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I. Main duties of the research unit in 2017

The fundamental goal of the Alfréd Rényi Institute of Mathematics is to pursue research of high international standing in pure mathematics. The institute is an important center of mathematics internationally. Fellows of the institute received several Hungarian and international recognitions in 2017. One researcher received the Széchenyi Prize, another one the Prima Prize. A younger fellow got the Erdős Prize from the Mathematics Section of the Hungarian Academy of Sciences, another one was awarded the Bolyai medal by the Board of the János Bolyai research fellowship, and two others obtained the Grünwald medal from the János Bolyai Mathematical Society. An emeritus research professor of the institute received the honorary doctorate from Eötvös University. It is an outstanding international recognition that four members of the institute got an invitation to speak at the International Congress of Mathematicians in 2018. Researchers of the institute continued to successfully apply for grants from the European Research Council; in 2017 two further grants were obtained: one Advanced Grant and one Consolidator Grant. In 2017 six research groups in the institute were funded by ERC. The institute is also very successful in the Momentum program of the Academy. In 2017 the seventh Momentum research group was launched, this time on a topic in number theory.

Researchers of the institute published 160 scientific papers during the year. The most important results have appeared in the most significant international mathematical periodicals (Annals of Mathematics, Duke Mathematical Journal, Journal of the European Mathematical Society, Annals of Probability, Mathematische Annalen, Journal für die reine und angewandte Mathematik, Inventiones Mathematicae, etc.).

The scientific tasks of the institute concentrate on fundamental research. However, significant efforts were devoted to some topics in applied mathematics as well. The main applied areas investigated in the institute are the theory of large networks (within this area their new direction is “deep learning” – a fundamental paradigm in artificial intelligence), cryptography, as well as bioinformatics, but mathematical statistics has also been applied in several related areas (e.g., in environmental science and astronomy).

The institute is organized in the framework of 9 scientific departments, 6 Momentum research groups, and the recently created research group on mathematics education. The research topics of the institute are continuously adjusted to the most recent developments in mathematics.

II. Outstanding research and other results in 2017

a) Outstanding research results

Low Dimensional Topology Momentum Research Group

They continued the studies regarding the very successful knot invariant Upsilon, and the invariant has been extended for knots which lie in closed three-manifolds having torsion first homology.

The Hitchin fibrations on moduli spaces of Higgs bundles have been studied. They determined the configuration of all singular fibers in many cases. This description revealed an interesting wall-crossing phenomenon.

The cobordism groups of fold maps into the plane have been computed, for example, in the case of oriented manifolds of dimension divisible by 4. Using this, they proved a formula about the signature of the manifold and properties of the singular set of the map.

By revising previous results regarding classification of tight and fillable contact structures on certain Seifert fibered manifolds, they reached some further understanding of the Ozsváth-Szabó contact invariants of contact three-manifolds which bound star-shaped plumbings. They identified the characteristic cohomology element which presents the contact invariant in the lattice homology.

They studied the Legendrian knot invariant LOSS and successfully generalized these invariants to links and applied them to prove some results on non-loose Legendrian and transverse links in over-twisted contact structures.

Automorph Momentum Research Group

They investigated the global maximum of cusp forms defined on n x n matrix groups, and where the maximum can occur within the matrix group. For $n = 3$ they gave a concrete upper bound in terms of the Laplace eigenvalue, while for general n they proved that the maximum can be bounded from above by a power of the Laplace eigenvalue in which the exponent is a cubic polynomial of n. They also succeeded in explicating certain subconvex bounds for automorphic L-functions (belonging to the case $n = 2$). These bounds are useful in the various arithmetic distribution problems. In another direction, they generalized a variant of Jensen's formula to automorphic functions on hyperbolic space, creating a link between the average value and the zeros of the function. They answered a question of Komornik-Pedicini-Pethő in the topic of non-integer base number systems and random power series: the peculiar infinite binary sequences discovered by these authors are in fact common and typical.

Groups and Graphs Momentum Research Group

They investigated point processes on locally compact groups. It turned out that every free pmp action can be realized by a point process. The Palm relation allows one to define the cost of a point process and relate it to rank gradient. Using spectral theory, they showed that a group is non-amenable if and only if the Poisson point process has spectral gap.

They showed that for higher rank locally symmetric spaces, the mod 2 homology grows subexponentially in the volume. The proof uses the surprising new result that every mod 2 homology class contains an element with sublinear total length in the volume.

They gave rigorous proofs for the observations of Liu, Slotine and Barabási on network controllability, which where based on statistical physics heuristics and numerical results. They showed that the directed matching ratio (the proportion of the size of a maximal directed matching) converges almost surely to a constant for the most widely used families of scale-free networks. They further proved that the directed matching ratio of random directed graphs given by a fixed degree sequence concentrates around its mean.

The general probabilistic model of information theory is a common generalization of classical and quantum information theory, in which the state space is an arbitrary convex body. It was shown that a system whose state space is an ellipsoid (of arbitrary dimension) can be simulated (in a well-defined sense) by a classical 2-state system. More generally, an upper bound was given for the number of classical states needed to simulate systems whose state spaces are unit balls of certain norms.

Large Networks Momentum Research Group

In 2017 one of the main research directions of the group was to develop limit theories for various kinds of structures. Beyond networks they studied the limits of various operators. They started to develop a generalization of graph limit theory in the frame of which only certain subgraph densities are relevant; however, they can be calculated in broader operator classes. They examined limits of dense random matrices. In particular, they obtained strong restrictions on the entry distributions of almost eigenvectors in the Ginibre ensemble.

They had a major breakthrough in their work concerning higher order Fourier analysis. They were able to give a full characterization for the characteristic factors of the Host-Kra seminorms in nilpotent group actions. They proved theorems that connect the ergodic theory and the higher order Fourier analysis. They developed a general limit theory that works for functions on Abelian groups and more generally for functions on nilspaces.

They made significant progress in Sidorenko's conjecture. They managed to prove the conjecture in an important case. This result shows a promising new way of studying the general case.

They put more effort into the development of the mathematical foundations of artificial intelligence. One interesting direction is to study non-linear dimension reductions and holographic functions. This is deeply connected to structural limit theories. In another, more practical direction, they developed a regularization method that helps to reduce the training set in deep learning.

They started a project on the reconstruction of incomplete data. This project is a continuation of a project from the past years in quadratic Fourier analysis but it also incorporates methods from deep learning. Quite surprisingly they were able to give a general explanation for sporadic phenomena observed by other people in graph reconstruction.

Financial Mathematics Momentum Research Group

They studied optimal investment problems under model uncertainty where investors aim at maximizing their satisfaction in the worst-case scenario. Solving such problems leads to hedging strategies which work well even if the statistical properties of the prices are difficult to determine from available data. They managed to establish the existence of optimal portfolios in various model classes in discrete and continuous time.

Participating in a project of the Alan Turing Institute, London, they continued work on stochastic gradient methods. The purpose is to study online algorithms for finding a global maximum in high-dimensional problems with many local maxima (e.g. finding the best parameters for tuning a neural network). They managed to make progress from independent data samples (such as a sequence of coin tossing) towards general, realistic, stationary data samples.

Random Spectra Momentum Research Group

Just like in the central limit theorem, random matrices converge to a random operator, the analogy of normal distribution. This operator was defined and it was shown that random unitary matrices converge there at high speed.

The classical bubble sort, applied to a sequence of numbers in reverse order, can be understood as a shortest path between the reverse and identity permutations in the adjacent swap Cayley graph of the symmetric group. A decade long conjecture suggests that a typical such path behaves in a different, unexpected way. A typical such path has a halfway permutation matrix which looks like the projection of a 3-dimensional sphere. It was shown that most of the ones in the permutation matrix are located within the circle. Later, the original conjectures were proved.

Local algorithms can be understood through so-called factor-of-iid processes on infinite regular trees. In these processes, the correlation decays exponentially. It was shown that the same holds for mutual information, with a surprisingly different exponential rate.

Benjamini asked whether the 3-dimensional space has invariant random subsets whose connectivity graph is a 3-regular tree. This was shown to be true, despite the fact that the two systems are crucially different.

A breakthrough was achieved in a classic problem of Erdős: how many points can be put in the *d*-dimensional space so that all triples form an acute triangle? The previous best lower bound was exponentially weak. The new construction differs from the known upper bound only by a factor of 2.

Didactics Research Group

The members of the Didactics Group worked in the Content Pedagogy Research Program of the Hungarian Academy of Sciences.

They organized didactical events for teachers of special mathematics classes, and coordinated the modernization and update of their curriculum.

In the spring they provided special mathematical workshops in 52 different high schools all around the country. In September they launched a 2-year-long talent nurturing program for the 70 most talented students.

They led extra maths groups of talented, underprivileged students in the 7th, 8th and 10th district of Budapest.

They organized 25 weekend maths camps for talented students in 2017. Approximately 250 students participated at least one of these weekend camps. In the summer they organized two summer mathematics camps: MaMuT (Camp of Mathematical Amusements) and MaMuT2. Talented students from age 10 to 18 with outstanding results in Hungarian and international mathematical competitions were invited to improve their mathematical knowledge and skills.

Many talented students were nurtured by the members of the group, in the form of private tutoring or small groups.

They taught the basic principles of discovery learning in mathematics at Eötvös Loránd University and at Budapest Semesters in Mathematics Education.

Department of Algebra

It was known that the Noether number of a subgroup is bounded by the Noether number of the full group. Now it has been proved that the Noether number of a proper subgroup is strictly smaller than the Noether number of the full group.

A presentation by generators and relations of the symmetric tensor powers of a commutative algebra was worked out. This was applied to get information on a minimal homogeneous generating system of the symmetric tensor power algebra in the graded case. The known result that the symmetric tensor power algebra is isomorphic to the coordinate ring of the scheme of n-dimensional semisimple representations of the original algebra was also recovered.

An explicit bound was proved for the degree of nilpotency of a finitely generated bounded nil algebra. The bound is polynomial both in the number of generators and in the nil index.

A simplified proof was obtained for the result that reduces the problem of representing finite lattices as intervals in subgroup lattices of finite groups to two particular cases: almost simple groups on one hand, and twisted wreath products on the other hand.

It has been shown that, over any firm semigroup, the categories of firm acts, of non-singular acts, as well as of those acts which correspond to a kind of module appearing in the Morita theory of idempotent rings, are equivalent with each other. This strengthens the view that the class of firm semigroups yields the most appropriate environment for Morita equivalence of semigroups.

A non-pointed version of combinatorial exactness structure has been proposed, which allows developing an abstract Kurosh-Amitsur type radical theory in the non-pointed context. This has led to the surprising observation that the theory of closure operators is a special case of the theory of radicals.

The cocommutative elements in the quantized coordinate rings of matrices, general linear groups, and special linear groups were shown to form a maximal commutative subalgebra. The Lie-theoretic relevance of the theorem mentioned above was realized, and the completeness of integrable systems in the analogous semiclassical limit Poisson algebras was proved.

Toric ideals of combinatorially defined classes of lattice polytopes were studied, with a focus on matroid polytopes and their generalizations. Some partial results were achieved on proving degree bounds for the generators in these classes and new methods were developed for approximating these along the way. Computer programs were written which made it possible to understand some interesting (but large) examples in detail.

Department of Algebraic Geometry and Differential Topology

The Algebraic Geometry and Differential Topology section initiated a new Vienna-Budapest algebraic geometry seminar which held four meetings in 2017.

They gave a construction of the height pairing for algebraic cycles over higher-dimensional function fields, generalizing a classical construction of Beilinson.

They have started investigating Tate-Shafarevich groups of algebraic tori defined over function fields of curves over number fields, and have obtained new finiteness results.

They provided a surgery description on the boundary of the Milnor fiber of a non-isolated hypersurface germ given by a stable immersion off the origin.

They proved that the link of a normal surface singularity is an *L*-space if and only if the singularity is rational.

They established a general surgery formula for the Seiberg-Witten invariant of negative definite plumbed 3-manifolds.

They introduced the Abel map for normal surface singularities and worked on the extension of the Brill-Noether theory (originally valid for curves) to surface singularities.

Department of Algebraic Logic

They proved the existence of two relation algebraic atom-structures that cannot be distinguished with first-order logic formulas, while the subalgebras generated by the atoms of their complex-algebras can be distinguished with such formulas.

They investigated the connections between relativistic and non-relativistic space-time structures, by means of modern mathematical logic. They proved that, while neither can be interpreted in the other in the old standard sense, each can be interpreted in the other by using the modern, more flexible many-sorted version of definability theory. However, relativistic and non-relativistic spacetimes are not definitionally equivalent in this new sense, either. This implies that Einstein's special theory of relativity cannot be brought into definitional equivalence with the classical theory even via using this more flexible notion of definability.

They investigated the connections between geometries and groups, by means of modern definability theory. They proved that 4-dimensional scale-free Minkowskian geometry is definitionally equivalent to the Poincaré-group in the new, many-sorted sense.

They streamlined definability theory for many-sorted first-order logic, e.g., the notions of interpretation and definitional equivalence. They have shown that the relation of definitional equivalence generalized by Barrett and Halvorson is not transitive for theories having overlapping languages.

Department of Analysis

Turán type converse Markov-Bernstein inequalities for derivatives of algebraic polynomials were investigated on convex domains. Optimal order lower estimates were verified in L_q-norm for a wide class of convex domains in the complex plane. They initiated for the first time in the literature the investigation of Markov-Bernstein type inequalities for the so called asymmetric weights.

The minimax problem arising from translates of a given concave kernel function on the circle was studied. The original conjecture was generalized by showing that the results can be extended to translates of totally different kernel functions, as well.

The development of effective extension, decomposition and dilation theories in the context of the so called anti-dual pairs was continued. This generality allows a broader range of applicability including non-metrizable spaces. As a by-product of this research, they characterized the normality of functionals by means of the Krein-von Neumann type extension theorem.

Recently Marcinkiewicz-Zygmund type inequalities were verified for univariate polynomials for the general class of doubling weights, and for multivariate polynomials on the ball and sphere with doubling weights. These considerations have been extended to essentially more general multidimensional domains, which in particular include polytopes, cones, spherical sectors, and tori.

Using precise estimates for the root distance of Laguerre orthogonal polynomials new bounds for the weighted Lebesgue constants of Lagrange interpolation were found.

The study of typical-multifractal properties was continued which resulted in finding multifractal properties of typical convex functions. Some open problems concerning almost everywhere convergence of translates of real functions were resolved.

Department of Discrete Mathematics

In the extremal theory of ordered graphs, a particularly interesting conjecture is that if the forbidden subgraph is ordered bipartite and cycle-free, then the extremal function is almost linear. Now it was proved for a large family of forbidden cycle-free ordered bipartite graphs. They determined the order of magnitude of the edges in some cases when two types of an ordered 6-cycle are forbidden.

The stability version of the classical Erdős-Gallai theorem was proved. It is the first result of this type concerning the extremal numbers of bipartite graphs.

A new field of extremal graph theory is when a subgraph is forbidden in a graph and the maximal number of copies of another subgraph has to be determined. They wrote several papers about these problems, e.g., improving the estimate on the number of triangles in graphs not containing pentagons. They managed to generalize a result of Cohen, Fachini and Körner which is about the maximal number of paths so that every pairwise union of paths contains a cycle of length four.

They generalized Tyshkevich's decomposition for splitted bipartite graphs and for directed graphs as well.

They discovered a spectacular extension of Mantel's theorem.

They showed that for $k > 2$, any k-colored graph G with sufficiently large minimum degree contains a monochromatic connected part with order at least $1/(k-1)$ times the order of G.

Exact and approximate values have been determined concerning the maximum length of total dominating sequences in various types of graph products.

They studied the matchings containing all colors in edge colored graphs. They found effective algorithms to find extremal structures of this type.

A new graph model was introduced in 2015 as the graph model of the connections between the neurons in the brain. In 2017, the critical values of percolation have been determined in the real graph model.

They proved the last missing case of the hypergraph version (Berge path) of the Erdős-Gallai theorem on paths. They proved various theorems on 3-uniform hypergraphs not containing any linear cycle and clique of 5 vertices.

The exact Turán number for several hypergraphs was determined. They determined the asymptotic value of the Turán number of a Berge k-path in the case when the hypergraph is connected.

They showed (in contrast to the graph case) that large families of uniquely saturated hypergraphs exist and gave good bounds on their possible orders.

They determined the order of magnitude of Turán numbers for all different types of four-cycles in triple systems.

They introduced and investigated the domination game for k-uniform hypergraphs. They determined the length of the game asymptotically (in k). They investigated the so-called Plurality problem (that was introduced by Aigner) in case of larger query sets. They could prove exact results in some cases.

They asymptotically determined the maximum number of edges in a 3-uniform, Berge-$K_{2,t}$-free (linear) hypergraph.

They extended the famous theorem of Tverberg to the case of positive and negative coefficients and determined the limit shape of integer vector partitions.

A dimension free version of the famous Caratheodory theorem was proved and extended to the case of weak epsilon nets and of densely covered points.

The art gallery problems have been studied, too. They proved a theorem showing a sharp bound on the ratio of the minimum number of point guards and the minimum number of vertical and horizontal mobile guards in a simple orthogonal art gallery.

Earlier bin-packing games have been generalized by introducing a matrix model. For the case of symmetric matrices the existence of Nash Equilibria has been proved.

They were able to eliminate the classification theorem of finite simple graphs from the analysis of Babai's famous Quasipolynomial Graph Isomorphism algorithm.

Department of Geometry

They improved the bounds about the minimal central sections of simplices. They also improved the upper bound on the density of planar sets avoiding unit distances.

They proved a new Erdős-Szekeres type theorem for packings of convex discs.

They minimized the multiplicity of an arrangement of spherical zones depending on the common width of the zones.

They gave a bounded region D and a convex disk K such that in every covering of D with the minimum number of copies of K, two copies must cross.

According to a 40-year old well known conjecture of Karzanov and Lomonosov, if an n-vertex hypergraph has no k pairwise crossing edges, then its number of edges is at most linear. They managed to improve Lomonosov's upper bound to almost linear.

A classic conjecture of Erdős and Hajnal states that for every nontrivial hereditary property T, every graph n vertices with property T has a clique or an independent set of size on that is a power of n. In the special case where the property is that the VC-dimension of the graph is bounded, they nearly verified this conjecture.

80 years after Erdős and Szekeres formulated their conjecture about convex n-gons, Suk has achieved a major breakthrough by essentially verifying it. The researchers of the institute generalized this result to families of non-crossing convex bodies and slightly improved Suk's bound.

The most important inequality for the crossing numbers of graphs is the Crossing Lemma. It does not hold for multigraphs in general. They proved that under some natural and simple conditions, the Crossing Lemma can be still generalized for multigraphs.

Department of Set Theory and Topology

They have developed a general method based on trees of elementary submodels in order to present highly simplified proofs of to numerous results in infinite combinatorics. While countable elementary submodels have been employed in such settings already, they significantly broadened this framework by developing the corresponding technique for countably closed models of size continuum. The applications range from various theorems on paradoxical decompositions of the plane, to coloring sparse set systems, results on graph chromatic number and constructions from point-set topology.

One of the oldest open questions concerning resolvability of topological spaces is whether crowded regular pseudocompact spaces are resolvable. They proved that any such space is even c-resolvable, provided that every disjoint collection of open subsets in it has size at most a finite successor of c (the cardinality of the continuum). They also proved that it is consistent that every such space is c-resolvable.

They managed to improve upon the theory of hierarchies of function classes. Specifically, they constructed the first such rank on the Baire class 1 functions that is based on infinite games.

For nice Polish groups they completely described the cardinal invariants of the Haar null sets in the sense of Christensen.

Certain dichotomy theorems and games on generalized Baire spaces were studied. These games and dichotomies are related to perfect subsets of generalized Baire spaces. The uncountable analogue of the well-known Open Coloring Axiom was investigated for subsets of generalized Baire spaces. A variant concerning the existence of perfect homogeneous subsets was also examined. It was shown that these statements for analytic subsets of the generalized Baire space are equiconsistent with the existence of an inaccessible cardinal. An analogous consistency result for the Silver dichotomy for "simply definable" Borel equivalence relations on generalized Baire spaces was also obtained.

Department of Number Theory

Erdős conjectured 60-70 years ago that the sequence of the normalized prime gaps $d_n/\log n$ (where d_n is the n^{th} difference between consecutive primes) is everywhere dense in the set of non-negative real numbers. The researchers of the institute managed to prove that the density is at least 1/4 showing that for a large T the measure of the set of accumulation points up to T is at least $T/4$.

They generalized the Green function approach to other interesting arithmetic quotients. Of these the most interesting are quotients of the hyperbolic space by groups of isometries arising form 2 by 2 matrices over integers in an imaginary quadratic field.

They studied the uniform distribution of the linear flow on the d-dimensional torus, and disproved a 1989 conjecture of Drmota.

They completed the proof of a theorem stated by Granville about the connection between the Goldbach conjecture and the Riemann hypothesis.

They extended the investigations of block partitions of sequences to higher dimensions. They found a new, elementary proof of Selberg's formula for Kloosterman sums. They disproved a conjecture concerning the domination number of orientations of graphs, finding the exact order of magnitude. They gave an upper bound for the number of pairwise touching simplices in 3-space.

They finalized their results about the Lang-Trotter conjecture concerning primitive points on elliptic curves over finite function fields. They started the investigation of Kloosterman sums over matrix groups and gave an optimal estimation in the case of sums over GL_2 and GL_3.

Department of Probability Theory and Statistics

They worked out a new method for testing the hypothesis of complete independence of the coordinates of a multidimensional random vector. The developed method was tested via MTA cloud virtual computers. According to the results, the proposed test method is stronger than the techniques applied before.

The orthogonality of the measures corresponding to the different scale parameters of the Matérn model has been proven in the previously open 4-dimensional case. The exact form of the maximum likelihood estimator for the fractional Brownian motion with unknown mean was determined. The limit distribution of the eigenvalues was determined for some random matrices.

Random walk properties were investigated for random walks on a spider structure. Weak and strong laws and invariance results were proved for the local and occupation times of this random walk.

Fundamental research in information theory has been conducted for multiuser and/or asynchronous systems, and exponential error bounds were derived. Optimization problems involving generalized information measures have been studied, and the results were applied in mathematical finance.

They provided substantially improved mixing time bounds for certain Markov chains. This may lead to a better understanding of the behavior of the Metropolis algorithm when its setup is suboptimal. Through a Belgian collaboration they worked on the better bounding of the shortest reset word for synchronizing automata, and they achieved partial results.

They described the rate of decay of the mutual information of two distant nodes for factor of iid. processes on large random graphs as the distance increases to infinity.

They proved that two tree degree sequences have edge disjoint caterpillar realizations if and only if their sum is graphical, and the maximum summed degree is at most S+4, where S is the number of vertices that are leaves in at least one of the trees. They also proved that three tree degree sequences without common leaves always have edge disjoint caterpillar realizations. They proved that three tree degree sequences always have edge disjoint tree realizations if the minimum sum is 4 and the sum of the three sequences as well as the sum of any two sequences are all graphical. They gave linear bound conditions for bipartite degree sequences for which the swap Markov chain is rapidly mixing on the realizations.

They proved the central limit theorem for the random walk in divergence-free random drift field. The physical relevance of the problem stems from the fact that this type of random walk in random environments models the motion of particles drifting along with an incompressible turbulent flow in stationary regime. This work closes a longer saga: the proof has been attempted several times since the mid-eighties, but no complete proof has been produced so far. The proof relies on a non-trivial extension of Nash's celebrated moment bound to non-reversible setup with unbounded stream tensor.

They proved the central limit theorem for the Lorentz gas beyond the Boltzmann-Grad limit. This is a most relevant problem of classical statistical physics in which there has been no significant progress since the early eighties.

Applied research

The research carried out at the Rényi Institute has focused on exploratory (theoretical) research. On the applied research projects the research groups of the previous years have continued their work in database theory, cryptography, bioinformatics and other mathematical methods applied in life sciences, e.g., in neural networks.

Some projects involved the application of mathematical statistics, one of which was of astronomical connection. The so called Gamma Ray Burst provided 361 three-dimensional (3D) data about the past of the Universe. The project dealt with the mathematical statistical evaluation of these data. One other medical type statistical investigation fit the CRISPR-Cas system – giving the immune system of bacteria – into their mathematical immunology model.

In the first half of 2017 the deep learning research group, operating partly in the framework of the Large Networks Momentum/ERC research group – beyond the theoretical results detailed in the report of the research group – focused their attention on deep generative models, with the following main results:

- Observed that spectral methods can make dense autoencoders competitive with dense variational autoencoders. The experiments are ongoing to generalize these results to convolutional networks.
- Designed a radial basis neural network layer that can be interpreted as a simple vector-to-pixel graphics renderer. The layer is differentiable, thus it can be used in generative models where the harder reverse task, pixel-to-vector graphics transformation is required.
- Based on the Wasserstein distance, they have created a set of synthetic benchmark tasks that can be used to evaluate the performance of generative models. In the second half of the year the group turned to gradient-based regularization methods, first on generative, then discriminative models. On discriminative vision tasks they have found a class of gradient-based regularization methods that outperform several common regularization methods when applied to competitive baseline networks.

The Cryptology Research Group of the Rényi Institute particularly focused on the generalizations of previous results on secret sharing in 2017. One of their interesting results refutes the widely believed conjecture that large girth graphs have bounded complexity. Furthermore, they examined the complexities of unicycle graphs and some other special small graphs. They described and computed the entropy region for four variables at the supercomputer of NIIF. The results are striking and will be reported in this year. They participated on a cryptology conference, and the head of the research group was one of the plenary speakers of the Hungarian Science Festival.

The bioinformatics research group of the Rényi Institute initiated collaboration with several intuitions, including the University of Tennessee Health Science Center, Memphis, and the University of California at Berkeley, the University of Memphis and the University of Massachusetts Amherst. The main subject of the research initiative has been the understanding the role of oscillatory dynamics in the brain function. The research led to a series of publications, one of them in the Frontiers in Neural Circuits. The institute also continued several bioinformatics collaborations with the University of Notre Dame, the University of Colorado Denver and the Delft Institute of Applied Mathematics. The result of these collaborations was a published paper in Combinatorics, Probability and Computing, and another published paper in Electronic Journal of Combinatorics.

Career advancement of researchers

In 2017 one researcher defended his thesis for the Doctor of the Academy title, one part time researcher obtained habilitation, and five young researchers received the PhD degree, three others obtained nostrification of their foreign degrees. At the end of the year 10 members of the Academy, 36 doctors of HAS, 45 researchers with PhD or CSc worked at the institute, 28 researchers have not yet obtained a degree. Besides the regular employees 13 emeritus research professors (7 academicians, 6 with DSc title) take part in the scientific work of the institute. The institute puts great emphasis on involving young talents – working towards their PhD or having just obtained the degree – into the research work of the institute. In 2017 an additional 6 young researchers were employed in the new or vacant positions offered by the Academy. Altogether 19 young researchers worked in the institute in 2017. The institute has an agreement with the Central European University (CEU). In this framework 24 doctoral students were supervised by members of the institute.

b) Science and society

Unfortunately, most of the research topics in pure mathematics are not suitable for discussions for the general public. On the other hand, the international success of the researchers has underlined the importance of the research conducted in the institute, even in the media.

The researchers of the institute play an important role in popularizing mathematics, giving lectures for high school and university students. The institute regularly organizes an open house during the Festival of Hungarian Science, where high school students and their teachers can get information about the mathematics profession. Among the main events of the Festival of Hungarian Science in 2017 a fellow of the institute gave a lecture on cyber security. A young fellow gave several talks on popularizing mathematics including some at the Valley of the Arts festival.

Members of the institute take part in fostering mathematical talent. In 2017 they have organized several mathematical camps and other events for students interested in the subject. The institute plays a role in giving scientific background for the teachers of specialized mathematics classes in high schools.

III. A presentation of national and international R&D relations in 2017

National relations

Researchers of the institute teach part time at many universities both in Budapest and in other cities (Eötvös University, Budapest University of Technology, Péter Pázmány Catholic University, University of Szeged, Pannon University, etc.). They play an important role in doctoral schools and in Masters programs. 13 fellows of the institute are core members of doctoral schools in various universities, they supervise 52 doctoral students. Especially important is the collaboration between the institute and the Department of Mathematics and its Applications of the Central European University. The lecturers and the supervisors of the Masters and doctoral programs of CEU mainly belong to the institute, including the department chair and the leader of the doctoral program. Also a large part of lecturers of the Budapest Semesters in Mathematics English language study abroad program for North American students belongs to the institute. This program helps to bring the fame of Hungarian mathematics to American universities, and serves as a role model for other international programs. For the institute, the close contact with the new generation of mathematicians is of foremost importance. In this spirit 57 members of the institute (53 percent of all researchers) were active in teaching at universities in 2017; that included supervising one student research project, 11 BSc and 16 MSc theses.

As part of the renewal program of the Academy, the institute restarted its guest researcher program, which enables university professors and lecturers to spend one or two semesters in the institute freed from their teaching duties. As part of this program, three people from Eötvös University joined the research teams of the institute in 2017.

The weekly seminars in the institute are attended regularly by researchers from other institutions, among them some people from universities outside Budapest as well. This way these seminars influence the whole mathematical scene in Hungary.

Members of the Rényi Institute traditionally take part in various Hungarian scientific committees well over proportion. In particular, the Section of Mathematics of the Hungarian Academy of Sciences (MTA) and its committees, the Hungarian Research Fund (NKFIH), and the János Bolyai Mathematical Society (BJMT) can be mentioned. The president of the Section of Mathematics of MTA (in the first half of the year), the chairman of the Mathematics Committee, one of the vice-chairmen and the secretary of the Bioinformatics Committee, the president of the BJMT, the chairman and the secretary of the Scientific Section of BJMT, the vice-chairman of the Applied Mathematics Section of BJMT are all researchers of the MTA Rényi Institute.

International relations

The researchers of the institute have very extensive international relations. Among the coauthors of the members of the institute one finds mainly foreign mathematicians. Joint projects and jointly organized conferences are also typical.

In 2017 thirty-two people from the institute were involved in organizing international conferences, some of them even on several occasions. Completion of the construction work in the institute's building made it possible to organize five conferences and summer schools in 2017 in the Rényi Institute.

Presently an emeritus research professor of the institute serves as the president of the European Set Theory Society.

The visits in the framework of the bilateral exchange programs between the Hungarian Academy of Sciences and its partner institutions successfully contributed to the cooperation with foreign partners. With the help of these programs fruitful joint research projects, useful exchange of information, and conference participations were made possible.

Researchers of the institute took part in altogether eleven international scientific committees. Names of the institute's researchers can be found 161 times on the list of editorial board of various international journals. In 2017 the researchers gave altogether 269 talks at international meetings, many of these were given as an invited or plenary lecture.

In 2017 ten researchers spent more than half a year abroad at the following institutions: the University of Chicago (USA), the City University of New York (USA), National Science Foundation (USA), Auburn University (USA), Mathematical Sciences Research Institute (USA), École Polytechnique Fédérale de Lausanne (Switzerland), the University College London (UK), the University of Bristol (UK), Lancaster University (UK), the University of Toronto (Canada).

Financed by the ERC and Momentum grants or from other sources, 11 foreign researchers worked in the institute for a longer period; a further 15 foreign researchers spent 1-6 months in the institute (altogether accounting for 146 months). They came from the Czech Republic, Italy, Germany, Poland, Slovenia, Canada, Vietnam, Australia, Egypt, USA, Iran, Taiwan, China, and Sweden. The number of foreign visitors of the institute – not counting the conference participants – was 80 in 2017.

IV. Brief summary of national and international research proposals, winning in 2017

National grants

The Rényi Institute, similar to the practices of the previous years, successfully participated in the national NKFIH researcher-initiated project proposals. In 2017 a further 3 research projects won support and one young research fellow started his NKFIH postdoc research project. The institute was successful as well on the newly introduced excellence driven NKFIH project calls. In the call for proposals for research teams with significant achievements of internationally outstanding impact two members of the institute won support. Furthermore, in the call "Frontline" – Research Excellence Programme, among 12 winning proposals nationwide, one was awarded to a researcher of the institute to continue the financing of his research group originally established via an ERC Advanced Grant and a Momentum grant of the Academy.

The institute was very successful in 2016 as well in the researcher-initiated project applications of NKFIH resulting in almost trebling the amount of this type of the institute's income. The numerous winning projects of 2017 – some with high support intensity – would allow the maintenance of this level; however, due to the increasing delay in the acceptance of the research reports as well as in the prefinancing of the running projects, the cash flow of this support seriously declined in 2017.

In the project calls of the Hungarian Academy of Sciences one young researcher received a Premium Post Doctorate Research grant and one colleague was awarded a Momentum project. These projects altogether – even with terminating, earlier Momentum projects – kept close to the earlier record of the financial support level of Academy and other national (non-NKFIH) projects.

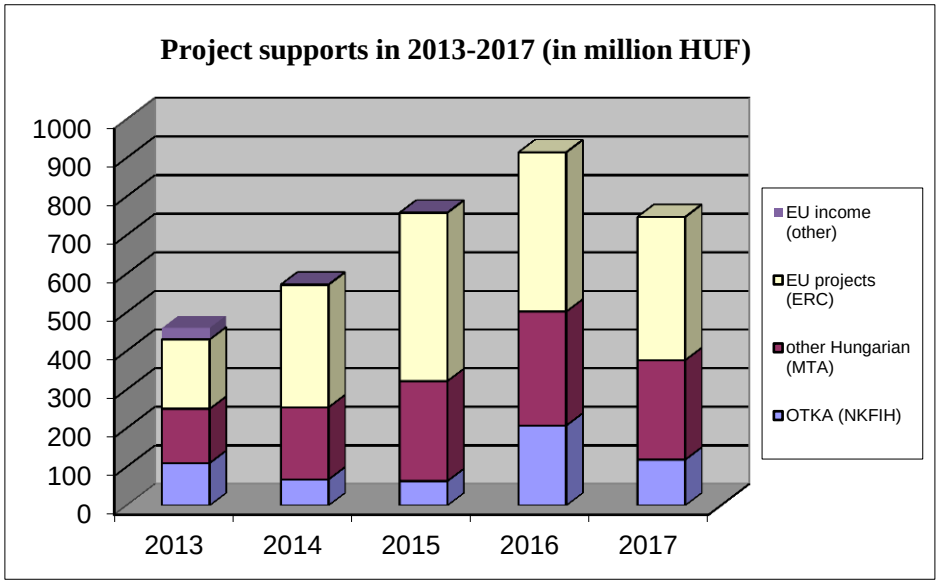
International grants

The most promising and successful international calls for the explanatory (theoretical) research projects of the Rényi Institute are EU European Research Council (ERC) calls and the mobility (Marie Curie) calls of the European Union. 2017 was a successful year in these projects as well, with an Advanced ERC grant proposal submitted and evaluated "A" in 2016 winning support together with a Consolidator ERC grant proposal supported as well in 2017. With these two new projects the number of financed ERC projects of the institute raised to 8.

In 2017 two earlier ERC projects terminated, and so their last financial support will only arrive in 2018. This results in a slight decline in the overall EU grant level of the institute. This will, however, be corrected in 2018 with the two starting ERC grants and the arrival of the last installment of the terminated projects. With the prefinancing of the running ERC projects the institute had no cash flow problem to cover their expenses.

Altogether the total research grant income of the institute in 2017 was slightly less than in the previous year. This was due to the earlier detailed decrease in the Hungarian NKFIH and European ERC grant levels. As both of them were caused by temporary, limited reasons which will hopefully be mended in 2018, the volume of the research grant income of the institute is foreseen to be stable in the near future.

The following diagram shows the amount of project support received during the last 5 years.



V. List of important publications in 2017

1. Abért M, Bergeron N, Biringer I, Gelander T, Nikolov N, Raimbault J, et al. (7): On the growth of L^2-invariants for sequences of lattices in Lie groups. ANNALS OF MATHEMATICS, 185:(3) 711-790 (2017) http://real.mtak.hu/64994/

2. Akopyan A, Bárány I, Robins S: Algebraic vertices of non-convex polyhedra. ADVANCES IN MATHEMATICS, 308: 627-644 (2017) http://real.mtak.hu/55890/

3. Andréka H, van Benthem J, Németi I: On a new semantics for first-order predicate logic. JOURNAL OF PHILOSOPHICAL LOGIC, 46:(3) 259-267 (2017) http://real.mtak.hu/74512/

4. Backhausz Á, Virág B: Spectral measures of factor of i.i.d. processes on vertex-transitive graphs. ANNALES DE L INSTITUT HENRI POINCARE-PROBABILITES ET STATISTIQUES, 53:(4) 2260-2278 (2017) http://real.mtak.hu/74276/

5. Blomer V, Buttcane J, Maga P: Applications of the Kuznetsov formula on GL(3) II: the level aspect. MATHEMATISCHE ANNALEN, 369:(1-2) 723-759 (2017) http://real.mtak.hu/71655/

6. Chau HN, Rásonyi M: Skorohod's representation theorem and optimal strategies for markets with frictions. SIAM JOURNAL ON CONTROL AND OPTIMIZATION, 55:(6) 3592-3608 (2017) http://real.mtak.hu/72708/

7. Csóka E, Lippner G: Invariant random perfect matchings in Cayley graphs. GROUPS GEOMETRY AND DYNAMICS, 11:(1) 211-243 (2017) http://real.mtak.hu/44112/

8. Domokos M: Degree bound for separating invariants of abelian groups. PROCEEDINGS OF THE AMERICAN MATHEMATICAL SOCIETY, 145:(9) 3695-3708 (2017) http://real.mtak.hu/70000/

9. Elekes M, Vidnyánszky Z: Characterization of order types of pointwise linearly ordered families of Baire class 1 functions. ADVANCES IN MATHEMATICS, 307: 559-597 (2017) http://real.mtak.hu/70249/

10. Erdélyi M, Zábrádi G: Links between generalized Montréal-functors. MATHEMATISCHE ZEITSCHRIFT 286:(3-4), 1227-1275 (2017) http://real.mtak.hu/44133/

11. Erdős PL, Pálvölgyi D, Tardif C, Tardos G: Regular families of forests, antichains and duality pairs of relational structures. COMBINATORICA, 37:(4) 651-672 (2017) http://real.mtak.hu/44134/

12. Hladký J, Komlós J, Piguet D, Simonovits M, Stein M, Szemerédi E: The approximate Loebl-Komlós-Sós conjecture I – IV. SIAM JOURNAL ON DISCRETE MATHEMATICS, 31:(2) 945-1148 (2017)
http://real.mtak.hu/74259/
http://real.mtak.hu/74258/
http://real.mtak.hu/74257/
http://real.mtak.hu/74256/

13. Kozma G, Tóth B: Central limit theorem for random walks in doubly stochastic random environment: H-1 suffices. ANNALS OF PROBABILITY, 45:(6) 4307-4347 (2017) http://real.mtak.hu/73986/

14. Némethi A: Links of rational singularities, L-spaces and LO fundamental groups. INVENTIONES MATHEMATICAE, 210:(1) 69-83 (2017) http://real.mtak.hu/73786/

15. Ozsváth PS, Stipsicz AI, Szabó Z: Concordance homomorphisms from knot Floer homology. ADVANCES IN MATHEMATICS, 315: 366-426 (2017) http://real.mtak.hu/59037/

16. Guralnick RM, Maróti A, Pyber L: Normalizers of primitive permutation groups. ADVANCES IN MATHEMATICS 310: 1017-1063 (2017) http://real.mtak.hu/48131/