10th Emléktábla Workshop

Combinatorics in Quarantine 08.31. - 09.03.2020.

6th Emléktábla Workshop — Finite Geometry 07. 28-31. 2014.

5th Emléktábla Workshop — Combinatorial Search 08. 12-15. 2013.

9th Emléktábla Workshop — Matching Theory 06.17. - 06.21.2019.

3rd Emléktábla Workshop — Even more combinatorics 06. 27-30. 2011.

2nd Emléktábla Workshop — More combinatorics 01. 24-27. 2011.

4th Emléktábla Workshop — Positional Games 08. 06-09. 2012.

7th Emléktábla Workshop — Discrete Geometry 06.29. - 07.02. 2015.

8th Emléktábla Workshop — Combinatorial Geometry 06.18. - 06.22.2018.

1st Emléktábla Workshop — Hypergraphs, Set Systems 07. 26-29. 2010.

Preliminary Schedule

Monday

Arrival in the hotel from 12:00 but no lunch!
14:00 Short presentations of problems
14:45: Creation of groups and work

Meals

-9:30 Breakfast
12:30- Lunch
18:00- Dinner

Tuesday 17:00 Presentations of progress and regrouping
Thursday 14:00 Presentations of progress and bye

List of Participants

Péter Ágoston, MTA-ELTE CoGe
János Barát, University of Pannónia, Veszprém
Zoltán Blázsik, ELTE
Endre Csóka, Rényi Institute
Gábor Damásdi, MTA-ELTE CoGe
Dániel Gerbner, Rényi Institute
Ervin Győri, Rényi Institute
Olivér Janzer, Cambridge
Balázs Keszegh, Rényi Institute and MTA-ELTE CoGe
Zoltán Nagy, ELTE
Márton Naszódi, MTA-ELTE CoGe
Dömötör Pálvölgyi, MTA-ELTE CoGe
Balázs Patkós, Rényi Institute
Addisu Paulos, Rényi Institute
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Chuanqi Xiao, Rényi Institute

Contributed Problems

Supersaturation of Tree posets

by Balázs Patkós

A subfamily $\mathcal{G}$ of $\mathcal{F} \subseteq 2^{[n]}$ is a copy of a poset P in $\mathcal{F}$ if there exists an bijection $i : P \rightarrow \mathcal{G}$ such that $p \leq_P q$ implies $i(p) \subseteq i(q)$. If there is no copy of P in $\mathcal{F}$, then $\mathcal{F}$ is *P-free*. The main conjecture of the area of forbidden subposet problems states that if $e(P)$ denotes the maximum integer m such that for any $k \leq n$ the family $\binom{[n]}{k} \cup \binom{[n]}{k+1} \cup \cdots \cup \binom{[n]}{k+m-1}$ is P-free, then for the maximum size $La(n,P)$ of a P-free family in $2^{[n]}$, we have $\lim_{n\to\infty} \frac{La(n,P)}{\binom{n}{\lfloor n/2 \rfloor}} = e(P)$.

This conjecture has been proved for several classes of posets, but is wide open in general. One of the nicest results confirming the conjecture in some special case is due to Bukh [1] and proves the conjecture if P is a tree poset, i.e., its Hasse diagram is a tree.

For those posets P for which the above conjecture has been verified, one can study the corresponding *supersaturation* problem: what is the minimum number of copies of P in a family $\mathcal{F} \subseteq 2^{[n]}$ of size $La(n,P) + E$. This was done by Kleitman [3] for P_2, and, after works by several authors, by Samotij [5] for P_k, the chain on k elements. I managed to settle this problem for the butterfly poset B [4], but only for a very tiny range of values of E.

Recently, with Gerbner, Nagy, and Vizer [2], we have come up with the following stronger form of the above conjecture: let $m(n,P)$ denote the number of copies of P in the middle $e(P)+1$ layers of $2^{[n]}$. For any poset P and positive real ε, there exists $\delta > 0$ such that any family $\mathcal{F} \subseteq 2^{[n]}$ of size at least $(e(P)+\varepsilon)\binom{n}{\lfloor n/2 \rfloor}$ contains at least $\delta m(n,P)$ copies of P.

We have verified this stronger conjecture for some posets including tree posets of height 2, and monotone tree posets, but it would be nice to obtain this quantitative version of Bukh's theorem in full generality, i.e., for all tree posets.

References

[1] B. Bukh, Set families with a forbidden subposet, Electronic Journal of Combinatorics, 16(1) (2009) P142.

[2] D. Gerbner, D. Nagy, B. Patkós, M. Vizer, Supersaturation, counting, and randomness in forbidden subposet problems. arXiv preprint arXiv:2007.06854. 2020 Jul 14.

[3] D. Kleitman, A conjecture of Erdős-Katona on commensurable pairs among subsets of an n-set. In Theory of Graphs, Proc. Colloq. held at Tihany, Hungary 1966 Sep (pp. 187-207).

[4] Patkós, Supersaturation and stability for forbidden subposet problems. Journal of Combinatorial Theory, Series A. 2015 Nov 1;136:220-37.

[5] W. Samotij, Subsets of posets minimising the number of chains. Transactions of the American Mathematical Society. 2019;371(10):7259-74.

Reducing strongly self-dual planar graphs to K_4

by Márton Naszódi

Let G be a planar, 3-connected graph. We call G *strongly self-dual*, if there is an isomorphism $\tau : G \to G^*$ such that

(1) For every pair of vertices u, v, we have that $u \in \tau(v) \Leftrightarrow v \in \tau(u)$, and
(2) For every vertex u, $u \notin \tau(u)$.

Our goal is to reduce every strongly self-dual graph to K_4 using certain operations that we describe here. The **motivation** is the main open problem in a recent paper [2].

Definition. A *simple* Δ*-*Y *transformation* is the following operation on a 3-connected planar graph G. Take a triangle in G. Introduce a new vertex and connect it to the three vertices of the triangle. Remove the edges of the triangle. "Clean" the resulting graph, that is, if a vertex of the original (now removed) triangle became of degree 2, then remove the vertex and connect its two neighbors by an edge. It is not hard to see that we obtain another 3-connected planar graph.

A *simple* Y*-*Δ *transformation* is more or less the opposite (or dual) of a simple Δ-Y transformation, you can guess the definition, or look at the figure.

Steinitz' theorem states that every 3-connected planar graph is the edge graph of a 3-dimensional polyhedron. The proof has two parts. In the combinatorial part, it is shown that every such graph can be reduced to K_4 using finitely many simple Δ-Y and Y-Δ transformations. The geometric part is about geometric realizability of these two graph-operations, cf. [1, 3].

For an analogous geometric theorem, it would be very useful to know the answer to the following. We define a Δ*-*Y/Y*-*Δ *transformation pair* on a strongly self-dual graph G as performing a simple Δ-Y transformation on some triangle Δ in G, and then a simple Y-Δ transformation on the degree 3 vertex $\tau(\Delta)$.

Warm-up: *Does a* Δ*-*Y/Y*-*Δ *transformation pair turn a strongly self-dual graph into a strongly self-dual graph?*

Problem 1. *Can any strongly self-dual graph be reduced to* K_4 *using a finite sequence of* Δ*-*Y/Y*-*Δ *transformation pairs?*

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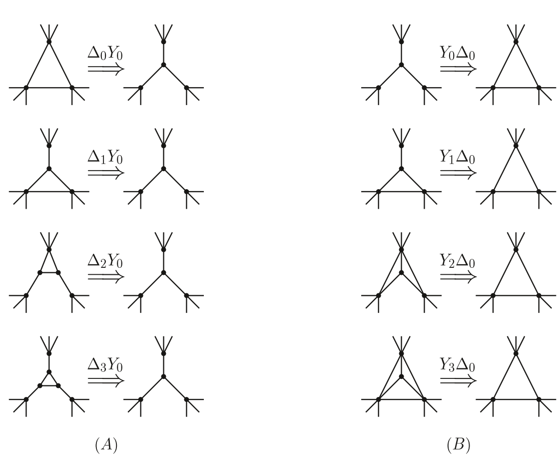

References

[1] Grünbaum, B. *Convex polytopes.* 2nd ed. Graduate Texts in Mathematics, 221. Springer-Verlag, New York, 2003.

[2] Montejano, L., Pauli, E., Raggi, M., Roldán-Pensado, E. *The Graphs Behind Reuleaux Polyhedra.* Discrete Comput Geom (2020). https://doi.org/10.1007/s00454-020-00220-0

[3] G. M. Ziegler. *Lectures on polytopes.* Graduate Texts in Mathematics, vol. **152**, Springer-Verlag, New York, 1995.

Supersaturation of the hexagon graph

by Zoltán Lóránt Nagy

Problem. Determine the order of magnitude of the minimum number of hexagons in n vertex graphs having m edges, in terms of m.

Background

The conjecture of Erdos-Simonovits and Sidorenko is well known, asserting that if m is quadratic, than the minimum number of bipartite subgraphs F is asymtotically attained in random graphs (with appropriate probability parameter p). Large family of Fs confirms the conjecture, including the hexagon graph. The assertion might also hold in a broader range of m, namely if $m \gg ex(n, F)$. This is the case for the quadrilateral, [6, 4] and for all the even cycles [5, 7]. However, a more elementary approach might be useful in case of the C_6. Also it would be interesting to construct graphs families above the threshold $ex(n, C_6)$ which has significantly less C_6 copies than a random graph with the same density.

Notes

• On the Turán number of C_6 or Theta graphs in general, see [3] and [1]. In fact, similar supersaturation should hold to Theta graphs than to even cycles.

• It might be useful to consider a bipartite variant of the problem, where the host graph is complete balanced bipartite. In this direction, a paper of Füredi might be a relevant read [2].

References

[1] Bukh, B., Tait, M. (2018). Turán number of theta graphs. arXiv preprint arXiv:1804.10014.

[2] Füredi, Z. (2013). On a theorem of Erdős and Simonovits on graphs not containing the cube. arXiv:1307.1062.

[3] Füredi, Z., Naor, A., Verstraëte, J. (2006). On the Turán number for the hexagon. Advances in Mathematics, 203(2), 476–496.

[4] He, J., Ma, J., Yang, T. (2019). Stability and supersaturation of 4-cycles. arXiv:1912.00986.

[5] Morris, R., Saxton, D. (2016). The number of C_{2l}-free graphs. Advances in Mathematics, 298, 534–580.

[6] Nagy, Z. L. (2019). Supersaturation of C_4: From Zarankiewicz towards Erdős-Simonovits-Sidorenko. European Journal of Combinatorics, 75, 19–31.

[7] Jiang, T., Yepremyan, L. (2017). Supersaturation of even linear cycles in linear hypergraphs. arXiv preprint arXiv:1707.03091.

Expander Steiner triple systems are abundant

by Zoltán Lóránt Nagy

Problem. Show that *whp* a Steiner triple system is δ-expander for some $\delta > 0$.

Background. The set of neighbours $N(U)$ of a vertex set of $U \subset V$ in a triple system $\mathcal{F}$ on a ground set V consists of vertices from $V \setminus U$ which form a triple of $\mathcal{F}$ together with two vertices of U. A triple system $\mathcal{F}$ on a vertex set V is expander, if every subset $U \subset V$ have the property that $\frac{|N(U)|}{|U|} \geq \delta > 0$, provided that U has cardinality $3 < |U| \leq |V|/2$ or U has 3 elements but it is not

contained in $\mathcal{F}$.
Observe that Steiner triple systems with nontrivial Steiner triple subsystems are not expanders.

Via an additive combinatorial construction, Blázsik and Z.L. Nagy proved the existence of almost 1-expander Steiner triple systems for infinitely many orders. [1]. My conjecture is that a much stronger result should hold, namely the majority of Steiner triple systems are expanders as $n = |V|$ grows.

Proof ideas. • Random process and probabilistic tools (Chernoff, Azuma), see e.g. [2, 3]

References

[1] Blázsik, Z. L., Nagy, Z. L. (2019). Spreading linear triple systems and expander triple systems. European Journal of Combinatorics, 89, 103155.
[2] Bohman, T., Warnke, L. (2019). Large girth approximate Steiner triple systems. Journal of the London Mathematical Society, 100(3), 895-913.
[3] Glock, S., Kühn, D., Lo, A., Osthus, D. (2020). On a conjecture of Erdős on locally sparse Steiner triple systems. Combinatorica, 40(3), 363-403.

Number of paths as a function of codegrees

by D. Gerbner, Z. L. Nagy and M. Vizer

Background. With D. Gerbner and Z. L. Nagy in [1] we introduced for 2 different simple graphs F and G the function $satex(n, F : m, G)$ that is the minimum number of subgraphs G in an n-vertex graph having at least $m(= m(n))$ copies of F as a subgraph. This function can be considered as a common extension of the so-called generalized Turán function and the supersaturation problem. Let us denote by P_k the path on k vertices.

Problem 1. Determine the order/asymptotics of $satex(n, K_{1,t} : m, P_{2k+1})$ for all $k, t \geq 1$.

Problem 2. Determine the order/asymptotics of $satex(n, K_{2,t} : m, P_{2k+1})$ for all $k, t \geq 1$.

Problem 3. Determine the order/asymptotics of $satex(n, K_{2,t} : m, P_{2k})$ for all $k, t \geq 1$.

Notes. • On the generalized Turán problem of $K_{2,t}$ and paths, see [2].

References

[1] Gerbner, D., Nagy, Z. L., Vizer, M. (2020) Unified approach to the generalized Turán problem and supersaturation. arXiv preprint arXiv: `https://arxiv.org/abs/2008.12093`
[2] Gerbner, D., Palmer, C. (2020) Counting copies of a fixed subgraph in F-free graphs, European Journal of Combinatorics, accepted.

Extremal graph theory for unit distance graphs

by Dömötör Pálvölgyi

A graph is a UDG if its vertices can be mapped into (different) points in the plane such that the ends of any edge are one unit apart. Denote by $u(n)$ the maximum number of edges a UDG on n vertices can have. The best estimates are $n^{1+c/\log\log n} \leq u(n) \leq O(n^{4/3})$.

Denote by $\mathrm{ex}_u(n, F)$ the number of edges a UDG on n vertices can have without an F; obviously, for every F $\mathrm{ex}_u(n, F) \leq u(n)$. The only result we are aware of is the simple $\mathrm{ex}_u(n, K_3) = \Theta(u(n))$.

Problem. *Study* $\mathrm{ex}_u(n, F)$.

Another natural question is to study the possible number of occurrences of some fixed graph F in a UDG. Denoting this by $\mathrm{ex}_u(\#F, n)$, the only result we are aware of is again the easy $\mathrm{ex}_u(\#K_3, n) = O(u(n))$ and the recent almost sharp bounds for $\mathrm{ex}_u(\#P_k, n)$ [2]. For more related results, see [1].

Problem. *Study* $\mathrm{ex}_u(\#F, n)$.

References

[1] P. Brass, W. O. J. Moser, J. Pach: Research Problems in Discrete Geometry (2005).

[2] N. Frankl, A. Kupavskii: Almost sharp bounds on the number of discrete chains in the plane. `https://arxiv.org/abs/1912.00224`

Intersection of two longest paths

by Dömötör Pálvölgyi

Conjecture (Hippchen [2])**.** *In a k-connected graph, any two longest paths intersect in at least k vertices.*

This has been proved only for $k \leq 4$ [1].

References

[1] J. Gutiérrez: On the intersection of two longest paths in k-connected graphs. `https://arxiv.org/abs/2008.02163`

[2] T. Hippchen: Intersections of longest paths and cycles, MSc thesis, 2008.

Structure of rainbow K_k-free complete graphs

by Dömötör Pálvölgyi

It is a classic result of Gallai that the vertices of any rainbow K_3-free edge-colored complete graph can be non-trivially partitioned such that at most two colors occur between all the parts, and only one color between any two parts. This has proved extremely useful in the study of various problems, and a generalization would be useful, e.g., for multicolor Erdős-Hajnal problems.

Problem. *Prove that any rainbow K_k-free edge-colored complete graph can be non-trivially partitioned such that at most $\binom{k}{2}-1$ colors occur between all the parts, and at most $f(t)$ colors between any t parts, where f is ... and furthermore also satisfies ...*

References

[1] T. Gallai: Transitiv orientierbare Graphen, Acta Math. Acad. Sci. Hungar. 18 (1967), 25–66.

[2] A. Gyárfás, G. Simonyi, Edge colorings of complete graphs without tricolored triangles, Journal of Graph Theory 46 (2004), 211–216.

Infinite Sperner's theorem

by Dömötör Pálvölgyi

For an infinite antichain $\mathcal{A}$ of finite subsets of the positive integers, denote by $\mathcal{A}_n$ its sets whose largest element is n, and let $a_n = |\mathcal{A}_n|$. From Kraft's inequaity, $\sum a_n 2^{-n} \leq 1$.

Problem. *Can this be sharpened?*

Denote the sum of the first few terms by $f_n = \sum_{i=1}^{n} a_i$. Sudakov-Tomon-Wagner [1] proved that $\liminf f_n 2^{-n} n \log n = 0$ always holds, but there is an $\mathcal{A}$ for which $\liminf f_n 2^{-n} n \log^{46} n > 0$.

Conjecture (Sudakov-Tomon-Wagner [1]). *There is an $\mathcal{A}$ for which* $\liminf f_n 2^{-n} n \log^{1.01} n > 0$.

References

[1] B. Sudakov, I. Tomon, Á. Zs. Wagner: Infinite Sperner's theorem. https://arxiv.org/abs/2008.04804

The saturation problem in the Ramsey theory of graphs

by Balázs Keszegh

Let $\mathcal{G}$ be the family of (labeled) complete graphs whose edges are colored with c colors (numbered by $1, 2, \dots, c$). Given $G, G' \in \mathcal{G}$, we say G' *extends* G if G is a proper subgraph of G', i.e., G' can be obtained from G by iteratively adding a new vertex and colored edges connecting the new vertex with each of the existing vertices. A member G of $\mathcal{G}$ is called $(k_1, k_2, \dots, k_c)$-*saturated* if for every $i \in [c]$, the graph G does not contain a monochromatic K_{k_i} of color i, but every $G' \in \mathcal{G}$ that extends G contains a monochromatic K_{k_i} of color i for some i.

Clearly the size of the largest $(k_1, k_2, \dots, k_c)$-saturated graph in $\mathcal{G}$ is equal to the usual Ramsey number minus one. Let $sat_{\mathcal{G}}(k_1, \dots k_c)$ denote the size of the smallest $(k_1, k_2, \dots, k_c)$-saturated $G \in \mathcal{G}$.

Theorem 1. *[1] For two colors,*

$$sat_{\mathcal{G}}(k, l) = (k-1)(l-1),$$

and for more than two colors,

$$(k_1 - 1)(k_2 + \dots + k_c - 2c + 3) \le sat_{\mathcal{G}}(k_1, \dots, k_c) \le (k_1 - 1) \cdots (k_c - 1).$$

In the latter lower bound we can exchange k_1 with any other k_i.

Problem. *Improve the above bounds on $sat_{\mathcal{G}}(k_1, \dots, k_c)$.*

For further reading see [1] and [2].

References

[1] Gábor Damásdi, Balázs Keszegh, David Malec, Casey Tompkins, Zhiyu Wang and Oscar Zamora: Saturation problems in the Ramsey theory of graphs, posets and point sets, arXiv:2004.06097 (2020)

[2] Tuan Tran: Two problems in graph Ramsey theory, arXiv:2008.07367 (2020)

Proper coloring 3-uniform hypergraphs with restricted intersections

by Balázs Keszegh and Dömötör Pálvölgyi

Problem. *Given a 3-uniform hypergraph $\mathcal{H}$ such that every hyperedge has one of its vertices associated with it and for every pair of hyperedges $H_1, H_2 \in \mathcal{H}$, if $H_1 \cap H_2 = \{v\}$, then v is associated with at least one of H_1 and H_2. Can we always properly color with two colors such a hypergraph?*

If v is associated with both hyperedges H_1 and H_2, then finding a proper two-coloring is easy.

On 3-uniform hypergraphs of girth six

by Ervin Győri and Nika Salia

Definition 1. *A* Berge cycle *of length ℓ in a hypergraph is an ordered set of ℓ distinct vertices $\{v_1, \ldots, v_\ell\}$ and ℓ distinct hyperedges $\{e_1, \ldots, e_\ell\}$ such that $\{v_i, v_{i+1}\} \subseteq e_i$ with indices taken modulo ℓ. We denote the class of Berge cycles of length ℓ by $\mathcal{B}C_\ell$.*

Definition 2. *The Turán number $ex_r(n, \mathcal{H})$ of a hypergraph $\mathcal{H}$ is the maximum number of edges that an n-vertex graph can have without containing $\mathcal{H}$ as a subgraph. If a class $\mathbb{H}$ of hypergraphs is forbidden, then the Turán number is denoted by $ex(n, \mathbb{H})$.*

Theorem 3.

$$(1 - o(1))\frac{n^{3/2}}{3\sqrt{3}} \leq \mathrm{ex}_3(n, \mathcal{B}C_4) \leq (1 + o(1))\frac{n^{3/2}}{\sqrt{10}}.$$

The lower bound comes from a construction originating in Bollobás and Győri [1]. They take a C_4-free bipartite graph with color classes of size $n/3$ and $\frac{(2n/3)^{3/2}}{2\sqrt{2}} = \frac{n^{3/2}}{3\sqrt{3}}$ edges asymptotically. For every vertex v in one of the color classes, they take an additional vertex v' and add it to every edge in the graph incident to v. This results in a 3-uniform hypergraph on n vertices with $\frac{n^{3/2}}{3\sqrt{3}}$ hyperedges asymptotically, with girth at least six.

The upper bound comes from a recent manuscript of Ergemlidze, Győri, Methuku, Salia and Tompkins [2]. We suggest two open problems.

Conjecture 1. $\mathrm{ex}_3(n, \{\mathcal{B}C_3, \mathcal{B}C_4, \mathcal{B}C_5\}) = (1 + o(1))\frac{n^{3/2}}{3\sqrt{3}}$

Conjecture 2. $\mathrm{ex}_3(n, \mathcal{B}C_4) = (1 + o(1))\frac{n^{3/2}}{3\sqrt{3}}$

References

[1] B. Bollobás and E. Győri. *Pentagons vs. triangles.* Discrete Mathematics **308**(19), (2008): 4332–4336.

[2] Ergemlidze, Győri, Methuku, Salia and Tompkins, On 3-uniform hypergraphs avoiding a cycle of length four, arxiv.org/abs/2008.11372.

Local lemma for games

by Endre Csóka

We consider simple Maker-Breaker versions of games such as gomoku (in Hungarian: amőba). Namely, Maker and Breaker alternately puts his own symbol to an empty vertex of a hypergraph. If Maker puts his symbol to all vertices of a hyperedge, then Maker wins, otherwise Breaker wins.

There is a useful lemma for finite games. A hyperedge without the symbol of Breaker is called active. After the move of Maker, if

$$\text{potential function} = \sum_{\text{active hyperedge}} 2^{-|\text{empty vertices on the hyperedge}|} < 1, \tag{0.1}$$

then Breaker can win by always greedily minimizing the potential function.

This lemma cannot be directly applied for games on an infinite graph.[1] However, the playing experience of gomoku makes the impression that if the length of the winning lines were 6 instead of 5, then Maker would never be able to win. A "*low density of the potential*" should somehow imply that Breaker can win. In the following game, a good strategy of Breaker could be translated into a generalization of the previous simple lemma to a game theoretical local lemma (in 1 dimension).

Problem. We define a two-player game as follows. The state of the game (not including the playing order) can be written as a (potential) function $P\colon \mathbb{Z} \to [0, \infty)$ and an infinite multiset of offers $\mathcal{O}$ with every $f \in \mathcal{O}$, $f\colon \mathbb{Z} \to [0, \infty)$ and $0 \le f \le P$ and f is supported on at most two consecutive integers. The initial state is, say, $P\colon \mathbb{Z} \to 2$ with an infinite number of offers of the form $..., 0, 0, 0, 1, 1, 0, 0, 0, ...$, infinitely many at every position.

The two players take turns alternately, for an infinite number of rounds. At the turn of the first player, he chooses an offer, supported on some integers k and $k + 1$, adds it to P, deletes this offer from $\mathcal{O}$, and may increase every other offer at k and $k + 1$ as long as the offers still satisfy the conditions.[2] At the turn of the second player, she chooses an offer, supported on some integers k and $k + 1$, decreases P by the offer, deletes the offer from $\mathcal{O}$, and then the *first player* can decrease every other offer at k and $k + 1$ so as every offer should satisfy the conditions.

The first player wants to maximize and the second player wants to minimize the supremal potential function value in the history of the infinite game. Question: what can the players achieve?

[1] Unless if we split the vertices of the graph into finite classes and intersect each hyperedge with one of the classes.
[2] Variant: these values can be at most doubled.

Dichromatic number, directed Hajós join

by János Barát

Let D_1 and D_2 be two disjoint directed graphs and let $u_1v_1 \in E(D_1)$ and $v_2u_2 \in E(D_2)$. We obtain the Hajós join of D_1 and D_2 by deleting u_1v_1 and v_2u_2, merging v_1 and v_2 to a new vertex v and adding the arc u_1u_2.

Question 1. *How can a bidirected C_5 be constructed from $\overleftrightarrow{K_3}$'s by only using Hajós joins and identifying non-adjacent vertices?*

A k-coloring of a digraph D is a mapping of the vertices to $\{1, \ldots, k\}$ such that each color class is acyclic. The dichromatic number of a digraph D is the minimum integer k such that D admits a k-coloring.

Question 2. *Which oriented graphs have dichromatic number 2?*

References

[1] Bang-Jensen, Bellitto, Stiebitz, Schweser, Hajós and Ore constructions for digraphs `https://arxiv.org/abs/1908.04096`

Crumby coloring

by János Barát

A *crumby coloring* of a graph is a 2-coloring of its vertices in blue and red such that
(1) the subgraph induced by the blue vertices has maximum degree at most 1,
(2) the the subgraph induced by the red vertices has minimum degree at least 1, and
(3) the subgraph induced by the red vertices contains no path on 4 vertices.

Question 1. *Does every bipartite graph of maximum degree* 3 *possess a crumby coloring?*

Question 2. *Does every outerplanar graph of maximum degree* 3 *possess a crumby coloring?*

The same question has a negative answer for non-planar, non-bipartite graphs.

References

[1] Bellitto, Klimosova, Merker, Witkowski, Yuditsky, Counterexamples to Thomassen's conjecture on decomposition of cubic graphs `https://arxiv.org/abs/1908.06697`