On a refinement of Marcinkiewicz-Zygmund type inequalities

The classical Marcinkiewicz-Zygmund inequality frequently used in various areas of analysis states that for any univariate trigonometric polynomial $t_n$ of degree at most $n$ and every $1 \leq q < \infty$ we have

$$\int |t_n|^q \sim \frac{1}{n} \sum_{s=0}^{2n} |t_n \left( \frac{2\pi s}{2n+1} \right)|^q$$

(1)

where the constants involved in the above equivalence relation depend only on $q$.

The main goal of this talk is to present a refinement of Marcinkiewicz-Zygmund type inequalities of the form

$$\frac{1}{2} \sum_{j=0}^{nm-1} (x_{j+1} - x_{j-1}) w(x_j)|t_n(x_j)|^q = (1 + O(m^{-2})) \int_{-\pi}^{\pi} w(x)|t_n(x)|^q dx,$$

where $t_n$ is any trigonometric polynomial of degree at most $n$, $2 \leq q < \infty$, $\max_{0 \leq j \leq N-1}(x_{j+1} - x_{j}) = O\left(\frac{1}{mn}\right)$, $m, n \in \mathbb{N}$ and $w$ is a Jacobi type weight. Moreover, the term $O(m^{-2})$ is shown to be in general sharp. In addition, similar multivariate results will be also discussed.