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Title: Thom polynomials relative to prescribed maps between codimension-zero submanifolds

Abstract: Thom polynomials are universal cohomological obstructions to the appearance of singularities of given types in differentiable maps. As an application, various invariants of immersions have been expressed in terms of singularities of extensions of immersions (known as singular Seifert surfaces). To place these results in a unified framework, I would like to aim in this talk to establish the foundation of a relative version of Thom polynomial theory. Our result consists of three parts.

(1) We introduce the notion of relative Thom polynomials, which are relative cohomological obstructions for extensions of prescribed maps between codimension-zero submanifolds that avoid singularities of given types.

(2) We show a structure theorem for relative Thom polynomials when the prescribed map is a framed immersion. It expresses them as the sum of the naive substitution of Kervaire's relative characteristic classes into the absolute Thom polynomial and a correction term. As a consequence, the correction term forms a regular homotopy invariant of the prescribed map.

(3) We determine correction terms in several cases, not only reinterpreting earlier works as formulas of relative Thom polynomials but also applying our framework to the type A_1. We observe that these terms vanish or recover classical invariants.

If time permits, I would like to talk about future directions.