On non-isometric extensions of some GR space-times – a branching perspective

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One focus in the debate over determinism of GR has been the initial value problem. By the celebrated Choquet-Bruhat and Geroch (1969) theorem, an initial data set admits a unique, up to isometry, maximal Cauchy development. By definition, this is a maximal globally hyperbolic space-time satisfying Einstein's Field Equations and appropriately related to the initial data set. The theorem does not prohibit the existence of multiple non-isometric developments of an initial data set that are *not* globally hyperbolic. Such multiple developments are extensions of a maximal globally hyperbolic space-time; the most studied examples of this sort are non-isometric extensions of the Gowdy polarized space-time or of the Taub-NUT space-time. Since non-isometric extensions of a maximal globally hyperbolic space-time have isometric regions (i.e., isometric to the original maximal globally hyperbolic space-time), they witness indeterminism of GR, in the sense of J. Butterfield's (1989) definition of determinism.

Non-isometric extensions present a conundrum for a branching-style analysis of indeterminism, however. Given the presence of closed timelike curves, the branching framework needs to be generalized beyond Belnap's (1992) theory by relaxing the postulate of antisymmetric causal ordering. With this relaxation accomplished we still face what seems to be the main problem: since branching is committed to thinking in terms of "little" objects facing alternative possible future evolutions, the challenge is to find in GR candidates for little objects with bifurcate alternative possible paths. Here a feature of Taub-NUT as well as of the polarized Gowdy space-times can help: each can be extended to a non-Hausdorff manifold, whose maximal Hausdorff sub-manifolds are identifiable with the (non-isometric) extensions of the original space-time. The non-Hausdorff manifold thus can be viewed as providing a modal format that accommodates all possible GR space-times developing from a given initial data set. This instinct led Müller (2013) and Placek (2014) to develop a "topological" version of branching, in which a possible history is identified with a maximal Hausdorff sub-manifold of a base manifold (typically non-Hausdorff). On this analysis, non-isometric extensions of a maximal globally hyperbolic space-time come out as alternative possible histories, providing evidence for indeterminism, in agreement with the verdict of Butterfield's (1989) definition. But with this topological turn, do we have candidates for locate objects with bifurcate alternative possible paths? The non-Hausdorff manifold encompassing non-isometric extensions of the Taub-NUT space-time (or non-isometric extensions of the polarized Gowdy space-time) contains no bifurcate geodesics. Even more generally, Hájíček's (1971) theorem suggests that there are no bifurcate curves in these manifolds. We thus face a dilemma: on the one hand, since the extensions are non-isometric Hausdorff manifolds, it looks as if GR were indeterministic. On the other hand, since no curve bifurcates in a non-Hausdorff manifold encompassing the non-isometric extensions, it looks as if no object had alternative possible evolutions, which prompts one to say "determinism". But how can the world be both globally indeterministic and locally deterministic?

References

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