Connection between Neutron Star Observables and the Quantum Nature of Nuclear Matter

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Abstract.

The recent discovery of the gravitational waves provides a new method to study the interior of compact astrophysical objects, such as neutron stars. The high-accuracy measurements of neutron star mass gives constraint for the nuclear models of compact stars interior, which may further restricted by the gravitational wave data. Neutron star mergers, which are the most common predicted sources of gravitational waves, are very sensitive to the nuclear equation of state and different phases of high-density nuclear matter. Investigation of these compact star "fingerprints" are one of the most active areas of this field. Equation of state zoo of the compact star interior has wide variety. Especially, the applied models have strong impact on the final observables of the objects. We study the effect of quantum fluctuations on these physical observables, using the Functional Renormalization Group (FRG) method in effective field theories of the nuclear matter. Within this framework we explored the effect of the running self interaction coupling in a simple model of Fermions coupled to a fluctuating scalar field with Yukawa-coupling. We calculated the phase diagram and the equation of state in this model, and compared the results to mean field and one-loop calculations.We extracted the mass-radius relation for a static, spherically symmetric compact star corresponding to our model, which was compared to other results as well. Here we present our results and the latest extended models on the effect of quantum fluctuations in neutron star mass and radii.

1. Introduction

Describing the behavior of nuclear matter and providing equation of state is still an active research field, with many challenges. At low densities and high temperatures accelerator experiments and lattice QCD provides information, but states characterized by high density and low temperature are inaccessible for direct measurements. Nuclear matter in this extreme state is present in compact astrophysical objects such as neutron-, quark-, or hybrid stars, and by modelling these objects one can get more insight into the nuclear equation of state for high densities. Studying the inner structure of compact stars is a challenge due to the lack of direct probes or measurements of their interior. Recent spectroscopic radius measurements using X-ray data analysis [1] and even the gravitational-wave discoveries [2, 3] may provide additional data, which led us to a more reliable description of super-dense nuclear matter.

The above mentioned task is shaded by the *masquarade* problem: different equation of state (EoS) results similar observeables of compact astrophysical obejcts [4]. On the other hand, model calcualtions for the phase structure of nuclear matter shows the importance of correct treatmnet of quantum fluctuations in the bosonic sector [5], so considering bosonic quantum fluctuations in neutron star EoS may weaken the masquerade problem.

In this work we follow Refs. [6, 7, 8] and use a simple model of Yukawa interaction between one fermionic and bosonic degree of freedom where the effect of bosonic quantum fluctuations induced by self interaction is considered. We use the Wetterich equation to compute thermodynamic quantities in Local Potential Approximation (LPA) with the optimized Litim regulator [9]. The calculated EoS is studied by solving the Tolman – Oppenheimer – Volkov (TOV) equations and calculating the mass-radius relation of compact stars in different levels of approximation for quantum fluctuations. Comparison of the results with other high density low temperature EoS is given, and the magnitude of the fluctuations is shown in various observables.

2. The interacting Fermi gas in the FRG framework

The functional renormalization group (FRG) method interpolates smoothly between microscopic scale and the macroscopic observable quantities in a general system. The FRG formalism led us to calculate the effect of quantum fluctuations on macroscopic thermodynamical quantities like pressure and energy density, by using the FRG method at zero temperature and finite chemical potential, which is a key for investigating the matter of compact stars.

The FRG method accounts for the quantum effects by introducing a regulator R_k in the action, which acts as a mass term, and suppresses modes below scale k. This makes, the action Γ_k and all quantities derived from it, scale dependent, which is determined by the Wetterich equation (1) as explained in Refs. [10, 11, 12],

$$\partial_k \Gamma_k = \frac{1}{2} \int \mathrm{d}p^D \, \mathrm{STr} \left[(\partial_k R_k) \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right], \tag{1}$$

where $\Gamma_k^{(2)}$ is the second derivative matrix of the effective action. The term 'STr' denotes the normal *trace* operation but includes a negative sign for fermionic fields and sums over all indices. The observable quantities are derived from the low-scale IR effective action which is computed by integrating the Wetterich equation (1), from the classical limit at the UV-scale $k = \Lambda$ to the IR scale $k = 0$, where quantum effects are taken into account. The parameters of the UV (classical) action $\Gamma_{k=\Lambda}$, has to be chosen in a way that the known IR quantities are correctly reproduced.

To study the effect of quantum fluctuations on EoS and on compact star observables, we use a simple Yukawa-type model with one bosonic and one fermionic degree of freedom described by the bare action, (2).

$$\Gamma_k[\varphi, \psi] = \int \mathrm{d}^4 x \left[\bar{\psi}(i\partial\!\!\!/ - g\varphi)\psi + \frac{1}{2}(\partial_\mu \varphi)^2 - U_k(\varphi) \right]. \tag{2}$$

The effect of bosonic fluctuations is characterized by the scale dependent effective potential U_k. Following [6] the Wetterich equation for U_k at finite temperature is,

$$\partial_k U_k = \frac{1}{2} \mathrm{STr} \ln \left[R_k + \Gamma_k^{(2)} \right] = \frac{k^4}{12\pi^2} \left[\frac{1 + 2n_B(\omega_B)}{\omega_B} + 4 \frac{-1 + n_F(\omega_F - \mu) + n_F(\omega_F + \mu)}{\omega_F} \right], \tag{3}$$

where n_B and n_F are the Bose – Einstein and the Fermi – Dirac distributions, respectively

$$n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta \omega_{B/F}}}, \text{ while } \omega_B^2 = k^2 + \partial_\varphi^2 U, \;\; \omega_F^2 = k^2 + g^2 \varphi^2, \;\; \text{and } \; \beta = \frac{1}{T}. \tag{4}$$

Equation (3) for the running of the effective potential U_k is solved by the method described in Ref. [7] at zero temperature. The resulting EoS is studied in Ref. [8] and the corresponding neutron stars are compared to other models. In the next chapter we show the effect of quantum fluctuations on compact star observables and study how they manifest in different levels of approximations.

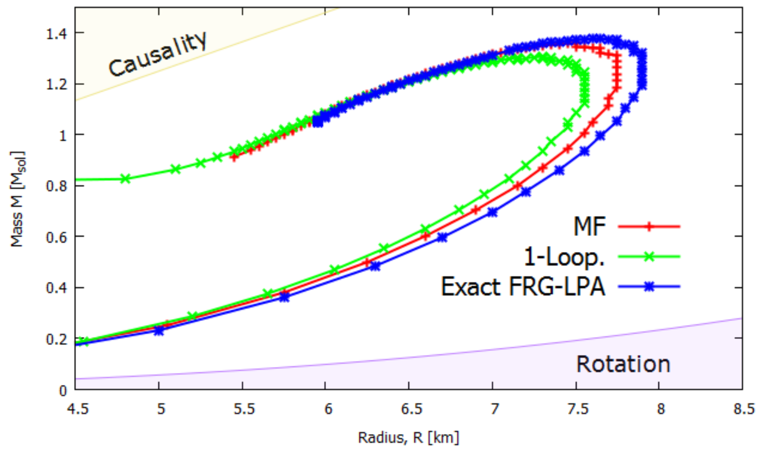

Figure 1: Comparison of the M-R diagram corresponding to different approximations

3. Results: The effect of quantum fluctuations on compact star observables

In Ref. [8] we calculated the mass-radius relation for the neutron stars corresponding to our simplified model with quantum fluctuations. The results showed that the mass and the radius of the compact stars depend on the level of approximation for the quantum fluctuations as it is shown on Fig. 1.

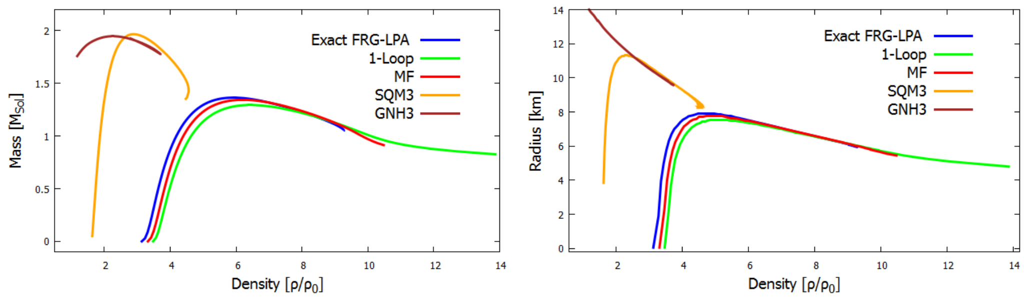

Figure 2: The density dependece of neutron star mass and radius for the FRG based calculation is compared to the GNH3 and SQM3 models which are taken from Ref. [1]. The density is the nuclear saturation density units $\rho_0 = 0.153\,\mathrm{fm}^{-3}$

The mean field calculation (MF) contains no quantum fluctuations, the one-loop model (1-Loop) is the simplest case which considers the effect of fluctuations, and the FRG method (Exact FRG-LPA) takes into account sufficiently high order terms so that the solution to the Wetterich equation converges. Fig. 1 shows that taking into account quantum fluctuations increases the radius and mass of the corresponding neutron star, but only if sufficiently high terms are considered, which is demonstrated by the fact that the one-loop solution has smaller radius and mass than the mean field one. It is also apparent from Fig. 1 that the effect of fluctuations is the most relevant at the high mass stars, and it disappears at the unstable and low star mass (approximately below $0.4\,\mathrm{M}_\odot$) part of the M-R diagram

We calculated how the mass and radius of neutron stars depend on the averge density of the star using the TOV equations, and compared our model to some other EoS taken from Ref. [1], which are typically used in compact star models. Since our model does not contain any repulsive force besides the fermionic behaviour of the matter, the resulting objects are much more dense, but the situation is similar to Fig. 1: including bosonic fluctuations improves the situation, but they have to be considered up to sufficiently high order as in the exact FRG calculation (considering one loop only worsen the situation). The FRG calculations shows that fluctuations lower the density corresponding to the highest mass stars and increases maximum neutron star mass, by approximately 5 percent. The difference between the Exact FRG-LPA, MF and 1-Loop calculations disappears above a given average density which is approximately $8\,\rho_0$ and $6\,\rho_0$ corresponding to mass and radius calculations. It is also apparent that considering fluctuations only in one loop order has an opposite effect to observables than the correct high order FRG-LPA calculation, therefore low order corrections are not enough to understand the physical effects of quantum corrections.

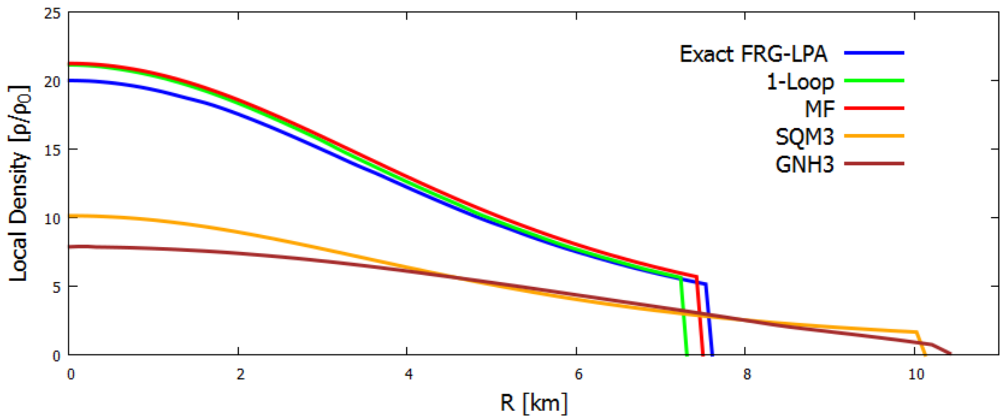

Figure 3: Matter density in neutrons stars as a function of distance from the centre calculated in the interacting Fermi gas model and EoS from Ref. [1]. The mass of the resulting stars are: Mean field: 1.37 $M_\odot$, one-loop: 1.31 $M_\odot$, FRG-LPA: 1.38 $M_\odot$, GNH3: 1.96 $M_\odot$, SQM3: 1.91 $M_\odot$

To see exactly how quantum corrections influence neutron star observables we calculated the matter density in the neutron star as a function of distance from the centre for a spherically symmetric and static case. The results for the highest mass stars corresponding to each EoS is shown on Fig. 3. The Fermi gas model starts at higher densities because of reasons mentioned above, but the FRG calculation shows that the quantum fluctuations lower the density and make the curve less steeper, overall much more similar to the sophisticated GNH3 and SQM3 models. Fig 3 also shows that although the MF and 1-Loop EoS start at higher densities they do not produce higher star mass, because the integration reach the edge of the star defined by $p = 0$ earlier than FRG-LPA, GNH3 and SQM3. The curve corresponding to the GNH3 model is also less steeper than the SQM3 curve and it produces a higher mass star too, so including quantum fluctuations in other models may increase neutron star mass, since the density curve becomes less steep.

4. Summary

Based on Refs. [6, 7, 8] using the FRG method, we demonstrated the effect of bosonic quantum fluctuations on neutron star observables considering the simplest interacting Fermi-gas model and compared our results to other EoS as well. We highlighted that quantum fluctuations improve on the model if they considered in sufficiently high precision.

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