Undecidability of relativity theories

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In the last years, Andréka, Madarász and Németi developed a first-order model theory for the special relativity theory. Their theories formalise the basic principles of the common approach to the relativistic kinematics.

In this talk we demonstrate that most of the resulting theories are undecidable - even the coordinatizing field has a decidable first-order theory. This fact shows that the mentioned theories capture more from the relativity theory than the first-order theory of Minkowskian geometry which is shown to be decidable e.g. by Rob Goldblatt. The main reason for the undecidability is that these theories allow to exist objects living on a space-time trajectory no coinciding with any geodetics of the Minkowskian geometry (e.g. periodically moving bodies). Utilizing such an object we can execute a Gödel-like reasoning to find a sentence speaking about its own truth - and it gives hereditaly undecidability of these theories.

On the other hand, if we restrict the expressive power of the possible models by cutting out such objects (+ adding some natural symmetry axioms and requiring that the coordinatizing field is real-closed) then we can create a decidable extension which has enough expressive power to deal with the common classroom treatise of the special relativity.