GEOMETRIC RAMSEY THEORY ON $\mathbb{Z}^d$: SIMPLICES, BARYCENTERS AND BEYOND

Abstract. Szemerédi's theorem suggests a general philosophy: large sets must contain structured patterns. In a geometric direction, Bourgain showed that any subset of $\mathbb{R}^d$ of positive upper density contains isometric copies of a fixed simplex at all sufficiently large scales. I will begin by recalling this result and explaining how later work, including my theorem on k-point configurations and joint work with Cook and Pramanik, extends this picture to the discrete setting and naturally leads to refined notions of distance and uniformity.

In the second part of the talk I will introduce a general framework for geometric Ramsey problems in which a configuration splits into a rigid part-an isometric copy of a simplex with respect to a distance coming from a convex polynomial form-and additional points given by linear relations, such as face barycenters or cube-like sums $y_I = \sum_{i \in I} y_i$. This viewpoint unifies a range of patterns defined by geometric and linear relations, in both the Euclidean and the discrete settings.

Finally, I will outline recent and ongoing results for these configurations in three parallel settings: finite fields (as a model for the arithmetic and Gowers-norm aspects), Euclidean space (via oscillatory integrals and multilinear operators), and the integer lattice (using the circle method and transference). Time permitting, I will also mention quantitative questions and open problems.