Goal directed proofs and diagrams suitable for applications in the philosophy of science

In this paper we raise the following general question: how can humans *reach* proofs of various forms?

Hintikka (1999) differentiated between *definitory rules*, which can be interpreted as the rules of inference, and *the strategy*, that is the heuristic rules used when pursuing a proof. While the rules of inference have received ample attention from logicians, the strategy has been neglected for the most practical reasons – including heurisitics into the proof formalism seems to be a complicated and laborous task. However, Batens and Provijn (2001) argue that Hintikka's distinction into definitory rules and the strategy is not very deep. They also attempted to include the heuristic rules into the rules of interence, in other words to *push down* the strategy into the definitory rules without any loss to the metatheory.

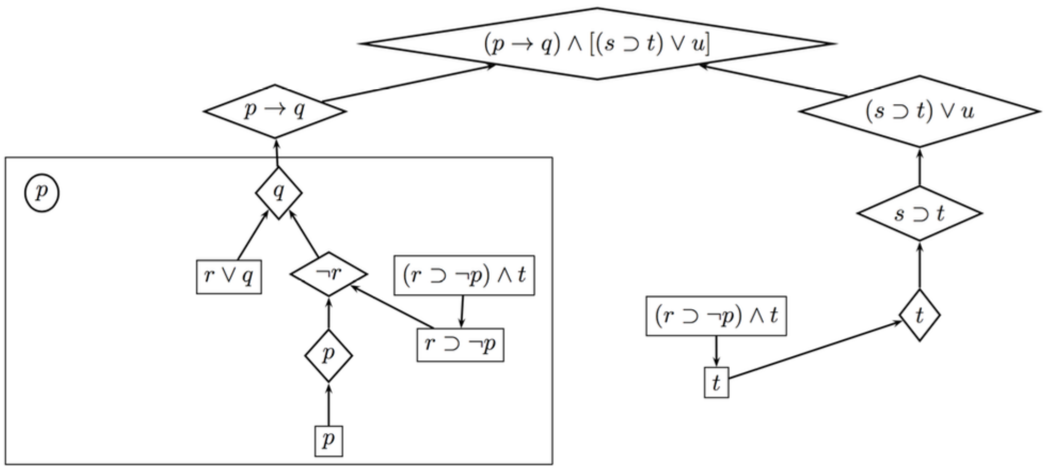
To that aim Batens and Provijn (2001) utilize a goal directed proof procedure which found applications in the philosophy of science, specifically in clarifying the pragmatic aspects of the process of explanation (Batens and Meheus, 2001).

We shall (i) present the motivation and certain technical aspects of the goal directed proof method as presented by Gabbay (2000), Batens and Provijn (2001) and Batens (2002), and (ii) discuss the proof diagrams developed further by Batens (2006) and also Verdée and De Bal (2015).

Goal-directedness in proof theory has found some interesting applications in non-classical logics. Verdée and De Bal (2015) devised a relevant logic NTR that explains the relevance characteristics of the lines of goal directed proofs. They have also developed a diagrammatic proof method for NTR based on methods derived from Batens (2006).

In a goal directed proof a candidate conclusion, called *the goal of the proof*, is proven from a set of premisses. We write down the successive lines starting with the goal and gradually justifying all *subgoals*. We continue the procedure as far as possible using the premisses. Lines of goal directed proofs are of conditional character. We also ensure that the conditions of lines are always relevant for the consequent and *vice versa*. This characteristic holds even in case of goal directed proofs for classical logic.

The diagrams capture the relevant entailment relation in a classical logic context without weakening classical logic. Below we discuss an example of a diagrammatic proof of a goal formula (placed at the top of the diagram) stating that $(p \rightarrow q) \wedge [(s \supset t) \vee u]$. The diagram proves that this goal formula is a logical consequence of (and moreover is entailed by) $(r \supset \neg p) \wedge t$ together with $r \vee q$.



Example. A goal directed diagrammatic proof (image credit: Peter Verdee)

In the context of a theory which contains, among other formulas, the formula $r \vee q$, the diagram informs us that the goal formula can be explained by $r \vee q$ together with t and $\neg r$. The diamond-shaped nodes are the result of analyzing the goal and its subgoals, while the rectangle-shaped nodes are premisses and elements derived from the analysis of the premisses. The circle node is a hypothesis that can be used only inside the given frame.

Moreover, all arrows in the diagrams denote entailment relations. If two arrows end up in the same node it means that the contents of the departure nodes together entail the content of the arrival node. The analyzing steps are always depicted in the up-down-direction, even if the entailment relation is down-up.

We shall argue that goal directed diagrammatic proofs are a suitable means of including the strategy of pursuing proofs into the rules of interence. Moreover, no damage to the metatheory is inflicted. We shall conclude with more practical examples and the summary of those features of goal directed proof diagrams which are desirable from the point of view of philosophy of science: capturing the entailment relation (and even the relevant entailment), providing a natural position for proof heuristics and the resulting enhanced applicability to the process of explanation.

References

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