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Title: Multifold degeneracy points of quantum systems and singularities of matrix varieties

Abstract: In the talk we answer a physically motivated question using the tools of local algebraic geometry.

In quantum mechanics and condensed matter physics, several systems are described by a parameter-dependent Hamiltonian, which is a smooth map of a manifold M to the space of hermitian matrices. A point of M is called degeneracy point if the corresponding hermitian matrix has coincident eigenvalues — in other words, the energy levels cross each other. In the case of a three dimensional parameter space M, the generic degeneracy points are called Weyl points; they are isolated 2-fold degeneracy points (i.e., two eigenvalues coincide). Multifold degeneracy points may appear due to the symmetries of the system. This happens for example in the case of a spinful particle in an external magnetic field, or in the electronic band structure of a crystalline material with certain lattice symmetries. A generic symmetry breaking perturbation splits the multifold degeneracy point into Weyl points. Our main target quantity is the number of these Weyl points.

We complexify the variety of matrices with coincident eigenvalues. The matrices corresponding to a multifold degeneracy point are singular points of this variety. We compute the multiplicity at these points to obtain an upper bound for the number of Weyl points born from a multifold degeneracy point. Although for this we adapt classical notions from algebraic geometry, they are put together in an unusual way to fit our setup. In particular, we show that the variety of matrices with eigenvalues of geometric multiplicity at least two is not Cohen–Macaulay, hence, the computation of the multiplicity requires comparing our variety with classical determinantal varieties. The corresponding Hilbert sequence is computed via the Gulliksen–Negard resolution.

Joint work in progress with György Frank, Dániel Varjas, Alex Hof and András Pályi.