20th Emléktábla Workshop

Extremal combinatorics 07.19 - 07.23.2026.

# Preliminary Schedule for the Extremal Combinatorics group:

**Sunday**

Arrival at the hotel before 12:30.
12:30-14:00 Lunch (at the hotel)
15:00 Short introduction/presentation of problems

**Monday-Wednesday**

7:30-8:30 Breakfast (at the hotel)
9:00-17:00 Working in groups of 4-5
12:30-14:00 Lunch (at the hotel)
16:45 Presentations of daily progress
18:00 Dinner (your choice)

**Thursday**

7:30-8:30 Breakfast (at the hotel)
9:00-12:30 Working in groups of 4-5
12:30-14:00 Lunch (at the hotel)
14:15 Summary of the progress

| Name | University/Institute |
|---|---|
| Nóra Almási | Technical University of Budapest |
| Péter Ágoston | Charles University, Prague |
| Martin Balko | Charles University, Prague |
| János Barát | Rényi Institute |
| Pál Bärnkopf | Eötvös University, Budapest |
| Csongor Beke | University of Cambridge, UK |
| Zoltán L. Blázsik | University of Szeged |
| Vladimir Boskovic | IPhT Paris-Saclay |
| Michael L. Bruner | University of Montana |
| Ignacy Buczek | Jagiellonian University, Kraków |
| Recep Altar Ciceksiz | Umea University |
| Gábor Damásdi | Rényi Institute |
| Andrea Freschi | Rényi Institute |
| Jakob Führer | Johannes Kepler University Linz |
| Dániel Gerbner | Rényi Institute |
| Jelena Glišić | Charles University, Prague |
| Aleksandr Grebennikov | IST Austria |
| Andrzej Grzesik | Jagiellonian University, Kraków |
| Anna Gujgiczer | Technical University of Budapest |
| Ervin Győri | Rényi Institute |
| Máté Jánosik | Eötvös University, Budapest |
| Attila Jung | Eötvös University |
| Hilal Hama Karim | Rényi Institute |
| Nina Kamčev | University of Zagreb |
| Balázs Keszegh | Rényi Institute |
| Lenka Kopfová | IST Austria |
| Benedek Kovács | Eötvös University, Budapest |
| Gaurav Kucheriya | Charles University, Prague |
| Nathan Lemons | Los Alamos National Lab. |
| Binlong Li | Northwestern Polytechnical University, China |
| Kenneth Moore | Rényi Institute |
| Gábor V. Nagy | University of Szeged |
| Kartal Nagy | Eötvös University, Budapest |
| Zoltán Lóránt Nagy | Eötvös University, Budapest |
| Cory Palmer | University of Montana |
| Dömötör Pálvölgyi | Eötvös University, Budapest |
| Balázs Patkós | Rényi Institute |
| Laurentiu Ioan Ploscaru | Rényi Institute |
| Magdalena Prorok | AGH University of Kraków |
| Farhood Rostamkhani | IST Austria |
| Nika Salia | Jagiellonian University, Kraków |
| Dániel Simon | Rényi Institute |
| Balázs Szabó | Eötvös University, Budapest |
| Dávid Szabó | Rényi Institute |
| Géza Tóth | Rényi Institute |
| Tadej Petar Tukara | University of Zagreb |
| Benedek Váli | University of Szeged |
| Máté Vizer | Technical University of Budapest |
| Yiting Wang | IST Austria |
| Kristóf Zólomy | Eötvös University, Budapest |

# Large Almost Monochromatic Subsets in $k$-Uniform Hypergraphs

by Martin Balko

For 3-uniform hypergraphs, Conlon, Fox, and Sudakov proved that for any $\epsilon > 0$ and $\ell$, there is a constant $c = c(\ell, \epsilon) > 0$ such that every $\ell$-coloring of the triples of an $N$-element set contains a subset $S$ of size $c\sqrt{\log N}$ with at least a $1 - \epsilon$ fraction of its triples having the same color [1]. It remains an open question to fully generalize these bounds and the corresponding bounds on complete multipartite hypergraphs for higher uniformities $k \geq 4$ [1].

**Problem 1.** *Extend the* 3*-uniform discrepancy result to uniformity* $k \geq 4$ *[1]. Specifically, does there exist an absolute constant* $\delta = \delta(k, \ell) > 0$ *(independent of* $\epsilon$*) and a constant* $c = c(k, \ell, \epsilon) > 0$ *such that every* $\ell$*-coloring of the* $k$*-tuples of an* $N$*-element set contains a subset of size* $s = c(\log N)^{\delta}$ *containing at least* $(1 - \epsilon)\binom{s}{k}$ $k$*-tuples of the same color?*

The following is a preliminary result from Conlon, Fox, and Sudakov regarding this direction:

**Theorem 1.** *For all* $k, \ell,$ *and* $\epsilon > 0$ *there is a* $\delta = \delta(k, \ell, \epsilon) > 0$ *such that every* $\ell$*-coloring of the* $k$*-tuples of an* $N$*-element set contains a subset of size* $s = (\log N)^{\delta}$ *which contains at least* $(1-\epsilon)\binom{s}{k}$ $k$*-tuples of the same color [1].*

**Problem 2.** *In the preliminary result above, the exponent* $\delta$ *depends on* $\epsilon$ *[2]. Because this result is obtained by bounding the Ramsey numbers of complete multipartite* $k$*-uniform hypergraphs [2], can we find a tighter upper bound for the* $\ell$*-color Ramsey numbers of complete multipartite* $k$*-uniform hypergraphs (for* $k \geq 4$*) that successfully removes the dependence of* $\delta$ *on* $\epsilon$*?*

# References


[1] D. Conlon, J. Fox, and B. Sudakov. Hypergraph Ramsey numbers, submitted.

[2] P. Erdős, A. Hajnal. Ramsey-type theorems. *Discrete Appl. Math.* **25** (1989), 37–52.

## Frank number

by János Barát

In an orientation $O$ of a graph $G$, an edge $e$ is deletable if $O - e$ is strongly connected. For a 3-edge-connected graph $G$, Hörsch and Szigeti defined the Frank number as the minimum $k$ for which $G$ admits $k$ orientations such that every edge $e$ of $G$ is deletable in at least one of the $k$ orientations. They conjectured the Frank number is at most 3 for every 3-edge-connected graph $G$. They proved the Petersen graph has Frank number 3, but this was the only example with this property. Barát and Blázsik [1] showed an infinite class of graphs having Frank number 3. Each of these graphs contain the Petersen graph as a minor (or in some other sense).

**Problem 1.** *Construct or find* 3*-edge-connected graphs* $G$ *such that* $G$ *has Frank number* 3*, but* $G$ *has no Petersen minor.*

Studying the Frank number, we found that the following was always true. Can you prove or disprove this observation?

**Problem 2.** *For every cubic* 3*-edge-connected graph* $G$*, there exists a strongly connected orientation* $D$ *of* $G$ *such that* $D - a$ *is strongly connected for at least half of the arcs* $a$ *of* $D$*.*

A counterexample would immediately give us a graph with Frank number larger than 2.

Goedgebeur, Máčajová and Renders [2] proved the following:

**Theorem 1.** *Every* 3*-edge-connected graph has Frank number at most* 4 *and every graph which is* 3*-edge-connected and* 3*-edge-colourable has Frank number* 2*.*

# References

[1] János Barát, Zoltán L. Blázsik. Improved upper bound on the Frank number of 3-edge-connected graphs. European Journal of Combinatorics, Volume 118, 2024, 103913, https://doi.org/10.1016/j.ejc.2023.103913.

[2] Jan Goedgebeur, Edita Máçajová, Jarne Renders. The Frank number and nowhere-zero flows on graphs. European Journal of Combinatorics, Volume 126, 2025, 104127, https://doi.org/10.1016/j.ejc.2025.104127.

## The Erdős-Lovász Cover Number Problem

by János Barát

A hypergraph is *intersecting* if any two of its hyperedges have a common vertex. A *cover* of a hypergraph $\mathcal{H}$ is a set of vertices meeting every edge.

Let $g(r)$ be the minimum number of edges in an $r$-uniform intersecting hypergraph with cover number $r$. Erdős and Lovász [2] proved the lower bound $g(r) \geq 8r/3 - 3$. It was one of the 3 favourite problems of Erdős whether the upper bound was linear. This was proved by Kahn [3].

**Problem 1.** *Can you improve the lower bound on $g(r)$ to $3r$?*

About 10-15 years ago I was obsessed with this problem. Recently, Varun Sivashankar [4] posted a solution on arXiv.

**Theorem 1.** *The following bounds hold.*

*(i) For every positive integer $r$, $g(r) \geq 3r - 4$.*

*(ii) For every $\varepsilon > 0$, there is $r_0$ such that, for every $r \geq r_0$,*

$$g(r) \geq \left(\frac{61}{20} - \varepsilon\right) r.$$

His acknowledgement reads as follows: "The proof was discovered with the help of ChatGPT 5.5 Pro. Theorem 1 was formalized in Lean with the help of Harmonic's Aristotle: part (i) was formalized in full, and part (ii) was formalized conditional on Kahn's theorem."

**Problem 2.** *Can you determine $g(6)$?*

There are many related problems to work on. I think we should dig deep and familiarize with the recent developments.

# References


[1] J. Barát, *Intersecting and 2-intersecting hypergraphs with maximal covering number: the Erdős–Lovász theme revisited*, J. Combin. Des. 29 (2021), no. 3, 193–209.

[2] P. Erdős and L. Lovász, *Problems and results on 3-chromatic hypergraphs and some related questions*, in *Infinite and Finite Sets*, Colloq. Math. Soc. János Bolyai, Vol. 10, North-Holland, Amsterdam, 1975, pp. 609–627.

[3] J. Kahn, *On a problem of Erdős and Lovász. II. $n(r) = O(r)$*, J. Amer. Math. Soc. 7 (1994), no. 1, 125–143.

[4] V. Sivashankar. An Improved Lower Bound for the Erdős-Lovász Cover Number Problem. `https://arxiv.org/pdf/2606.24878`

## Reed's Conjecture for Triangle-Free Graphs

Pál Bärnkopf

Reed's famous conjecture [2] states that every graph $G$ satisfies $\chi(G) \leq \left\lceil \frac{\Delta(G)+\omega(G)+1}{2} \right\rceil$, where $\chi(G)$, $\Delta(G)$, and $\omega(G)$ denote the chromatic number, the maximum degree, and the clique number of $G$, respectively. This would strengthen Brooks' theorem in a striking way. For triangle-free graphs ($\omega(G) = 2$), Reed's conjecture reduces to the following.

**Problem 1.** *Every triangle-free graph satisfies* $\chi(G) \leq \left\lfloor \frac{\Delta(G)}{2} \right\rfloor + 2$.

This bound is far from best possible asymptotically. Johansson proved that triangle-free graphs satisfy $\chi(G) = \mathcal{O}\left(\frac{\Delta}{\log \Delta}\right)$. On the other hand, one of the strongest explicit bounds currently known is due to Kostochka, who proved $\chi(G) \leq \frac{2\Delta(G)}{3} + 2$. The smallest unresolved case of the triangle-free version of Reed's conjecture appears to be $\Delta = 5$.

**Problem 2.** *Is every triangle-free graph with maximum degree at most* 5 4*-colorable?*

# References

[1] G. Abrishami, A. Erfanian, A note on Reed's conjecture for triangle-free graphs, Discrete Mathematics, 346(12), 2023.

[2] B. A. Reed, $\omega$, $\Delta$ and $\chi$, J. Graph Theory 27 (1998) 177–212

## Edge-colorability of 5-edge-connected 5-regular graphs

Pál Bärnkopf

A graph $G$ is Class 1 if it is $\Delta(G)$-edge-colorable, otherwise it is Class 2. It is known that $r$-edge-connected $r$-regular Class 2 graphs exist for all integers $r \geq 2$, with $r \neq 5$ (see, for example, [1]). Surprisingly, it seems that no 5-edge-connected 5-regular Class 2 graph is known.

**Problem 1.** *(Ma, Mattiolo, Steffen, Wolf [1]) Is there any* 5*-edge-connected* 5*-regular Class 2 graph?*

If the answer is negative, then it would have far-reaching consequences, as it would imply several well-known conjectures, including the Fan–Raspaud Conjecture, the 5-Cycle Double Cover Conjecture, and the Berge–Fulkerson Conjecture.[1]

## References

[1] Y. Ma, D. Mattiolo, E. Steffen, I. H. Wolf Pairwise disjoint perfect matchings in r-edge-connected r-regular graphs. SIAM Journal on Discrete Mathematics, 37(3) (2023),1548–1565.

[2] Open problems of the 32nd Workshop on Cycles and Colourings https://arxiv.org/pdf/2411.10046

## The connection between the chromatic numbers of a hypergraph and its 1-intersection graph

by Zoltán L. Blázsik

Let $\mathcal{H}$ be an arbitrary hypergraph. The 1-intersection graph, denoted by $\mathcal{H}^{[1]}$, of $\mathcal{H}$ is a graph such that the vertices are the hyperedges of $\mathcal{H}$ and two hyperedges are adjacent in $\mathcal{H}^{[1]}$ if they have exactly one element in common. Gyárfás et al. [1] asked the following question:

**Problem 1.** *Let $r \geq 3$ and let $\mathcal{H}$ be an $r$-uniform hypergrap whose 1-intersection graph $\mathcal{H}^{[1]}$ is $k$-colorable for some $k \geq 2$. Is it true that $\mathcal{H}$ is also $k$-colorable? In other words does $\chi(\mathcal{H}) \leq \chi(\mathcal{H}^{[1]}) = k$ hold?*

They proved the affirmative answer for $r = 3$ in [1], but for larger $r$ values the question is still open. On the other hand, Lemons and Blázsik [2] proved the following:

**Theorem 1.** *For any hypergraph $\mathcal{H}$ if $\chi(\mathcal{H}^{[1]}) = 2$ then $\chi(\mathcal{H}) = 2$, too. And if $\chi(\mathcal{H}^{[1]}) = 4$ then $\chi(\mathcal{H}) \leq 4$.*

Important to notice that without any constraints the inequality $\chi(\mathcal{H}) \leq \chi(\mathcal{H}^{[1]})$ does not hold in general. See for example $\mathcal{H} = K_{2m}$. But for all the known counterexamples the chromatic number of the 1-intersection graph is an odd number.

**Problem 2.** *Is it true that $\chi(\mathcal{H}) \leq \chi(\mathcal{H}^{[1]})$ holds if $\chi(\mathcal{H}^{[1]})$ is even? Or can we find some assumptions on $\mathcal{H}$ which ensures $\chi(\mathcal{H}) \leq \chi(\mathcal{H}^{[1]})$ in general?*

# References

[1] A. Gyárfás, A.W.N. Riasanovsky, M.U. Sherman-Bennett. Chromatic Ramsey number of acyclic hypergraphs. Discrete Mathematics, 340 (3), 373-378, 2017.

[2] Z. L. Blázsik, N. W. Lemons. The connection between the chromatic numbers of a hypergraph and its 1-intersection graph. Discrete Mathematics, 349 (2), 114810, 2026.

## ODD SQUARES IN THE HYPERCUBE

by Gábor Damásdi, who proposed the problem

Let $Q_n$ denote the $n$-dimensional hypercube. Write $f(n) = ex(Q_n, C_4)$ for the maximum number of edges in a subgraph of $Q_n$ containing no copy of $C_4$. A classical problem of Erdős asks whether

$$f(n) \leq \left(\frac{1}{2} + o(1)\right) n2^{n-1}.$$

Equivalently, Erdős conjectured that the edge Turán density of $C_4$ in the hypercube is 1/2 [1].

The Erdős question seems hard, so here is a parity variant which is much more understood. Let $g(n)$ be the maximum number of edges one can pick so that every square of the hypercube contains an odd number of picked edges. Since such an $F$ contains either one or three edges from each square, it cannot contain all four edges of any square. Therefore $g(n) \leq f(n)$. On the other hand the known small values of $f(n)$ are

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(n)$ | 1 | 3 | 9 | 24 | 56 | 132. |

The nontrivial cases $n = 5, 6$ are due to Emamy-K., Guan and Rivera-Vega [2], and to Harborth and Nienborg [3], respectively. The corresponding values found for the odd-square problem are the same, see [5]:

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $g(n)$ | 1 | 3 | 9 | 24 | 56 | 132. |

Moreover, in some dimensions there are extremal $C_4$-free examples which do not satisfy the odd-square condition, but in all checked small dimensions there is also an extremal $C_4$-free example which does satisfy it.

**Problem 1.** *Is it true that*

$$g(n) = f(n)$$

*for every $n$?*

(The parity condition also has a natural formulation in the language of signed graphs. This question was also posed, in the equivalent frustration-index formulation, by Zhouningxin Wang.)

# References


[1] P. Erdős, On some problems in graph theory, combinatorial analysis and combinatorial number theory, in: *Graph Theory and Combinatorics*, B. Bollobás, ed., Academic Press, 1984, 1–17.

[2] M. R. Emamy-K., P. Guan and P. I. Rivera-Vega, On the characterization of the maximum squareless subgraphs of the 5-cube, *Congr. Numer.* 88 (1992), 97–109.

[3] H. Harborth and H. Nienborg, Maximum number of edges in a six-cube without four-cycles, *Bull. Inst. Combin. Appl.* 12 (1994), 55–60.

[4] P. Brass, H. Harborth and H. Nienborg, On the maximum number of edges in a $C_4$-free subgraph of $Q_n$, *J. Graph Theory* 19 (1995), 17–23.

[5] S. Laplante, R. Naserasr, A. Sunny and Z. Wang, Sensitivity conjecture and signed hypercubes, *ACM Trans. Comput. Theory* 18(1) (2026), Article 6, 1–25.

## When is the Füredi graph extremal?

by Dániel Gerbner

Füredi [2] constructed a graph $F_t(n)$ that aymptotically contains the most edges among $n$-vertex $K_{2,t}$-free graphs. This graph also contains asymptotically the most copies of triangles [1], paths and cycles [4], $K_{2,s}$ [5]. I characterized [3] the trees $T$ such that $F_t(n)$ contains the most copies of $T$ asymptotically. What about other graphs?

**Problem 1.** *Find more graphs $H$/characterize the graphs $H$ such that $ex(n, H, K_{2,t}) = (1 + o(1))N(H, F_t(n))$, i.e., $F_t(n)$ aymptotically contains the most copies of $H$.*

# References

[1] N. Alon and C. Shikhelman. Many $T$ copies in $H$-free graphs. Journal of Combinatorial Theory, Series B, 121:146–172, 2016.

[2] Z. Füredi. New asymptotics for bipartite Turán numbers. Journal of Combinatorial Theory, Series A, 75(1):141–144, 1996.

[3] D. Gerbner. Generalized Turán problems for $K_{2,t}$. The Electronic Journal of Combinatorics, 30:P1.34, 2023.

[4] D. Gerbner and C. Palmer. Counting copies of a fixed subgraph in $F$ -free graphs. European Journal of Combinatorics, 82:103001, 2019.

[5] D. Gerbner and B. Patkós. Generalized Tur´an problems for complete bipartite graphs. Graphs and Combinatorics, 38(5):164, 2022.

## The strong chromatic number of star-free 3-graphs

by Dániel Gerbner

It is well-known that the largest chromatic number of $F$-free graphs is at most $|V(F)|-1$ if $F$ is a forest and unbounded otherwise. Loh [1] proved an analogous result for the weak chromatic number of uniform hypergraphs, where weak coloring means that no color class contains a hyperedge. In a strong coloring, no hyperedge is incident to two vertices of the same color. It is not hard to see that the largest strong chromatic number of $F$-free hypergraphs is unbounded unless $F$ is a single edge or $F$ is a 3-uniform star, i.e., $F$ consists of $k$ 3-edges with pairwise intersections $v$ for some vertex $v$ and integer $k$. For such hypergraphs, we proved [2] the upper bound $2k^2 + k/2 - 1$, and the lower bound $2k^2 - 2k - 2$ for infinitely many $k$.

**Problem 1.** *Close the gap.*

Note that a slight improvement on the upper bound is easy.

# References

[1] P.-S. Loh, A note on embedding hypertrees, Electron. J. Combin., 16 (2009).

[2] Y. Wang, M. Duan, D. Gerbner, and H. Hama Karim, On the largest chromatic number of F-free hypergraphs, preprint, 2026, arXiv:2604.21551.

## Size Ramsey problems for hypergraphs and arithmetic progressions

by Nina Kamčev

We write $G \to H$ if every red-blue colouring of the edges of $G$ contains a monochromatic copy of $H$, and say that $G$ *is Ramsey for* $H$ if this holds. Let $r(G)$ be the usual Ramsey number of $G$, and $r(n) = r(K_n)$. A classical result, due to Chvátal, says that if $G \to K_n$, then $|E(G)| \geq \binom{r(n)}{2}$. I.e., the graph $K_{r(n)}$ minimises the number of edges subject to being $K_n$-Ramsey.

We suspect that the analogous statements do not hold in more general settings, e.g. APs and hypergraphs. Specifically, we have the following problems.

**Problem 1** (Graham; see [1])**.** *Let $W(k)$ be the usual two-colour van der Waerden number, i.e. the smallest $N$ such that every two-colouring of $[N]$ contains a monochromatic $k$-term arithmetic progression. Let $W^*(k)$ be the minimum size of a finite set $A \subset \mathbb{Z}$ such that every two-colouring of $A$ contains a monochromatic $k$-term arithmetic progression.*

*Is $W(k) - W^*(k)$ unbounded as $k \to \infty$?*

By definition, $W^*(k) \leq W(k)$, and we do not know if the inequality is strict in general. The first non-trivial evidence that the inequality can be strict is $W^*(4) \leq 27 < 35 = W(4)$.

**Problem 2** (Dudek–La Fleur–Mubayi–Rödl [2])**.** *Let $k \geq 3$, let $N = r(K_n^{(k)})$, and let $K_N^{(k)-}$ be the complete $k$-uniform hypergraph on $N$ vertices with one edge removed. Is it true that $K_N^{(k)-} \to K_n^{(k)}$?*

The first non-trivial case is known by a computational result of McKay: we have $R(K_4^{(3)}) = 13$, and $K_{13}^{(3)-} \to K_4^{(3)}$.

It would also be interesting to understand whether the analogue of Chvátal's theorem still holds in the canonical Ramsey setting.

**Problem 3.** *Let $\mathrm{CR}(n)$ be the least $N$ such that every colouring of the edges of $K_N$ with arbitrary colours contains an $n$-vertex clique whose edge-colouring is canonical, in the sense of Erdős–Rado [3]. If a graph $G$ has the property that every edge-colouring of $G$ contains a canonical $K_n$-copy, is it true that*

$$|E(G)| \geq \binom{\mathrm{CR}(n)}{2}?$$

# References


[1] E. Croot and V. F. Lev. Open problems in additive combinatorics. In *Additive Combinatorics*, CRM Proc. Lecture Notes 43, Amer. Math. Soc., Providence, RI, 2007, 207–233.

[2] A. Dudek, S. La Fleur, D. Mubayi and V. Rödl. On the size-Ramsey number of hypergraphs. *J. Graph Theory* 86 (2017), no. 1, 104–121.

[3] P. Erdős and R. Rado. A combinatorial theorem. *J. London Math. Soc.* 25 (1950), 249–255.

[4] B. D. McKay. A class of Ramsey-extremal hypergraphs. *Trans. Comb.* 6 (2017), no. 3, 37–43.

## Local thrackles

by Balázs Keszegh

We call a simple topological graph with no plane path of length 3 a *local thrackle*. Note that every thrackle is a local thrackle, but the two notions do differ. In [1] the following are shown:

**Theorem 1.** *A local thrackle on $n$ vertices and with $x$-monotone edges has at most $\frac{4}{3}n$ edges, while there exist local thrackles on $n$ vertices with $x$-monotone edges and with $\frac{6}{5}n - O(1)$ edges.*

**Theorem 2.** *A local thrackle on $n$ vertices has at most $O(n^{4/3})$ edges, while there exist local thrackles on $n$ vertices and with $2n - O(\sqrt{n})$ edges.*

For larger paths we only know the following:

**Theorem 3.** *Every $n$-vertex simple topological graph not containing a plane path of length $k$ has at most $O_k(n^{2-8/k^4})$ edges.*

**Problem 1.** *Improve the above bounds.*

# References

[1] B. Keszegh, A. Suk, G. Tardos, J. Zeng, Unavoidable patterns and plane paths in dense topological graphs. Proceedings of SoCG 2026, Proceedings of SoCG 2026, LIPIcs 367 (2026), 63:1-63:15 `https://arxiv.org/abs/2512.04795`

## Balanced Hyperplane Covers of the Boolean Cube

by Zoltán Lóránt Nagy

Let $Q_n = \{0,1\}^n$, and write $Q_n^* = Q_n \setminus \{0\}$. Alon and Füredi [1] proved that $n$ affine hyperplanes are necessary and sufficient to cover $Q_n^*$ while avoiding the origin. For example, the $n$ coordinate hyperplanes $x_i = 1$ do the job, but each has size $2^{n-1}$.

We are interested in a balanced version of this equality case. For an affine hyperplane $H$, call

$$|H \cap Q_n|$$

its load, and define $F(n)$ to be the smallest possible value of the largest load among all $n$-hyperplane covers of $Q_n^*$ avoiding the origin. By averaging,

$$F(n) \geq \frac{2^n - 1}{n}.$$

On the other hand, the Hamming-weight layers

$$x_1 + \cdots + x_n = j, \qquad j = 1, \ldots, n,$$

cover $Q_n^*$, avoid the origin, and give

$$F(n) \leq \binom{n}{\lfloor n/2 \rfloor} = O(2^n/\sqrt{n}).$$

Thus the trivial exponent $1/2$ is immediate, while the averaging lower bound suggests that exponent 1 might be the natural limit.

**Problem 1.** *Does there exist an absolute constant $C$ such that*

$$F(n) \leq C\frac{2^n}{n}$$

*for infinitely many $n$, or perhaps for all $n$?*

A weaker version is already interesting.

**Problem 2.** *Is there some $\beta > 1/2$ and an absolute constant $C_\beta$ such that*

$$F(n) \leq C_\beta \frac{2^n}{n^\beta}$$

*for infinitely many $n$?*

Small-dimensional searches suggest the answer might be affirmative even for Problem 1.

# References

[1] N. Alon and Z. Füredi, Covering the cube by affine hyperplanes. European Journal of Combinatorics 14(2), 79–83, 1993.

## Legitimate Colorings of Finite Affine and Projective Spaces

by Zoltán Lóránt Nagy

Let $\Pi$ be a finite incidence geometry, and let $\mathcal{L}$ denote its set of lines. A coloring of the points of $\Pi$ is called legitimate if no two distinct lines have the same color distribution. In other words, for any two distinct lines $L_1, L_2 \in \mathcal{L}$, there is a color $c$ such that

$$|L_1 \cap C_c| \neq |L_2 \cap C_c|,$$

where $C_c$ denotes the set of points of color $c$. Let $\chi_L(\Pi)$ be the minimum number of colors in a legitimate coloring of $\Pi$.

Alon and Füredi [1] studied legitimate colorings of finite projective planes. In particular, they proved that for projective planes $PG(2,q)$, the number $\chi_L(PG(2,q))$ is bounded from above by an absolute constant, independent of $q$.

We ask for higher-dimensional analogues. For finite affine and projective spaces over $\mathbb{F}_q$, define

$$A_q(n) = \chi_L(AG(n,q)), \qquad P_q(n) = \chi_L(PG(n,q)).$$

How do these functions grow with the dimension $n$ and the field size $q$?

Even the affine spaces over $\mathbb{F}_3$ seem to be of particular interest. In this case every affine line has three points, so a legitimate coloring of $AG(n,3) = \mathbb{F}_3^n$ assigns distinct unordered color triples to all affine lines.

For this case, a simple double counting argument (from below) and alteration method (from above) gives

$$0.95 \cdot 3^n \geq A_3(n) \geq (1-o(1))3^{\frac{2n}{3}}.$$

More generally, one may ask the same question for arbitrary finite fields.

**Problem 1.** *For fixed $q$, determine the order of magnitude of*

$$A_q(n) = \chi_L(AG(n,q))$$

*as $n \to \infty$. How does the answer depend on $q$? Improve on $O_q(q^{4n/(q+1)}) = A_q(n) \geq ((q-2)! + o(1))^{1/q}\, q^{2n/q}$.*

The projective version is equally natural.

**Problem 2.** *For fixed $q$, determine the order of magnitude of*

$$P_q(n) = \chi_L(PG(n,q))$$

*as $n \to \infty$, and improve $O_q(v_P(n)^{4/(q+2)}) \geq P_q(n) \geq ((q-1)! + o(1))^{1/(q+1)} \left(\frac{q^{n+1}-1}{q-1}\right)^{2/(q+1)}$.*

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## Isosceles triangles in the square grid

by Zoltán Lóránt Nagy

Let

$$\alpha_{\mathrm{iso}}(n) = \max\{|A| : A \subseteq [n] \times [n] \text{ contains no non-degenerate isosceles triangle}\}.$$

Here an isosceles triangle is counted with a distinguished apex: distinct points $a, b, c \in A$ with

$$\|a - b\|^2 = \|a - c\|^2.$$

Such problems were asked by Erdős and Purdy, see [2]. Degenerate collinear triples may be added to the forbidden family; they are not the main issue in the questions below. We only know that $\Omega(n/\log n) = \alpha_{\mathrm{iso}}(n) = O(n^2 \exp(-c \log n/\log\log n))$ for some $c > 0$ [1, 3].

**Problem 1.** *Is it true that*

$$\alpha_{\mathrm{iso}}(n) \gg n?$$

*Equivalently, does there exist an absolute constant $c > 0$ such that every sufficiently large $n \times n$ grid contains an isosceles-triangle-free subset of size at least $cn$?*

A natural formulation uses perfect matchings. Identify a point $(i, j)$ with the edge $ij$ of $K_{n,n}$. A perfect matching $P$ is the graph of a permutation and hence gives exactly $n$ grid points. For $a \in P$, put

$$L_a(P) = \sum_{\rho} \binom{|P \cap C(a, \rho)|}{2}, \qquad C(a, \rho) = \{x \in [n]^2 : \|x - a\|^2 = \rho\},$$

and let

$$T(P) = \sum_{a \in P} L_a(P).$$

Thus $T(P)$ is the number of isosceles triples in the permutation graph, counted by apex.

**Problem 2** (Energy form)**.** *Is there a perfect matching $P \subseteq K_{n,n}$ such that*

$$T(P) = O(n)?$$

*If so, then Problem 1 follows: keep each point of $P$ independently with a small fixed probability $\theta$, and then delete one point from every surviving isosceles triangle. Since*

$$\mathbb{E}|P_\theta| = \theta n, \qquad \mathbb{E}T(P_\theta) \le \theta^3 T(P) = O(\theta^3 n),$$

*choosing $\theta$ sufficiently small leaves $\Omega(n)$ points after deletion.*

The random perfect matching has $\mathbb{E}T(P) \asymp n \log n$.

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## $C$-diversity of hypergraphs

by Cory Palmer

The $C$*-diversity* of an intersecting $r$-uniform hypergraph $\mathcal{H} \subseteq \binom{[n]}{r}$ is defined as

$$d_C(\mathcal{H}) := |\mathcal{H}| - C \cdot \Delta(\mathcal{H})$$

where $\Delta(\mathcal{H})$ is the maximum degree of $\mathcal{H}$.

When $C < 3/2$, the $C$-diversity is maximized by a "two out of three" hypergraph which is all $r$-edges intersecting a fixed 3-set in at least two vertices. When $3/2 \leq C < 7/4$, the $C$-diversity is maximized by the following hypergraph: fix a Fano plane and take every $r$-edge that includes an edge of the Fano plane (see [3]).

For $C \geq 7/4$, the question is open. A likely extremal construction is an analogue of the Fano construction with an order-3 finite projective plane.

**Problem 1.** *Determine the maximum $C$-diversity for $C \geq 7/4$.*

A related question:

**Problem 2** (Kupavskii-Zakharov [1])**.** *Determine the largest possible $C > 1$ such that*

$$C \cdot |\mathcal{H}| - (C-1) \cdot \Delta(\mathcal{H}) \leq \binom{n-1}{r-1}.$$

It would also be interesting to improve the superexponential thresholds on $n$ from [3]. Perhaps as:

**Problem 3.** *Use the "spread approximation method" of Kupavskii and Zakharov [2] to improve the threshold on $n$ for the existing $3/2 \leq C < 7/4$ results.*

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## Odd Area

by Dömötör

Define the odd area of a measurable set $S \subset \mathbb{R}^d$, as the least $\mu$ such that the area of the symmetric difference of any odd number of translates of $S$ is always at least $\mu$. In other words, the odd area is the smallest possible measure of points of $\mathbb{R}^d$ that are covered by an odd number of translates in an odd-size family. It is a classic puzzle for intervals/squares that their odd area is always their area, while the general definition appeared in Pak [2] and later Pinchasi [3] popularized the problem a lot and conjectured that, similarly, the odd area of a unit disk is always at least $\pi$. However, this has been very recently disproved in [5] with computer help. But the following question is left open.

**Problem 1.** *Is the odd area of a disk positive?*

Also, one might want to generalize the counterexample to other sets.

**Problem 2.** *For which sets is their odd area equal to their area?*

What happens if instead of $\mathbb{R}^2$ we work on a torus?

**Problem 3.** *Is the odd area of a disk on a torus positive?*

In the last question if the torus is so small that the disk overlaps with itself, then the problem is meant so that we take its "symmetric difference" with itself.

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## Orientations with Almost-Sources and Almost-Sinks

by Laurentiu Ploscaru

Given an orientation of a graph $G$, let $d^+(v)$ and $d^-(v)$ denote the outdegree and indegree of a vertex $v$, respectively.

**Problem 1.** *Does every planar triangulation admit an orientation such that, for every vertex $v$, one has*

$$\min\big(d^+(v), d^-(v)\big) \leq 1?$$

If above we require $\min\big(d^+(v), d^-(v)\big) = 0$, this is clearly impossible since there are planar graph which are not bipartite. On the other hand, if we require $\min\big(d^+(v), d^-(v)\big) \leq 2$, this is a very easy exercise as every planar graph has a vertex of degree at most 5 and one can induct.

The problem admits an equivalent formulation in terms of vertex partitions into pseudoforests. A pseudoforest is a graph in which every connected component contains at most one cycle.

For every graph $G$, the following statements are equivalent:

1. $G$ has an orientation such that $\min\big(d^+(v), d^-(v)\big) \leq 1$ for every vertex $v$.
2. The vertices of $G$ can be partitioned as $V(G) = A \cup B$ so that both induced subgraphs $G[A]$ and $G[B]$ are pseudoforests.

If such a partition exists for a triangulation on $n$ vertices, then $e(A) \leq |A|$ and $e(B) \leq |B|$, hence $e(A, B) \geq (3n - 6) - n = 2n - 6$, which is very close to $2n - 4$, the maximum number of edges a bipartite planar graph can have.

It is known that there are planar triangulations whose vertices cannot be partitioned into two sets, each inducing a forest, but what about pseudoforests?

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## Decomposing graphs into comparability graphs

Géza Tóth

For any graph $G$, let $c(G)$ be the smallest number $c$ with the property that $G$ can be obtained as the union of $c$ comparability graphs.

Similarly, let $p(G)$ be the smallest number $p$ with the property that $G$ can be obtained as the *edge-disjoint* union of $p$ comparability graphs.

Clearly, for any graph $G$, $p(G) \geq c(G)$. It is not hard to see that there are graphs $G$ with arbitrarily large $p(G)$ and $c(G)$. It is also true that there are perfect graphs with this property [1].

**Problem 1.** *Is there a perfect graph $G$ such that $p(G) > c(G)$?*

Marits, Gyárfás and Tóth (manuscript) constructed a (non-perfect) graph $G$ with $p(G) = 3$ and $c(G) = 2$. How far can this construction be improved?

**Problem 2.** *Is there a function $f$ such that for any graph $G$, $p(G) \leq f(c(G))$? In particular, is there a graph $G$ with $p(G) = 4$ and $c(G) = 2$?*

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## The Small-Color Regime for Color-Avoiding Monotone Paths

by Vizer Máté

For integers $k, q$ and $1 \le p < q$, let $A_k(n; q, p)$ be the least $N$ such that every $q$-coloring of the $k$-sets of $[N]$ contains a tight monotone path with $n$ edges whose edges use at most $p$ colors.

This problem is a color-avoiding version of the Moshkovitz–Shapira theorem on monochromatic monotone paths. Mulrenin, Pohoata and Zakharov proved that, in the range $p > q/2$, the usual tower-height upper bound drops by one. More recently, Choi, Lee and Tran showed that the true tower height can be much smaller: for every fixed $p$ and all sufficiently large $q$, the exact tower height of $A_k(n; q, p)$ is

$$\left\lceil \frac{k-1}{p} \right\rceil.$$

Their lower-bound construction, however, applies when $q$ is sufficiently large compared with $p$. Thus it does not settle the opposite, small-color regime $q < 2p$, which is precisely the original color-avoiding range of Mulrenin–Pohoata–Zakharov.

**Problem 1.** *Determine the tower height of $A_k(n; q, p)$ for fixed $k \ge 4$ and*

$$p < q < 2p.$$

*In particular, is it always true that*

$$A_k(n; q, p) \quad \textit{has tower height} \quad \left\lceil \frac{k-1}{p} \right\rceil ?$$

*Equivalently, can the Choi–Lee–Tran lower bound be extended to the small-color regime $q < 2p$, or do new phenomena occur when the number of available colors is only slightly larger than the number of colors allowed on the path?*

The first open cases already seem interesting. For example, what is the exact tower height of $A_k(n; 3, 2)$ for $k \ge 4$? More generally, what happens for one-color avoidance, that is, for $A_k(n; q, q-1)$?

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## Cycles of Length 0 Modulo $k$

by Vizer Máté

For an integer $k \geq 2$, let $c_{0,k}$ denote the smallest constant such that every $n$-vertex graph with at least $c_{0,k}n$ edges contains a cycle whose length is divisible by $k$.

This problem goes back to Burr and Erdős and has recently seen major progress. The odd-modulus case was solved very recently, while the even-modulus case remains open.

**Theorem 1** (Bai–Grzesik–Li–Prorok)**.** *For every odd integer $k$,*

$$c_{0,k} = k - 1.$$

The behavior for even $k$ appears substantially more complicated.

**Problem 1.** *Determine $c_{0,k}$ for even integers $k \geq 6$.*

*In particular, is it true that*

$$c_{0,k} = \frac{k-1}{2}$$

*for every even integer $k \geq 6$?*

A promising direction is to develop stability results describing the structure of graphs with close to $c_{0,k}n$ edges but containing no cycle of length divisible by $k$.

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# Number of distinct spanning trees in $(n, d, \lambda)$ graphs

Yiting Wang

How many non-isomorphic spanning trees are there in a $d$-regular graph? Lee [1] proved there are at least $e^{n/2000}$ non-isomorphic spanning trees in any connected $d$-regular graphs. In fact, he proved a stronger anti-concentration result, which directly implies the above result:

**Theorem 1** (Lee [1])**.** *Let $G$ be a $d$-regular graph on $n$ vertices and $T$ be an arbitrary spanning tree in $G$. Let $T'$ be a spanning tree of $G$ chosen uniformly at random from all spanning trees from $G$. Then,*

$$\mathbb{P}(T \simeq_{\text{iso}} T') \leq e^{-n/2000}.$$

The problem I am interested in (also one of the open problems in Lee's paper) is to find under which conditions of $d$ and $\lambda$ can we show the optimal anticoncentration bound for an $(n, d, \lambda)$-graph:

**Problem 1.** *Let $G$ be an $(n, d, \lambda)$-graph and $\varepsilon > 0$. Under which condition of $d$ and $\lambda$ (depending on $\varepsilon$) can we show that*

$$\mathbb{P}(T \simeq_{\text{iso}} T') \leq e^{-(1-\varepsilon)n},$$

*where $T$ is any arbitrary spanning tree in $G$ and $T'$ is a uniformly random spanning tree in $G$?*

This bound is asymptotically optimal up to subexponential factors. Indeed, it is easy to see the total number of labelled spanning trees is at most $d^{n-1}$. Moreover, there are $d$-regular graphs on $n$ vertices with $(d/e)^{(1-o_d(1))n}$ many Hamilton paths.

## References

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