Many-valued temporal logic for quantum mechanics

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"Past", "present" and "future" are unproblematic words in classical physics. As adjectives, they describe the location of events on a particular worldline relative to a reference event ("now") on that worldline. As tenses, they give the appropriate form of propositions uttered at the reference event and referring to other events on the worldline — which we are now taking to be the worldline of a localised, sentient, articulate physical system. In deterministic classical physics, any such statement is either true or false, and the logic of tensed statements, though more elaborate than simple propositional calculus [1], is classically bivalent. In an indeterministic classical world, however, this is not so clear; there is a venerable tradition (stemming, some say, from Aristotle) that future contingent propositions should obey a many-valued logic.

In quantum mechanics the situation is even less clear. There is an immediate problem with the concept of a worldline, but instead of the worldline of a localised system we can consider the system itself. Classically, each point on the worldline (equally, each value of the time coordinate in any frame of reference) corresponds to a unique internal state of the system, which, in the case of a sentient system, we can take to be an experience of the system. In quantum mechanics, because of both superposition and entanglement, this is not true. Experience states do not exhaust all states of the system, since

there can be superpositions of such states; but, being describable and distinguishable in classical terms, they must be orthogonal, and we can assume that they constitute a basis $|\eta_n\rangle$ for the state space of the experiencing system. But the system is not necessarily described even by a superposition of these experience states, because it may be (probably is) entangled with the rest of the universe. All we can say is that the state of the whole universe at any time t can be expanded in terms of the system's experience states as

$$|\Psi(t)\rangle = \sum_n |\eta_n\rangle|\Phi_n(t)\rangle \in \mathcal{H}_S \otimes \mathcal{H}'_{\rm U} \tag{1}$$

where $\mathcal{H}_{\rm S}$ and $\mathcal{H}'_{\rm U}$ are the state spaces of the system and the rest of the universe, and the $|\Phi_n(t)\rangle$ are non-normalised states of the rest of the universe, most of which will be zero.

In this situation, an utterance of the perceiving, recording system S belongs to a perspective, not only of a time t, as for a classical system, but also of one of the experiences η_n. But how are we to assess the truth of an assertion by this system, while perceiving the experience $\eta_{\rm now}$ at time $t = 0$, that its experience at a later time t will be $\eta_{\rm future}$? There is nothing in the physics of (1) to connect the present experience η_n with any unique future experience η_f, so no one statement about future experience is singled out as true while all others are false. However, there are varying degrees of connection between the present experience and possible future experiences. Any given present experience $|\eta_n\rangle$ goes together with a state $|\Psi_n(0)\rangle$ of the rest of the universe to form a component $|\eta_n\rangle|\Psi_n(0)\rangle$ of the universal state vector at time $t = 0$; this would evolve to $\mathrm{e}^{-iHt}\big(|\eta_n\rangle|\Psi_n(0)\rangle\big)$ at the later time t, and we can asssess the extent to which this evolved state will be, at time t, aligned with any given experience state $|\eta_f\rangle$, using the squared modulus of the inner product. I propose that this measure of alignment between the present state $|\eta_n\rangle$ and the future state $|\eta_f\rangle$ should be identified with the degree of truth of the future-tense statement "My experience at time t' will be η_n", uttered in the context of the experience η_n at time $t = 0$. I identify this degree of truth with the *probability* that this subject's experience at time t will be η_f.

This leads to a many-valued logic for a lattice of propositions which is generated by sublattices $\mathcal{L}_0$, containing propositions about the present, and $\mathcal{L}_t$, for each positive real number, containing propositions referring to a time t in the future. The contents of these sublattices are descriptions of the experiences of the subject S. Their truth values are determined by the context of a present experience and the universal Hamiltonian which determines how that experience will evolve. I will give axioms for these truth values, and outline the development of a full logic [2] on the basis of these axioms.

This logic concerns statements made by the perceiving system from inside the physical universe described by the universal state vector. But we can also consider statements about the whole universe made from an external, God's-eye view. The truth values of such statements are independent of time; from this external standpoint the evolution is deterministic and there is no place for probabilities. I will defend the view that the statements of the experiencing subsystem, despite their apparent subjectivity, are as valid as external statements about the whole universe. There are opposing ontologies, neither of which has overriding claim to being the ultimate truth.

So much for the present and the future. What about the past? At first sight there is no problem here: for a subject with memory, the past is part of present experience. Statements about the past are therefore a subset of the lattice $\mathcal{L}_0$ of statements about the present, subject to bivalent logic. However, attempts to model such a memory in quantum mechanics are limited by the watched-pot effect. It seems that it is not possible for a quantum system to contain information about all past times, but there is a restricted set of times which are possible labels for propositions about the past. This is the subject of current research, on which I will report progress.

References

[1] A. Prior. *Past, Present and Future.* Clarendon Press, Oxford, 1967.

[2] A. Sudbery. The logic of the future in quantum theory. *Synthese*, 2016. doi:10.1007/s11229-016-1142-9.