

Relative Lattice Cohomology

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ABSTRACT. Lattice cohomology theory encompasses a family of invariants associated to isolated singularities (closely related to Heegaard Floer theory). Fundamentally, it associates a “landscape” of numerical invariants to a geometric object and then studies the algebraic topology of the filtered space arising from its sublevel sets. The first prototype appeared in 2008 when Némethi introduced the Topological lattice cohomology of normal surface singularities, followed in 2023 by Ágoston and Némethi’s construction of the Analytic lattice cohomology of curve singularities.

This talk introduces a new “relative” theory interpolating between these two versions: given a curve singularity (C, o) embedded into a rational surface singularity (X, o) , I will present the construction of a well-defined (resolution-independent) relative lattice cohomology module $\mathbb{H}^*(X, C, o)$. I will then discuss two applications of this new tool: deriving a classical analytic invariant (the semi-group of values) of the curve from its embedded topology, and establishing a lattice-cohomological obstruction to embedding irreducible curve singularities into simple surface germs as Cartier divisors.