

A mathematical logic based approach to isotropy, homogeneity and special principle of relativity

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We formalize the isotropy of space, the homogeneity of space and time, and the special principle of relativity theory in first-order logic (FOL) and we investigate their interrelationships. Rindler [5] and Dixon [2] claim that “the principle of relativity is equivalent to the isotropy (of space) and the homogeneity (of space and time)”. We analyze this statement within the scope of FOL.

In [3, 4] we introduced a FOL language for space-time theories and a formula schema SPR^+ for the special principle of relativity. We formalize the isotropy of space, the homogeneity of space and time in similar ways and obtain formula schemas

- ISO for the isotropy of space,
- SHOM for the homogeneity of space, and
- THOM for the homogeneity of time.

In our statements, we will use the following formulas and axiom systems:

- ASync is a FOL formula expressing that there is a clock which gets out of synchronism.
- Rest(+) is a FOL formula expressing that if two observers do not move relative to each other, then the world-view transformation between them preserves the orientation of space and the direction of time.
- SpecRel is an axiom system for special relativity.
- NewtKin is an axiom system for Newtonian kinematics.

We obtain the following results if we assume some auxiliary axioms:

- $ISO \models SHOM$, i.e. the isotropy of space implies the homogeneity of space.
- $THOM \models SHOM$, i.e. the homogeneity of time implies the homogeneity of space.
- $(SHOM + ASync) \models THOM$, i.e. if there is a clock which gets out of synchronism, the homogeneity of space implies the homogeneity of time.
- $(SHOM + NewtKin) \not\models THOM$, i.e. the homogeneity of space does not imply the homogeneity of time in Newtonian kinematics.
- $(SHOM + THOM + NewtKin) \not\models ISO$ and $(SHOM + THOM + SpecRel) \not\models ISO$, i.e. the homogeneity of space-time does not imply the isotropy in special relativity theory and in Newtonian kinematics.
- $SPR^+ \models (ISO + THOM + SHOM)$, i.e. the special principle of relativity implies the isotropy of space and the homogeneity of space-time.
- $(ISO + THOM + SHOM) \not\models SPR^+$, i.e. the isotropy of space and the homogeneity of space-time do not imply the special principle of relativity.
- $(ISO + THOM + Rest(+)) \models SPR^+$, i.e. if we assume that the world-view transformations between observers which do not move relative to each other preserve the direction of time and the orientation space, then the isotropy of space and the homogeneity of time imply the special principle of relativity.
- If we replace SPR^+ with its weaker version SPR , then assumption $Rest(+)$ can be omitted from the last statement, i.e. $(ISO + THOM) \models SPR$.

This is only a sample of our approach (see the references in [1]) to the logical analysis of space-time theories in the axiomatic framework of modern mathematical logic. The aim of our research is to build a flexible hierarchy of axiom systems (instead of one axiom system only), analyzing the logical connections between the different axioms and axiomatizations. We try to formulate simple, logically transparent and intuitively convincing axioms. The questions we study include: What is believed and why? - Which axioms are responsible for certain predictions? - What happens if we discard some axioms? - Can we change the axioms, and at what price?

References

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