Isolation of graphs

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If $\mathcal{F}$ is a set of graphs and $F$ is a copy of a graph in $\mathcal{F}$, then we call $F$ an $\mathcal{F}$-graph. If $G$ is a graph and $D \subseteq V(G)$ such that $G - N[D]$ (the graph obtained from $G$ by deleting the closed neighbourhood $N[D]$ of $D$) contains no $\mathcal{F}$-graph, then $D$ is called an $\mathcal{F}$-isolating set of $G$. The size of a smallest $\mathcal{F}$-isolating set of $G$ is denoted by $\iota(G, \mathcal{F})$ and called the $\mathcal{F}$-isolation number of $G$.

The study of isolating sets was introduced by Caro and Hansberg [3]. It is a natural generalization of the classical domination problem. Indeed, $D$ is a $\{K_1\}$-isolating set of $G$ if and only if $D$ is a dominating set of $G$ (that is, $N[D] = V(G)$), so the $\{K_1\}$-isolation number is the domination number.

Let $G$ be a connected $n$-vertex graph. In [1], the speaker showed that if $G$ is not a triangle and $C$ is the set of all cycles, then $\iota(G, C) \leq \frac{n}{4}$ (that is, $V(G)$ has a subset $D$ such that $|D| \leq \frac{n}{4}$ and $G - N[D]$ contains no cycle). Together with Fenech and Kaemawichanurat [2], he also showed that $\iota(G, \{K_k\}) \leq \frac{n}{k+1}$ if $G$ is neither a $k$-clique nor a 5-cycle. The case $k=1$ is a classical domination bound of Ore [6], and the case $k=2$ is a result of Caro and Hansberg [3]. The bounds are sharp and settle two problems in [3].

Let $G$ be a maximal outerplanar $n$-vertex graph. Fisk’s short proof [5] of Chvátal’s Art Gallery Theorem (included in the book Proofs from THE BOOK) established that the domination number of $G$ is at most $\frac{n}{3}$. Kaemawichanurat and the speaker recently generalized this result by showing that $\iota(G, \{S_{k+1}\}) \leq \frac{n}{k+4}$, where $S_{k+1}$ is the star with $k+1$ leaves. Note that $D$ is a $\{S_{k+1}\}$-isolating set of $G$ if and only if the maximum degree of $G - N[D]$ is at most $k$.

References