Should the Principle of Relativity Speak only about Reference Frames instead of Coordinate Systems?

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[1] Joint research with Judit X. Madarász and Mike Stannett

Rindler's book:

$$\boxed{\text{SPR} \iff \text{Isotropy \& (spacetime) Homogeneity}}$$

Our CQG paper:

$$\boxed{\text{SPR} \;\not\Longrightarrow\; \text{Isotropy}}$$

SPR := Principle of Relativity

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SPR := Principle of Relativity

Who is right?

Who is right? Both of us!

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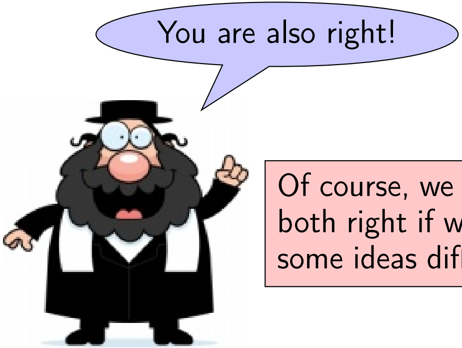


Of course, we can only be both right if we capture some ideas differently.

DIALOGO

DI

GALILEO GALILEI LINCEO

MATEMATICO SOPRAORDINARIO

DELLO STVDIO DI PISA.

E Filosofo, e Matematico primario del

SERENISSIMO

GR.DVCA DI TOSCANA.

Doue ne i congressi di quattro giornate si discorre sopra i due

MASSIMI SISTEMI DEL MONDO TOLEMAICO, E COPERNICANO;

Proponendo indeterminatamente le ragioni Filosofiche, e Naturali tanto per l'vna, quanto per l'altra parte.

CON PRIVILEGI.

IN FIORENZA, Per Gio:Batista Landini MDCXXXII.

CON LICENZA DE' SVPERIORI.

ha sempre fermo nel concetto, che la palla si parta dalla quiete nel venir cacciata dal fuoco fuor del pezzo; e partirsi dallo stato di quiete non può esser, se non supposta la quiete del globo terrestre, che è poi la conclusione di che si quistioneggia; replico per tanto, che quelli, che fanno la terra mobile, rispondono, che l'artiglieria, e la palla, che vi è dentro partecipano il medesimo moto, che ha la terra; anzi che questo insieme con lei l'hanno eglino da natura, e che però la palla non si parte altrimenti dalla quiete; ma congiunta co'l suo moto intorno al centro, il quale dalla proiezione in sù, non le vien nè tolto, nè impedito; & in tal guisa seguitando il moto vniuersale della terra verso Oriente sopra l'istesso pezzo di continuo si mantiene, sì nell'alzarsi, come nel ritorno, e l'istesso vedrete voi accadere facendo l'esperienza in naue di vna palla tirata in sù a perpendicolo con vna balestra, la quale ritorna nell'istesso luogo, muouasi la naue, ò stia ferma.

SAGR. Questo sodisfà benissimo al tutto; ma perchè hò veduto, che il Sign. Simplicio prende gusto di certe arguzie da chiappare (come si dice) il compagno, gli voglio domandare, se supposto per hora, che la Terra stia ferma, e sopra essa l'artiglieria eretta perpendicolarmente, e drizzata al nostro Zenit, egli ha difficultà nessuna in intender, che quello è il vero tiro a perpendicolo, e che la palla nel partirsi, e nel ritorno sia per andar per l'istessa linea retta, intendendo sempre rimossi tutti gli impedimenti esterni, & accidentarij.

Altra soluzione alla medesima instanza.

SIMP. Io intendo, che il fatto deua succeder così per appunto.

SAGR. Ma quando l'artiglieria si piantasse, non a perpendicolo, ma inclinata verso qualche parte, qual dourebbe esser il moto della palla? andrebbe ella forse, come nell'altro tiro per la linea perpendicolare, e ritornando anco poi per l'istessa?

SIMP. Questo non farebb'ella, ma vscita del pezzo seguiterebbe il suo moto per la linea retta, che continua la dirittura della canna, se non in quanto il proprio peso la farebbe declinar da tal dirittura verso terra.

SAGR. Talchè la dirittura della canna è la regolatrice del moto della palla: nè fuori di tal linea si muoue, ò muouerebbe, se'l peso proprio non la facesse declinare in giù; e però posta la canna a perpendicolo, e cacciata la palla in sù, ella ritorna per l'istessa linea retta in giù, perchè il moto della palla dependente dalla sua grauità, è in giù per la medesima perpendicolare, il viaggio dunque della palla fuor del pezzo, cōtinua la dirittura di quella particella di viaggio, che ella ha fatto dentro al pezzo: non stà così?

Proietti continuano il moto per la linea retta che segue la direzione del moto,

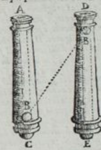
che fecero insieme col proiciente, mentre con esso erano cōgiunti.

SIMP. Così pare a me.

SAGR. Hora figurateui la canna eretta a perpendicolo, e che la terra si volga in se stessa co'l moto diurno, e seco porti l'artiglieria, ditemi qual sarà il moto della palla dentro alla canna, dato che si sia fuoco?

SIMP. Sarà vn moto retto, e perpendicolare, essendo la canna drizzata a perpendicolo.

SAGR. Considerate bene, perch'io credo, ch'e' non sarà perpendicolare altrimenti: sarebbe bene a perpendicolo, se la terra stesse ferma, perchè così la palla nō haurebbe altro moto, che quello, che le venisse dal fuoco. Ma quando la terra giri, la palla, che è nel pezzo, ha essa ancora il moto diurno, talchè sopraueuendole l'impulso del fuoco, ella cammina dalla culatta del pezzo alla bocca di due mouimēti, dal composto de' quali ne risulta, il moto fatto dal centro della grauità della palla essere vna linea inclinata. E per più chiara intelligenza, sia l'artiglieria AC. eretta, & in essa la palla B. è manifesto, che stando il pezzo īmobile, e datogli fuoco, la palla vscirà per la bocca A. & haurà co'l suo centro, cāminando per il pezzo, descritta la linea perpēdicolare BA. e quella dirittura andrà seguitādo fuor del pezzo, mouendosi verso il vertice. Ma quando la terra andasse in volta, & in conseguenza seco portasse l'artiglieria, nel tempo, che la palla cacciata dal fuoco si muouesse per la canna, l'artiglieria portata dalla terra, passerebbe nel sito DE. e la palla B. nello sboccare sarebbe alla gioia D. & il moto del centro della palla, sarebbe stato secondo la linea BD. non più perpendicolare, ma inclinata verso Leuante; e douendo (come già s'è concluso)



Posta la vertigine della terra, la palla nell' artiglieria eretta a perpendicolo, non si muoue per linea perpendicolare, ma per vna inclinata.

SPR according to Einstein in 1905:

§ 2. **Über die Relativität von Längen und Zeiten.**

Die folgenden Überlegungen stützen sich auf das Relativitätsprinzip und auf das Prinzip der Konstanz der Lichtgeschwindigkeit, welche beiden Prinzipien wir folgendermaßen definieren.

1. Die Gesetze, nach denen sich die Zustände der physikalischen Systeme ändern, sind unabhängig davon, auf welches von zwei relativ zueinander in gleichförmiger Translationsbewegung befindlichen Koordinatensystemen diese Zustandsänderungen bezogen werden.

2. Jeder Lichtstrahl bewegt sich im „ruhenden" Koordinatensystem mit der bestimmten Geschwindigkeit V, unabhängig davon, ob dieser Lichtstrahl von einem ruhenden oder bewegten Körper emittiert ist. Hierbei ist

$$\text{Geschwindigkeit} = \frac{\text{Lichtweg}}{\text{Zeitdauer}},$$

wobei „Zeitdauer" im Sinne der Definition des § 1 aufzufassen ist.

From Einstein's paper „Zur Elektrodynamik Bewegter Körper."

SPR is an

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(Hence it can be formulated several different ways.)

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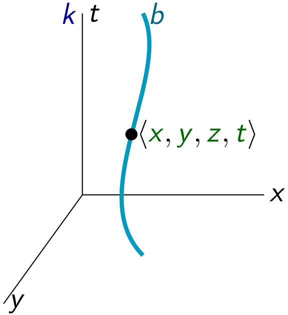
So the right question to ask is:

How are these formalizations related?

$\mathsf{W}(k, b, x, y, z, t) \leftrightsquigarrow$ „observer k coordinatizes body b at spacetime location $\langle x, y, z, t\rangle$."



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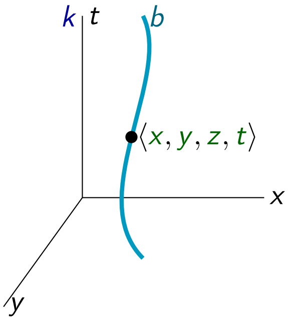
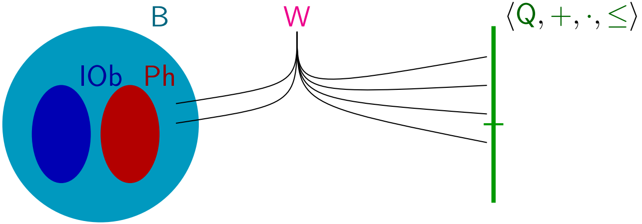


Worldline of body b according to observer k

$$\mathsf{wline}_k(b) = \{\langle x, y, z, t\rangle \in \mathsf{Q}^4 \;:\; \mathsf{W}(k, b, x, y, z, t)\}$$

Language: $\{ \mathsf{B}, \mathsf{IOb}, \mathsf{Q}, +, \cdot, \leq, \mathsf{W}\, \mathsf{Ph}, \text{Etc.} \}$



$\mathsf{B} \leftrightsquigarrow$ Bodies (things that move) $\mathsf{IOb} \leftrightsquigarrow$ Inertial Observers

$\mathsf{Q} \leftrightsquigarrow$ Quantities $+$, $\cdot$ and $\leq \leftrightsquigarrow$ field operations and ordering

$\mathsf{W} \leftrightsquigarrow$ Worldview (a 6-ary relation of sort BBQQQQ)

$\mathsf{Ph} \leftrightsquigarrow$ Photons (light signals)

AxOField:

The **structure of quantities** $\langle Q, +, \cdot, \leq \rangle$ is an ordered field,

- Rational numbers: $\mathbb{Q}$,
- $\mathbb{Q}(\sqrt{2})$, $\mathbb{Q}(\sqrt{3})$, $\mathbb{Q}(\pi)$, ...
- Computable numbers,
- Constructable numbers,
- Real algebraic numbers: $\mathbb{A} \cap \mathbb{R}$,
- **Real numbers**: $\mathbb{R}$,
- Hyperrational numbers: $\mathbb{Q}^*$,
- Hyperreal numbers: $\mathbb{R}^*$,
- Etc.

A principle of relativity

$\mathcal{S}$ – set of **experimental scenarios**.

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CoordSPR:

Every experimental scenario $\varphi \in \mathcal{S}$ is either realizable by every inertial observer or by none of them.

$$\text{For all } \varphi \in \mathcal{S}: \;\; \mathsf{IOb}(k), \mathsf{IOb}(k') \implies \big[\varphi(k, \bar{x}) \iff \varphi(k', \bar{x})\big].$$

Examples

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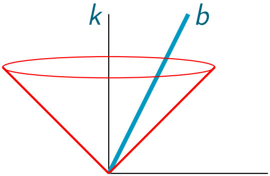

$$\varphi(k, c) \equiv (\forall b \in \mathsf{B})\left[\mathsf{speed}_k(b) < c\right]$$

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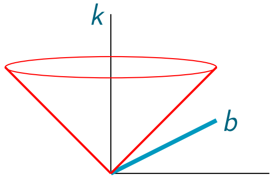

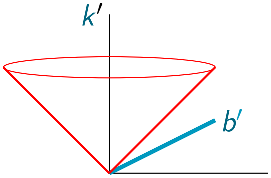

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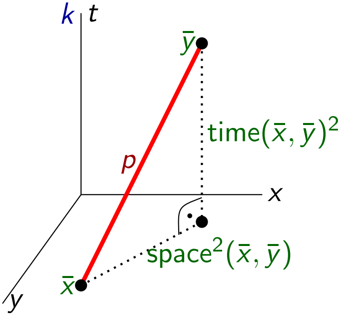
CoordSPR$^+$: when $\mathcal{S}$ contains all the formulas having only 1 free variable of sort bodies.

What does SPR imply?

AxLight:

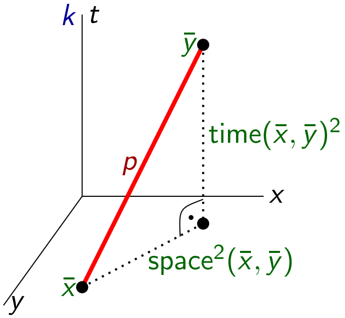
There is at least one inertial observer according to whom, any light signal moves with the same velocity in every direction.

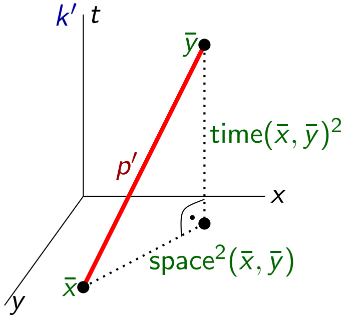


$$(\exists k \in \mathrm{IOb})(\exists c \in \mathrm{Q})(\forall \bar{x}\bar{y} \in \mathrm{Q}^4)\left[\begin{array}{c}(\exists p \in \mathrm{Ph})\big[\bar{x},\bar{y} \in \mathrm{wline}_k(p)\big] \\ \Updownarrow \\ \mathrm{space}^2(\bar{x},\bar{y}) = c^2 \cdot \mathrm{time}(\bar{x},\bar{y})^2\end{array}\right]$$

AxPh:

According to every inertial observer, any light signal moves with the same velocity in every direction.





$$(\forall \underline{k \in \mathrm{IOb}})(\exists c \in \mathrm{Q})(\forall \bar{x}\bar{y} \in \mathrm{Q}^4)\left[\begin{array}{c}(\exists p \in \mathrm{Ph})\big[\bar{x},\bar{y} \in \mathrm{wline}_k(p)\big]\\ \Updownarrow \\ \mathrm{space}^2(\bar{x},\bar{y}) = c^2 \cdot \mathrm{time}(\bar{x},\bar{y})^2\end{array}\right]$$

Proposition: (Assuming AxOField)

$$\mathsf{CoordSPR}^{+}, \mathsf{AxLight} \implies \mathsf{AxPh}$$

Proposition: (Assuming AxOField)

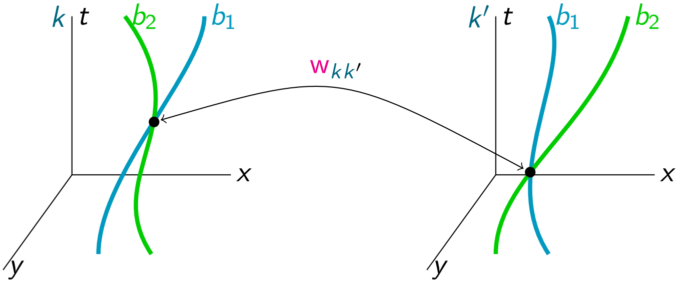
$$\mathsf{CoordSPR}_{\mathcal{S}}, \mathsf{AxLight} \implies \mathsf{AxPh}, \textit{ if}$$

$$\varphi(k,\bar{x},\bar{y}) \equiv (\exists p \in \mathsf{Ph})\big[\bar{x},\bar{y} \in \mathsf{wline}_k(p)\big] \in \mathcal{S}.$$

How to formalize Isotropy?

The **worldview transformation** $\mathsf{w}_{kk'}$ between observers k and k'

$$\mathsf{w}_{kk'}(x, y, z, t : x', y', z', t') \overset{def}{\Longleftrightarrow} \forall b \ \mathsf{W}(k, b, x, y, z, t) \iff \mathsf{W}(k', b, x', y', z', t').$$



Homogeneity:

Translations do not effect the outcomes of experiments (and the experiments can be translated).

$$\text{For all } \varphi \in \mathcal{S}: \quad \mathsf{w}_{kk'} \text{ „is a translation,"}$$

$$\mathsf{IOb}(k), \mathsf{IOb}(k') \implies \big[\varphi(k, \bar{x}) \iff \varphi(k', \bar{x})\big].$$

and

$$(\forall k \in \mathsf{IOb})(\forall T \text{ „translation" })(\exists k' \in \mathsf{IOb})\big[\mathsf{w}_{kk'} = T\big]$$

Isotropy:

Rotations do not effect the outcomes of experiments (and the experiments can be rotated).

For all $\varphi \in \mathcal{S}$: $\mathsf{w}_{kk'}$ „is a rotation restricted to space,"

$$\mathsf{IOb}(k), \mathsf{IOb}(k') \implies \left[\varphi(k, \bar{x}) \iff \varphi(k', \bar{x})\right].$$

and

$$(\forall k \in \mathsf{IOb})(\forall R \text{ „spatial rotation''})(\exists k' \in \mathsf{IOb})\left[\mathsf{w}_{kk'} = R\right]$$

Proposition: (assuming AxOField)

$$\mathsf{AxTriv}, \mathsf{CoordSPR} \implies \mathsf{Homogeneity}$$
$$\mathsf{AxTriv}, \mathsf{CoordSPR} \implies \mathsf{Isotropy}$$

AxTriv:

The rotated (around the time-axis) and translated versions of an inertial coordinate systems are also inertial coordinate system.

Frames vs. coordinate systems

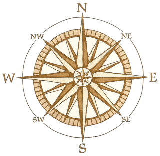
(isotropy), be repeated at *any time* (temporal homogeneity)—and the outcome will be the same.

All this is a direct consequence of the relativity principle. To see this, let us make a logical distinction between *inertial frames* and *inertial coordinate systems* (a distinction which later we shall generally ignore). The former are mere extensions (real or imagined) of non-rotating uniformly moving rigid bodies in a world without gravity, as in SR. An inertial coordinate system is such an IF *plus*, in it, a choice of standard coordinates x, y, z, t in standard units. For a given *frame* these *systems* differ from each other at most by spatial rotations and translations and time translations. Now, strictly speaking, Einstein's RP concerns inertial coordinate systems: *the laws of physics are invariant under a change of inertial coordinate system*. So we see at once that this equivalence of coordinate systems, when applied to just one IF, guarantees the homogeneity and isotropy of that IF.

It is perhaps less well known that, conversely, the homogeneity and isotropy of all inertial frames implies the RP, as has been especially stressed by Dixon. So it is really a question of taste which of the two is taken as axiom and which as consequence.

From Rindler's book „Relativity: Special, General, and Cosmological."

Coordinates vs frames



How can we introduce reference frames?

A **reference frame** is an equivalence class of observers:

$$k \sim h \stackrel{def}{\iff} \mathsf{w}_{kh} = T \circ R$$

for some rotation R around the time-axis and (spacetime) translation T.

FrameSPR:

Every $\varphi \in \mathcal{S}$ experimental scenario is either realizable in every inertial **frame of reference** or in none of them.

$$\text{For all } \varphi \in \mathcal{S}: \ \mathsf{IOb}(k), \mathsf{IOb}(k') \implies$$

$$\left((\exists h \in \mathsf{IOb}) \left[\begin{array}{c} k \sim h \\ \varphi(h, \bar{x}) \end{array} \right] \iff (\exists h' \in \mathsf{IOb}) \left[\begin{array}{c} k' \sim h' \\ \varphi(h', \bar{x}) \end{array} \right] \right)$$

Proposition: (Assuming AxOField)

$$\mathsf{FrameSPR}^{+}, \mathsf{AxLight}, \mathsf{AxRest} \implies \mathsf{AxPh}$$

AxRest:

Restricted to time or space the worldview transformation between any two inertial observers stationary with respect to each other is a similarity (i.e., isometry up to scaling).

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Proposition:

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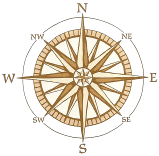
$$\mathsf{FrameSPR}^{+}, \mathsf{AxTriv}, \mathsf{AxRest}, \mathsf{Etc.} \quad \not\Longrightarrow \quad \mathsf{Isotropy}$$

CoordSPR $\overset{\Longrightarrow}{\not\Longleftarrow}$ FrameSPR



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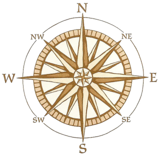

Thank you for your attention!