notin \mathcal{I}_k$ and any $f \in X$ with $X + e - f \in \mathcal{I}_k$ satisfies $f \succsim_k e$. The problem of finding a maximum cardinality matroid-kernel is a generalization of MAX-SMTI, and hence is NP-hard.

For the special case in which the two matroids are base-orderable, Yokoi [7] showed that a 1.5-approximate solution can be found efficiently. However, it is open whether this approximability extends to the case of general matroids.

Problem 1. *Is there an efficient algorithm to find a* 1.5*-approximate solution for the problem of finding a maximum matroid-kernel in the setting with ties?*

Note that the algorithm in [7] is defined for general matroids, and base-orderability is used only in the approximation ratio analysis. We have not yet found an instance for which the algorithm fails to find a 1.5-approximate solution. Therefore, there is a possibility that some new proof technique shows that the algorithm's approximation ratio is 1.5 for general matroids. Below, we explain the algorithm in [7], which can be regarded as a matroid generalization of Király's algorithm [5] for MAX-SMTI (with some ideas from [8]).

Given a pair of weakly ordered matroids $(E, \mathcal{I}_1, \succsim_1)$ and $(E, \mathcal{I}_2, \succsim_2)$, we first construct two ordered matroids $(E^*, \mathcal{I}_1^*, \succ_1^*)$ and $(E^*, \mathcal{I}_2^*, \succ_2^*)$ as follows. Let E be represented as $E = \{e_1, e_2, \ldots, e_m\}$. Let $E^* = \{\, x_i, y_i, z_i \mid i = 1, 2, \ldots, m \,\}$. For each $k = 1, 2$, let

$$\mathcal{I}_k^* = \{\, S^* \subseteq E^* \mid \ |\{x_i, y_i, z_i\} \cap S^*| \le 1 \ \text{ for any } e_i \in E \text{ and } \pi(S^*) \in \mathcal{I}_k \,\},$$

where $\pi(S^*) := \{\, e_i \in E \mid \{x_i, y_i, z_i\} \cap S^* \neq \emptyset \,\}$. That is, the matroid $(E^*, \mathcal{I}_k^*)$ is obtained from $(E, \mathcal{I}_k)$ by replacing each element e_i with its three parallel copies x_i, y_i, z_i.

The linear order $\succ_1^*$ on E^* is defined as follows. Take a tie $(e_{i_1} e_{i_2} \cdots e_{i_\ell})$ in $\succsim_1$. We replace it with a strict linear order of 2ℓ elements $x_{i_1} x_{i_2} \cdots x_{i_\ell} y_{i_1} y_{i_2} \cdots y_{i_\ell}$. Apply this operation to all the ties in $\succsim_1$, where an element not included in any tie is regarded as a tie of length one. Next, at the end of the resultant list, append the original list $\succsim_1$ with each e_i replaced with z_i and all the parentheses omitted. Here is a an example of of a weak order $\succsim_1$ and the resultant linear order $\succ_1^*$.

$$\begin{aligned} &\succsim_1\colon\ (\ e_2\ \ e_5\ )\ \ e_1\ \ (\ e_3\ \ e_4\ ) \\ &\succ_1^*\colon\ \ x_2\ x_5\ y_2\ y_5\ x_1\ y_1\ x_3\ x_4\ y_3\ y_4\ z_2\ z_5\ z_1\ z_3\ z_4 \end{aligned}$$

The linear order $\succ_2^*$ is defined in the same manner, where the roles of x_i and z_i are interchanged. Here is a an example of $\succsim_2$ and the resultant linear order $\succ_2^*$.

$$\begin{aligned} &\succsim_2\colon\ \ e_3\ \ (\ e_1\ \ e_2\ \ e_4\ )\ \ e_5 \\ &\succ_2^*\colon\ \ z_3\ y_3\ z_1\ z_2\ z_4\ y_1\ y_2\ y_4\ z_5\ y_5\ x_3\ x_1\ x_2\ x_4\ x_5 \end{aligned}$$

For the pair $(E^*, \mathcal{I}_1^*, \succ_1^*)$ and $(E^*, \mathcal{I}_2^*, \succ_2^*)$, we can find a matroid-kernel $X^* \subseteq E^*$ efficiently by the framework of Fleiner [2]. Let $X := \pi(X^*) = \{\, e_i \in E \mid \{x_i, y_i, z_i\} \cap X^* \neq \emptyset \,\}$ and output X.

It is shown in [7] that (i) X is a matroid-kernel of $(E, \mathcal{I}_1, \succsim_1)$ and $(E, \mathcal{I}_2, \succsim_2)$, and (ii) if the matroids are base-orderable, $|X|$ is at least $\frac{2}{3}$ of the size of a maximum matroid-kernel.

2 EF1 allocation under matroid constraints

Fair allocation of indivisible goods is a research area actively studied recently. Since a completely envy-free allocation may not exist when items are indivisible, several relaxed notions have been proposed. One of them is *envy freeness up to one good (EF1)* introduced by Budish [9].

Let $N = \{1, 2, \ldots, n\}$ be the set of agents and E be the set of items. Each agent i has a valuation $v_i : E \to \mathbf{R}_+$, and i values each bundle $X \subseteq E$ at $v_i(X) := \sum_{e \in X} v_i(e)$. An *allocation* is a subpartition $\mathcal{X} = (X_1, X_2, \ldots, X_n)$ of E, where X_i is the bundle allocated to $i \in N$. We call an allocation *complete* if all items are allocated. An allocation $\mathcal{X}$ is *envy free up to one good (EF1)* if, for every $i, j \in N$, we have $v_i(X_i) \geq v_i(X_j)$ or there is $e \in X_j$ such that $v_i(X_i) \geq v_i(X_j \setminus \{e\})$.

In the setting without constraints, a complete EF1 allocation always exists and can be found easily by the Round-Robin method. Settings with matroid constraints are studied in [10, 11]. When the agents have different constraints, the existence of such an allocation is not guaranteed even in a very restricted case (two agents, partition matroids, identical binary additive valuations) [11]. Here we focus on the case where all agents have the same constraints.

Let $(E, \mathcal{I})$ be a matroid defined on the set E of items. An allocation $\mathcal{X} = (X_1, X_2, \ldots, X_n)$ is called *feasible* if $X_i \in \mathcal{I}$ for every agent $i \in N$. We suppose that the matroid admits a complete feasible allocation. The main questions here are the following.

Problem 2. *Is there a complete feasible EF1 allocation? Can we find it efficiently?*

For the following special cases, the above questions are solved affirmatively in previous works.

- Partition matroid [10]
- Base-orderable matroid, identical valuations ($v_i = v_j$ for any $i, j \in N$) [10]
- Base-orderable matroid, two agents [11]
- Base-orderable matroid, three agents, binary values ($v_i(e) \in \{0, 1\}$ for any $i \in N$, $e \in E$) [11]

The algorithm for the first case explicitly uses the partition structure of the matroid. For the other three cases, base-orderability is explicitly used to define the algorithms. These algorithms repeat swapping elements from two bundles, and the bijection of symmetric exchange is needed here to show that there exists a swap for the improvement. To solve the problem for the general matroids, a novel approach will be required.

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