Bergman spaces on algebraic curves

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Let $U \subseteq \mathbb{C}^n$ be an open subset. The Bergman space $A^2(U)$ is the Hilbert space of holomorphic functions on U which are square-integrable with respect to the Euclidean volume form dV. A theorem of Wiegerinck asserts that in dimension one the Bergman space of an open subset $U \subset \mathbb{C}$ is either infinite-dimensional or trivial. Recently, this has been generalized to holomorphic vector bundles over the projective line by R. Szőke and later to vector bundles over any compact Riemann surface by A. Gallagher, P. Gupta, L. Vivas.

The topic of this talk will be to prove our even more recent result extending this dichotomy to the case of affine and projective algebraic curves. To do so I will recall the fundamentals of the theory of algebraic curves and introduce Bergman spaces. Then I discuss the extension of the results above to the case of certain singular volume forms on a Riemann surface associated to divisors and show how this yields versions of Wiegerinck's theorem for algebraic curves.

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