1 Nonnegative $k$-sums in a set of numbers. Alexey Pokrovskiy

Suppose that we have a set of numbers $x_1, \ldots, x_n$ which have nonnegative sum. How many subsets of $k$ numbers from $\{x_1, \ldots, x_n\}$ must have nonnegative sum?

By choosing $x_1 = n - 1$ and $x_2 = \cdots = x_n = -1$ we see that the answer to this question can be at most $\binom{n-1}{k-1}$. Manickam, Miklós, and Singhi conjectured that for $n \geq 4k$ this assignment gives the least possible number of nonnegative $k$-sums.

**Conjecture 1** (Manickam, Miklós, Singhi, [2, 3]). Suppose that $n \geq 4k$, and we have $n$ real numbers $x_1, \ldots, x_n$ such that $x_1 + \cdots + x_n \geq 0$. Then, at least $\binom{n-1}{k-1}$ subsets $A \subset \{x_1, \ldots, x_n\}$ of order $k$ satisfy $\sum_{a \in A} a \geq 0$.

Despite the apparent simplicity of the statement of Conjecture 1, it has been open for over two decades.

There have been several results establishing the conjecture when $n$ is large compared to $k$. Manickam and Miklós [2] showed that the conjecture holds when $n \geq (k-1)(k^k + k^2) + k$ holds. Tyomkyn improved this bound to $n \geq k(4e \log k)^k \approx e^{ck \log \log k}$. Alon, Huang, and Sudakov [1] showed that the conjecture holds when $n \geq 33k^2$. Subsequently Frankl gave an alternative proof of the conjecture in a range of the form $n \geq 3k^3/2$.

We will talk about a proof of the conjecture in a range which is linear in $k$.

**Theorem 1.** Suppose that $n \geq 10^{10}k$, and we have $n$ real numbers $x_1, \ldots, x_n$ such that $x_1 + \cdots + x_n \geq 0$. At least $\binom{n-1}{k-1}$ subsets $A \subset \{x_1, \ldots, x_n\}$ of order $k$ satisfy $\sum_{a \in A} a \geq 0$.

The method we use to prove Theorem 1 is inspired by an averaging argument which Katona used in his proof of the Erdős-Ko-Rado Theorem.

**References**

