On neat embeddability of cylindric algebras

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I would like to say thanks for Istvan Nemeti for calling my attention to the class relativized cylindric set algebras (Crs_α's) and to many other interesting areas of algebraic logic. He helped me much to prepare my first paper on Crs algebras. The result below is a part of my activity on this area.

As is well-known, the classical representation theorem of the theory of cylindric algebras is: $\mathcal{A} \in \mathrm{Gws}_\alpha$ if and only if $\mathcal{A} \in \mathrm{SNr}_\alpha \mathrm{CA}_{\alpha+\varepsilon}$, where $\varepsilon \geq \omega$, ε is a fixed ordinal, Gws_α is the class of generalized cylindric set algebras of dimension α, CA_α is the class of cylindric algebras of dimension α and $\mathrm{SNr}_\alpha \mathrm{CA}_{\alpha+\varepsilon}$ is the class of CA_α's that have the neat embedding property.

The following questions arise:

Is it possible to enlarge the class CA in $\mathrm{SNr}_\alpha \mathrm{CA}_{\alpha+\varepsilon}$ so that the theorem still be true?

Is it possible to generalize the theorem from the class Gws_α to the class D_α e.g.(D_α is a subclass of Crs_α for which $C_i D_{ij} = V$ where V is the unit of the algebra), and is it possible to replace the class CA in the hypothesis "$\mathcal{A} \in \mathrm{SNr}_\alpha \mathrm{CA}_{\alpha+\varepsilon}$" by a suitable class for this case?

We define a class M^α_β desired $(\alpha \leq \beta)$: M^α_β is the class for which

$$\mathrm{M}^\alpha_\beta \models \{\mathrm{C}_0, \mathrm{C}_1, \mathrm{C}_2, \mathrm{C}_3, \mathrm{C}_5, \mathrm{C}_7, {}_-\mathrm{C}_4, {}_-\mathrm{C}_6\}$$

where C_0 denotes the conjuction of the usual Boolean axioms, C_1,C_2,C_3,C_5 and C_7 are the usual cylindric axioms, ${}_-\mathrm{C}_4$ and ${}_-\mathrm{C}_6$ are definite weakenings of the usual cylindric axioms C_4 and C_6. The following theorem is true:

Theorem $\mathcal{A} \in \mathrm{ID}_\alpha$ if and only if $\mathcal{A} \in \mathrm{SNr}_\alpha \mathrm{M}^\alpha_{\alpha+\varepsilon}$.

Here we need the class F_α where $\mathrm{F}_\alpha \models \{\mathrm{C}_0,\mathrm{C}_1,\mathrm{C}_2,\mathrm{C}_3,\hat{\mathrm{C}_4},\mathrm{C}_5,\mathrm{C}_6,\mathrm{C}_7\}$ and $\hat{\mathrm{C}_4}$ is the property $c_i d_{in} c_j d_{jm} x = c_j d_{jm} c_i d_{in} x$.

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