

# Quantum Information Theory and Mathematical Physics 2016

16.09.2016–19.09.2016

## Program

	Friday	Saturday	Sunday	Monday
9.30 – 10.30	Cecilia Lancien	David Reeb	Ludovico Lami	Fumio Hiai
10.30 – 11.00	Coffee break			
11.00 – 12.00	Carlos Palazuelos	Alexander Müller-Hermes	Robert König	Anna Jencova
12.00 – 14.30	Lunch			
14.30 – 15.30	Guillaume Aubrun	Ion Nechita	Madalin Guta	Péter Vrana
15.30 – 16.00	Coffee break			

### Conference dinner:

Time: Saturday, September 17, from 19.00

Location: Trófea Étterem, Hauszmann Alajos u. 6/b, 1117 Budapest (XI. district)

# Abstracts

**Speaker:** Cecilia Lancien

**Title:** Random quantum correlations are generically non-classical

**Abstract:** Two observers performing binary outcome measurements on subsystems of a global system may obtain more strongly correlated results when they have a shared entangled quantum state than when they only have shared randomness. This well-known phenomenon of Bell inequality violation can be precisely characterized mathematically. Indeed, being a classical or a quantum correlation matrix exactly corresponds to being in the unit ball of some tensor norms. In this talk, I will start with explaining all this in details. I will then look at the following problem: given a random matrix of size  $n$ , can one estimate the typical value of its "classical" and "quantum" norms, as  $n$  becomes large? For a wide class of random matrices, the answer is yes, and shows a separation between the two values. This result may be interpreted as follows: in a typical direction, the borders of the sets of classical and quantum correlations do not coincide.

Based on joint work with C. Gonzalez-Guillen, C. Palazuelos, I. Villanueva.  
<http://arxiv.org/abs/1607.04203>

**Speaker:** Carlos Palazuelos

**Title:** Are random quantum strategies highly non-classical?

**Abstract:** Motivated by Cecilia's talk, we will study similar problems in the context of general Bell inequalities. Following a similar approach, we will first study how to describe the classical and the quantum value of a Bell inequality by means of tensor norms on Banach spaces; and we will see that new problems arise in this new setting. Then, we will study some questions (not all of them solved!) involving the use of random techniques.

**Speaker:** Guillaume Aubrun

**Title:** Two proofs of Stormer's theorem

**Abstract:** Stormer's theorem (any positivity-preserving map over the algebra of  $2 \times 2$  matrices is the sum of a CP map and a co-CP map) and its corollary about qubit entanglement (any PPT state on 2 qubits is separable) are fundamental results in quantum information theory. However the traditional arguments rely on long and seemingly ad hoc computations. We present two proofs: a calculation-free proof based on Brouwer's fixed point theorem and an argument (due to Hildebrand) based on a characterization of extreme self-maps of the Lorentz cone. (Joint with S. Szarek.)

**Speaker:** David Reeb

**Title:** A quantum Mrs. Gerber's Lemma?

**Abstract:** I will present a conjecture of a "Mrs. Geber's Lemma" with quantum side information, which is an entropy inequality that concerns the convolution of two two-valued

random variables. The case of classical side information was proven by Wyner and Ziv [IEEE Trans. Inf. Theor. 19, 769-773 (1973)]. A proof of the Lemma would show that polar codes for c-q-channels approach capacity with polynomial blocklength (akin to arXiv:1304.4321 and arXiv:1411.6993). I will present partial progress towards a proof (and some obstacles), based on recent quantitative improvements of the strong subadditivity inequality (e.g. arXiv:1410.0664 and arXiv:1512.02615).

**Speaker: Alexander Müller-Hermes**

**Title: Recent progress on the PPT<sup>2</sup> conjecture**

**Abstract:** The PPT<sup>2</sup> conjecture states that any composition of two maps each of which is both CP and coCP is entanglement-breaking (i.e. has separable Choi matrix). I will review some consequences of the PPT<sup>2</sup> conjecture for quantum communication scenarios and present the current state of the conjecture. In particular I will present a proof for the conjecture for channels acting on qutrits.

**Speaker: Ion Nechita**

**Title: On some analytical aspects of real Hadamard matrices**

**Abstract:** I will present a recent approach to real Hadamard matrices, using tools from analysis rather than the usual methods coming from combinatorics, design theory or number theory. The starting point is that Hadamard matrices of size  $N$  are the global maxima of some very simple polynomial functions on the orthogonal group. Banica, Collins and Schlenker tried to compute these maxima using random matrix techniques. Later, with Banica and Życzkowski, we studied the local maxima of the same functions, which are now called “almost Hadamard matrices”. My talk will focus on the numerous open problems related to Hadamard matrices and, more specifically, to the aforementioned “analytical” approach.

Notes for the lecture: <http://ion.nechita.net/wp-content/uploads/2016/09/had.pdf>

**Speaker: Ludovico Lami**

**Title: Schur complement in quantum optics**

**Abstract:** It is often the case in mathematics, that the right definition allows you to turn complicated problems into elementary questions. In matrix analysis, one of these magic keys is undoubtedly the Schur complement. This beautiful construction appears naturally in many different contexts in pure and applied linear algebra. After reviewing briefly the history of its formalisation [1], we give a concise list of some of its main applications. The main goal of this talk is to add quantum optics to that list, by showing that Schur complements provide a powerful tool in understanding various forms of quantum correlations in continuous variable systems.

As a first application, we discuss a simple proof of the notable fact that the Positive Partial Transpose (PPT) criterion is necessary and sufficient for separability of Gaussian states of 1 vs  $N$  modes, for arbitrary  $N$ . The original proof relies on two different steps: the first one is the explicit solution of the 1 vs 1 case given in [2], while the second one is the reduction

to this latter case for arbitrary  $N$  [3]. Instead, employing Schur complements yields a direct proof of few lines for all  $N$ . A link with the recently established equivalence between complete Gaussian extendability and separability for all bipartite Gaussian states is highlighted [4].

As a second application, following [5] we prove an important hierarchical relation between measures of correlations for bipartite Gaussian states. Namely, we show that the Rényi-2 Gaussian entanglement of formation can be upper-bounded by half the Rényi-2 mutual information for all bipartite Gaussian states with an arbitrary number of modes per party. According to [6], this relation is a crucial component for a consistent theory of correlations in composite systems, and yet it is violated for analogous measures based on the conventional von Neumann entropy. This further strengthens the prominent role of the Rényi-2 entropy in continuous variable Gaussian quantum information.

We believe that the range of applicability of Schur complements in quantum optics goes far beyond what explored in this talk, and we expect a variety of further applications in the near future stemming from this framework.

## References

- [1] F. Zhang, *The Schur complement and its applications*, Springer, 2005.
- [2] R. Simon, Peres-Horodecki separability criterion for continuous variable systems, Phys. Rev. Lett. **84**, 2726 (2000).
- [3] R. F. Werner and M. M. Wolf, Bound entangled Gaussian states, Phys. Rev. Lett. **86**, 3658 (2001).
- [4] B. V. Rajarama Bhat, K. R. Parthasarathy, and Ritabrata Sengupta, On the equivalence of separability and extendability of quantum states, arXiv:1601.02365 (2016).
- [5] L. Lami, C. Hirche, G. Adesso, and A. Winter, Schur complement inequalities for covariance matrices and monogamy of quantum correlations, arXiv:1607.05285 (2016).
- [6] N. Li and S. Luo, Total versus quantum correlations in quantum states, Phys. Rev. A **7**, 032327 (2007).

**Speaker: Robert König**

**Title: Geometric inequalities from phase space translations**

**Abstract:** We establish a quantum version of the classical isoperimetric inequality relating the Fisher information and the entropy power of a quantum state. The key tool is a Fisher information inequality for a state which results from a certain convolution operation: the latter maps a classical probability distribution on phase space and a quantum state to a quantum state. We show that this inequality also gives rise to several related inequalities whose counterparts are well-known in the classical setting: in particular, it implies an entropy power inequality for the mentioned convolution operation as well as the isoperimetric inequality, and establishes concavity of the entropy power along trajectories of the quantum heat diffusion semigroup. As an application, we derive a Log-Sobolev inequality for the quantum Ornstein-Uhlenbeck semigroup, and argue that it implies fast convergence towards the fixed point for a large class of initial states.

This is joint work with Stefan Huber and Anna Vershynina.

**Speaker: Madalin Guta**

**Title: Information Geometry and Metrology in Open Quantum Dynamical Systems**

**Abstract:** This talk deals with the problem of identifying and estimating dynamical parameters of continuous-time quantum open systems, in the input-output formalism. I will discuss several aspects of this problem:

The first aspect concerns the structure of the space of identifiable parameters for ergodic dynamics, assuming full access to the output state for arbitrarily long times. I will show that the equivalence classes of undistinguishable parameters are orbits of a Lie group acting on the space of dynamical parameters.

The second aspect concerns the information geometric structure on this space. I will show that the space of identifiable parameters is the base space of a principal bundle given by the action of the group, and carries a Riemannian metric based on the quantum Fisher information of the output. The metric can be computed explicitly in terms of the Markov covariance of certain fluctuation operators, and relate it to the horizontal bundle of the connection.

The third aspect concerns the asymptotic statistical structure of the output state. I will show that the output state satisfy local asymptotic normality, i.e. they can be approximated by a Gaussian model consisting of coherent states of a multimode continuous variables system constructed from the Markov covariance data.

The forth direction explores the properties of systems which are near a dynamical phase transition. I will show that this is often accompanied by a Heisenberg scaling of the quantum Fisher information for long times of the order of the correlation time. Related to this is the phenomenon of metastability which will be briefly discussed.

References:

- [1] Madalin Guta, Jukka Kiukas: Information geometry and local asymptotic normality for multi-parameter estimation of quantum Markov dynamics; arXiv:1601.04355
- [2] Katarzyna Macieszczak, Madalin Guta, Igor Lesanovsky, Juan P. Garrahan: Dynamical phase transitions as a resource for quantum enhanced metrology; arXiv:1411.3914, Phys. Rev. A 93, 022103 (2016)
- [3] Katarzyna Macieszczak, Madalin Guta, Igor Lesanovsky, Juan P. Garrahan: Towards a theory of metastability in open quantum dynamics; arXiv:1512.05801, Phys. Rev. Lett. 116, 240404 (2016)

**Speaker: Fumio Hiai**

**Title: Different quantum  $f$ -divergences and the reversibility of quantum operations**

**Abstract:** We first give a comprehensive survey of different quantum divergences, including standard  $f$ -divergences, maximal  $f$ -divergences, sandwiched Rényi divergences, and  $\alpha$ - $z$ -Rényi relative entropies. We next discuss the reversibility problem for those divergences, in connection with the equality case in the monotonicity inequality (or DPI) under quantum operations. This is joint work with Milán Mosonyi.

Slides of the talk: [http://math.bme.hu/~mosonyi/QIMP\\_2016/hiai\\_slides.pdf](http://math.bme.hu/~mosonyi/QIMP_2016/hiai_slides.pdf)

**Speaker:** Anna Jencova

**Title:** Quantum divergences and interpolation

**Abstract:** Some relations between quantum divergences and interpolation theory are reviewed, with focus on monotonicity and equality conditions.

**Speaker:** Péter Vrana

**Title:** Tensor ranks

**Abstract:**