Changeable Sets and their Possible Applications to the Foundations of Physics *

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1. Introduction. In spite of huge success of modern theoretical physics and the mightiness of the mathematical tools, applied by it, the foundations of theoretical physics remain unclear. So the well-known sixth Hilbert's problem of mathematically strict formulation of the foundations of theoretical physics remains actual at the present time. A lot of papers were devoted to this problem (for example see [1–11]), but completely it is not solved to this day.

In our opinion, main cause of the lack of productivity of many attempts to solve the sixth Hilbert's problem is the absence of a single abstract and systematic approach to this problem.

The idea of involving the set theory as the mathematical apparatus for solution of the sixth Hilbert's problem seems quite natural. Indeed, any picture of our surrounding reality looks like as a set of some objects. But, if we look more carefully at any set, appearing in our reality, we may notice some details, which distinguish it from the classical set. For example, let us consider the set of all cats of Kyiv region. This set is the set in the classical sense if and only if we observe it at the some fixed time point. But if we observe this set during the some time interval, we must state that this set has not constant composition. Indeed, any cat can change its properties such as geometrical position, weight, chemical composition and others. Also, the cats may born or die or cross the boundaries of Kyiv region. Moreover any cat changes some properties (such as, for example, its geometrical position) depending on the point of view on it (that is depending on the reference frame). We name sets of such type by **changeable sets**.

The problem of constructing the mathematical theory of changeable sets was emerged in the papers of Russian scientist Alexander Levich (see, for example [12, 13]). Some not very successful and unfinished attempts to construct the mathematical objects, similar to the changeable sets were made in the papers [14, 15].

In the next two sections we introduce the main concepts of the theories of changeable sets and kinematic changeable sets, developed in [24–27]. The most complete and detailed explanation of the theories of changeable sets and kinematic changeable sets can be found in the preprint [28].

2. Short Introduction to the Theory of Changeable Sets. In this section we are going to give the strict definition of changeable set. This operation will be made in two steps. In the first step we formulate the definition of base changeable sets.

2.1. Base Changeable Sets From the intuitive point of view base changeable sets may be interpreted as the simplest particular cases of changeable sets, that is as the changeable sets with a single reference frame.

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Let $\mathbb{T} = (\mathbf{T}, \leq)$ be any linearly (totally) ordered set (the sense of [29, p. 12]) and let $\mathcal{X}$ be any nonempty set. For any ordered pair $\omega = (t, x) \in \mathbf{T} \times \mathcal{X}$ we use the following denotations:

$$\mathsf{bs}\,(\omega) := x, \quad \mathsf{tm}\,(\omega) := t.$$

According to [30], the ordered triple of kind $\mathcal{B} = (\mathbf{B}, \mathbb{T}, \Leftarrow)$, where $\mathbf{B} \subseteq \mathbf{T} \times \mathcal{X}$, is named by ***base changeable set*** if and only if the following conditions are satisfied:

1. $\mathbf{B} \neq \emptyset$ and $\Leftarrow$ is reflexive binary relation on $\mathbf{B}$ (that is $\forall \omega \in \mathbf{B}\ \omega \Leftarrow \omega$);
2. for arbitrary $\omega, \omega_2 \in \mathbf{B}$ the conditions $\omega_2 \Leftarrow \omega_1$ and $\omega_1 \neq \omega_2$ cause the inequality $\mathsf{tm}\,(\omega_1) < \mathsf{tm}\,(\omega_2)$, where $<$ is the strict order relation, generated by the non-strict order $\leq$ of linearly ordered set $\mathbb{T} = (\mathbf{T}, \leq)$.

Remark 1. For an arbitrary base changeable set $\mathcal{B} = (\mathbf{B}, \mathbb{T}, \Leftarrow) = (\mathbf{B}, (\mathbf{T}, \leq), \Leftarrow)$ (where $\mathbf{B} \subseteq \mathbf{T} \times \mathcal{X}$) we use the following *denotations and terminology*:

$$\begin{aligned}
&\mathbb{B}\mathfrak{s}(\mathcal{B}) := \mathbf{B}; \qquad \underset{\mathcal{B}}{\leftarrow} := \Leftarrow; \\
&\mathbb{T}\mathbf{m}(\mathcal{B}) := \mathbb{T}; \qquad \mathbf{Tm}(\mathcal{B}) := \mathbf{T}; \\
&\mathfrak{B}\mathfrak{s}(\mathcal{B}) := \{x \in \mathcal{X} \mid \exists\, \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\ (\mathsf{bs}\,(\omega) = x)\} = \{\mathsf{bs}\,(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}\,.
\end{aligned} \tag{1}$$

- The set $\mathfrak{B}\mathfrak{s}(\mathcal{B})$ is named by the basic set or the set of all elementary states of $\mathcal{B}$.
- The set $\mathbb{B}\mathfrak{s}(\mathcal{B})$ is named by the set of all elementary-time states of $\mathcal{B}$.
- The set $\mathbf{Tm}(\mathcal{B})$ is named by the set of time points of $\mathcal{B}$.
- The relation $\underset{\mathcal{B}}{\leftarrow}$ is named by the base of elementary processes of $\mathcal{B}$.

Remark 2. In the cases, when the base changeable set $\mathcal{B}$ is known in advance we use the denotation $\leftarrow$ instead of the denotation $\underset{\mathcal{B}}{\leftarrow}$.

For the elements $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ the record $\omega_2 \underset{\mathcal{B}}{\leftarrow} \omega_1$ should be interpreted as "the elementary-time state ω_2 is the result of transformations (or the transformation prolongation) of the elementary-time state ω_1".

Definition 1. *Let $\mathcal{B}$ be a base changeable set. Any subset $S \subseteq \mathbb{B}\mathfrak{s}(\mathcal{B})$ we name by a* ***changeable system*** *of the base changeable set $\mathcal{B}$.*

The concept of changeable system may be considered as some abstract generalization of the notion of physical body, which, in the general case, has not constant composition over time.

2.2. Changeable Sets Changeable sets, to be introduced in this subsection, may be interpreted as abstractions of models of physical and other processes of macrocosm in the framework of observation in many, different, reference frames.

Definition 2. *Let $\overleftarrow{\mathcal{B}} = (\mathcal{B}_\alpha \mid \alpha \in \mathcal{A})$ be any indexed family of base changeable sets (where $\mathcal{A} \neq \emptyset$ is the some set of indexes). The system of mappings $\overleftarrow{\mathfrak{U}} = (\mathfrak{U}_{\beta\alpha} \mid \alpha, \beta \in \mathcal{A})$ of kind:*

$$\mathfrak{U}_{\beta\alpha} : 2^{\mathbb{B}\mathfrak{s}(\mathcal{B}_\alpha)} \longmapsto 2^{\mathbb{B}\mathfrak{s}(\mathcal{B}_\beta)} \qquad (\alpha, \beta \in \mathcal{A})$$

is referred to as ***unification of perception*** *on $\overleftarrow{\mathcal{B}}$ if and only if the following conditions are satisfied:*

1. *$\mathfrak{U}_{\alpha\alpha} A = A$ for any $\alpha \in \mathcal{A}$ and $A \subseteq \mathbb{B}\mathfrak{s}\,(\mathcal{B}_\alpha)$.*
   (Here and further we denote by $\mathfrak{U}_{\beta\alpha} A$ the action of the mapping $\mathfrak{U}_{\beta\alpha}$ to the set $A \subseteq \mathbb{B}\mathfrak{s}\,(\mathcal{B}_\alpha)$, that is $\mathfrak{U}_{\beta\alpha} A := \mathfrak{U}_{\beta\alpha}(A)$.)

2. *Any mapping $\mathfrak{U}_{\beta\alpha}$ is a monotonous mapping of sets, IE for any $\alpha, \beta \in \mathcal{A}$ and $A, B \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$ the condition $A \subseteq B$ assures $\mathfrak{U}_{\beta\alpha} A \subseteq \mathfrak{U}_{\beta\alpha} B$.*

3. *For any $\alpha, \beta, \gamma \in \mathcal{A}$ and $A \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$ the following inclusion holds:*

$$\mathfrak{U}_{\gamma\beta}\mathfrak{U}_{\beta\alpha} A \subseteq \mathfrak{U}_{\gamma\alpha} A. \qquad (2)$$

*In this case we name the mappings $\mathfrak{U}_{\beta\alpha}$ ($\alpha, \beta \in \mathcal{A}$) by **unification mappings**, and the triple of kind:*

$$\mathcal{Z} = \left(\mathcal{A}, \overleftarrow{\mathcal{B}}, \overleftarrow{\mathfrak{U}}\right)$$

*we name by **changeable set**.*

Remark 3 (Some remarks to definition 2).

- Unification mappings in the definition of changeable set indicate how any changeable system from one reference frame must be looked out in other reference frame.
- The second condition of the definition of changeable set is dictated by the natural desire "to see" a subsystem of a given changeable system in a given reference frame as the subsystem of "the same" changeable system in other reference frame.
- At first glance the inclusion (2) in the third condition of the definition of changeable set must be an equality. Replacement of the equal sign by the sign inclusion is motivated by the permission to "distort the picture of reality" during "transition" to other reference frame in the our abstract theory. We suppose, that during this "transition" some elementary-time states may turn out to be "invisible" in other reference frame. It is known, that in relativity theory some events, visible in any inertial reference frame, may become invisible in some non-inertial frames. The third condition of the definition of changeable set also leads to some interesting effect. It turns out, that in abstract changeable sets structures, like to parallel worlds, may appear. In more details this situation is described in the works [31], [28, Section 12].

Remark 4 (on denotations). Let $\mathcal{Z} = \left(\mathcal{A}, \overleftarrow{\mathcal{B}}, \overleftarrow{\mathfrak{U}}\right)$ be a changeable set, where $\overleftarrow{\mathcal{B}} = (\mathcal{B}_\alpha \mid \alpha \in \mathcal{A})$ is an indexed family of base changeable sets and $\overleftarrow{\mathfrak{U}} = (\mathfrak{U}_{\beta\alpha} \mid \alpha, \beta \in \mathcal{A})$ is an unification of perception on $\overleftarrow{\mathcal{B}}$. Further we will use the following terms and notations:

1) The set $\mathcal{A}$ will be named the ***index set*** of the changeable set $\mathcal{Z}$, and it will be denoted by $\mathcal{I}nd(\mathcal{Z})$.
2) For any index $\alpha \in \mathcal{I}nd(\mathcal{Z})$ the pair $(\alpha, \mathcal{B}_\alpha)$ will be referred to as ***reference frame*** of the changeable set $\mathcal{Z}$.
3) The set of all reference frames of $\mathcal{Z}$ will be denoted by $\mathcal{L}k(\mathcal{Z})$:

$$\mathcal{L}k(\mathcal{Z}) := \{(\alpha, \mathcal{B}_\alpha) \mid \alpha \in \mathcal{I}nd(\mathcal{Z})\}.$$

Typically, reference frames will be denoted by small Gothic letters ($\mathfrak{l}, \mathfrak{m}, \mathfrak{k}, \mathfrak{p}$ and so on).
4) For $\mathfrak{l} = (\alpha, \mathcal{B}_\alpha) \in \mathcal{L}k(\mathcal{Z})$ we introduce the following denotations:

$$\mathtt{ind}(\mathfrak{l}) := \alpha; \quad \mathfrak{l}^{\hat{}} := \mathcal{B}_\alpha.$$

Thus, for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ the object $\mathfrak{l}^{\hat{}}$ is a base changeable set. Further, when it does not cause confusion, for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ the symbol "$\hat{}$" will be omitted in the denotations $\mathfrak{Bs}(\mathfrak{l}^{\hat{}})$, $\mathbb{Bs}(\mathfrak{l}^{\hat{}})$, $\mathbf{Tm}(\mathfrak{l}^{\hat{}})$, $\mathbb{Tm}(\mathfrak{l}^{\hat{}})$, $\underset{\mathfrak{l}^{\hat{}}}{\leftarrow}$, and the denotations $\mathfrak{Bs}(\mathfrak{l})$, $\mathbb{Bs}(\mathfrak{l})$, $\mathbf{Tm}(\mathfrak{l})$, $\mathbb{Tm}(\mathfrak{l})$, $\underset{\mathfrak{l}}{\leftarrow}$ will be used instead.

5) For any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ the mapping $\mathfrak{U}_{\mathtt{ind}(\mathfrak{m}),\mathtt{ind}(\mathfrak{l})}$ will be denoted by $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle$. Hence:

$$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle = \mathfrak{U}_{\mathtt{ind}(\mathfrak{m}),\mathtt{ind}(\mathfrak{l})}.$$

In the case, when the changeable $\mathcal{Z}$ set is known in advance, the symbol $\mathcal{Z}$ in the above notations will be omitted, and the denotation "$\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle$" will be used instead.

6) In the case, when it does not cause confusion, we will use the denotation $\leftarrow$ instead of the denotation $\underset{\mathfrak{l}}{\leftarrow}$.

7) For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ we reserve the terminology, introduced in Remark 1 (where the symbol $\mathcal{B}$ should be replaced by the symbol "$\mathfrak{l}$" and the phrase "base changeable set" should be replaced by the phrase "reference frame").

Definition 3. *We say, that a changeable set $\mathcal{Z}$ is* ***precisely visible*** *if and only if for any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ and for any element $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ there exist a unique element $\omega' \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ such, that $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\omega'\}$.*[1]

Let $\mathcal{Z}$ be any precisely visible changeable set and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ be any reference frames of $\mathcal{Z}$. For any $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ we denote by $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle \omega$ (or by $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega$) the unique (in accordance with Definition 3) element $\omega' \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ such, that $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\omega'\}$. Hence, we have $\forall \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\ \langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega\}$. The mapping $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle : \mathbb{B}\mathfrak{s}(\mathfrak{l}) \mapsto \mathbb{B}\mathfrak{s}(\mathfrak{m})$ we name as the ***precise unification mapping*** of $\mathcal{Z}$.

Assertion 1 (See at [28])**.** *Let $\mathcal{Z}$ be any precisely visible changeable set, and $\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k(\mathcal{Z})$ be arbitrary reference frames of $\mathcal{Z}$. Then:*

1. $\forall \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}) \quad \langle !\, \mathfrak{l} \leftarrow \mathfrak{l} \rangle \omega = \omega$;
2. $\forall A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l}) \quad \langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle A = \{\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega \mid \omega \in A\}$;
3. $\forall \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}) \quad \langle !\, \mathfrak{p} \leftarrow \mathfrak{m} \rangle \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega = \langle !\, \mathfrak{p} \leftarrow \mathfrak{l} \rangle \omega$.

3. Kinematic Changeable Sets. Kinematic changeable sets are mathematical objects, in which changeable sets are equipped by different geometrical or topological structures, namely metric, topological, linear, Banach, Hilbert and other spaces. Such mathematical objects may be used for construction of physical models, acting in the framework of some space environment and including the spatial movement of bodies. For simplicity we restrict our consideration to the case, where the geometrical environment of changeable set is generated by linear normed space[2]. Moreover, here we consider only kinematic changeable sets with constant (unchanging over time) geometry.

3.1. Main definitions

Definition 4. *Let $\mathcal{Z}$ be any changeable set. An indexed family of kind $\mathcal{G} = \left(\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}, k_{\mathfrak{l}}\right) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z})\right)$ will be named by geometric environment of the changeable set $\mathcal{Z}$, if and only if for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ the following conditions are satisfied:*

1. $\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}\right)$ *is a linear normed space over real field $\mathbb{R}$ or complex field $\mathbb{C}$.*
2. $k_{\mathfrak{l}} : \mathfrak{B}\mathfrak{s}(\mathfrak{l}) \mapsto \mathfrak{X}_{\mathfrak{l}}$ *is a mapping from $\mathfrak{B}\mathfrak{s}(\mathfrak{l})$ to $\mathfrak{X}_{\mathfrak{l}}$.*

In this context the pair $\mathfrak{C} = (\mathcal{Z}, \mathcal{G}) = \left(\mathcal{Z}, \left(\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}, k_{\mathfrak{l}}\right) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z})\right)\right)$ will be named by ***vector kinematic changeable set****. Taking into account, that we consider only vector*

[1] In some papers (see, for example, [28, Definition I.12.3]) it had been given another, different, definition of precisely visible changeable set notion. Using [28, Corollary I.12.5 and Assertion I.12.11] it can be proved, that Definition 3 is equivalent to the definition, given in [28].

[2] More general variants of geometrical environments of changeable sets are considered, for example, in the papers [25,26,28]. But the accepted restrictions are quite sufficient for understanding the results, presented in this work.

*kinematic changeable sets in this article, further we use the terms "**kinematic changeable set**" or "**kinematic set**" instead of "vector kinematic changeable set".*

*We say, that the kinematic set $\mathfrak{C} = (\mathcal{Z}, \mathcal{G})$ is **precisely visible** if and only if the changeable set $\mathcal{Z}$ is precisely visible.*

Remark 5. Let, $\mathfrak{C} = \left(\mathcal{Z}, \left(\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}, k_{\mathfrak{l}}\right) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z})\right)\right)$ be any kinematic set. The sets:

$$\mathcal{L}k(\mathfrak{C}) := \mathcal{L}k(\mathcal{Z}); \qquad \mathcal{I}nd(\mathfrak{C}) := \mathcal{I}nd(\mathcal{Z})$$

will be named by the set of all reference frames and the the set of indexes of kinematic set $\mathfrak{C}$ (correspondingly).

Further we use the following denotations for arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C}) = \mathcal{L}k(\mathcal{Z})$:

1. We keep all denotations, introduced for reference frames of changeable sets (namely $\mathtt{ind}(\mathfrak{l})$, $\mathfrak{l}^{\hat{}}$, $\mathfrak{Bs}(\mathfrak{l})$, $\mathbb{B}\mathfrak{s}(\mathfrak{l})$, $\underset{\mathfrak{l}}{\leftarrow}$, $\mathbf{Tm}(\mathfrak{l})$, $\mathbb{T}\mathbf{m}(\mathfrak{l})$) together with abbreviated variants of these denotations, introduced in item 6) of Remark 4 and terminology, described in item 7) of Remark 4 (where the symbol "$\mathcal{Z}$" should be replaced by "$\mathfrak{C}$").

2. For unification mappings we use the following denotation:

$$\langle \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C} \rangle := \langle \mathfrak{m} \leftarrow \mathfrak{l},\, \mathcal{Z} \rangle\,,$$

and, in the case of precisely visible kinematic set $\mathfrak{C}$, we use the denotation:

$$\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C} \rangle\, \omega := \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle\, \omega \quad (\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}))\,.$$

3. Denote: $\mathbf{Zk}(\mathfrak{l};\, \mathfrak{C}) := \mathfrak{X}_{\mathfrak{l}}$, $\|\cdot\|_{\mathfrak{l},\mathfrak{C}} := \|\cdot\|_{(\mathfrak{l})}$, $\mathfrak{q}_{\mathfrak{l}}(x, \mathfrak{C}) := k_{\mathfrak{l}}(x) \in \mathfrak{X}_{\mathfrak{l}} = \mathbf{Zk}(\mathfrak{l};\, \mathfrak{C})$ $(x \in \mathfrak{Bs}(\mathfrak{l}))$.

The set $\mathbf{Zk}(\mathfrak{l};\, \mathfrak{C})$ will be named by set of ***coordinate values*** for reference frame $\mathfrak{l}$ in kinematic set $\mathfrak{C}$.

4. In the cases, when the kinematic set $\mathfrak{C}$ is known in advance, we will use the **abbreviated variants of denotations** $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle$, $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega$, $\mathbf{Zk}(\mathfrak{l})$, $\|\cdot\|_{\mathfrak{l}}$ and $\mathfrak{q}_{\mathfrak{l}}(x)$ instead of $\langle \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C} \rangle$, $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C} \rangle\, \omega$, $\mathbf{Zk}(\mathfrak{l};\, \mathfrak{C})$, $\|\cdot\|_{\mathfrak{l},\mathfrak{C}}$ and $\mathfrak{q}_{\mathfrak{l}}(x, \mathfrak{C})$ (correspondingly).

3.2. Theorem on Multi-image for Kinematic Sets

Definition 5. *The ordered triple $(\mathbb{T}, \mathcal{X}, U)$ will be referred to as **evolution projector** for base changeable set $\mathcal{B}$ if and only if:*

1. $\mathbb{T} = (\mathbf{T}, \leq)$ is linearly ordered set;

2. $\mathcal{X}$ is any set;

3. U is a mapping from $\mathbb{B}\mathfrak{s}(\mathcal{B})$ into $\mathbf{T} \times \mathcal{X}$ $(U : \mathbb{B}\mathfrak{s}(\mathcal{B}) \mapsto \mathbf{T} \times \mathcal{X})$.

Definition 6 ([31])**.** *Let $\mathcal{B}$ be any base changeable set. We will say, that elementary-time states $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ are **united by fate** in $\mathcal{B}$ if and only if at least one of the conditions $\omega_2 \leftarrow \omega_1$ or $\omega_1 \leftarrow \omega_2$ is satisfied.*

Theorem 1 (on image for base changeable sets [24, 28])**.** *Let $(\mathbb{T}, \mathcal{X}, U)$ be any evolution projector for base changeable set $\mathcal{B}$. Then there exist only one base changeable set $U[\mathcal{B}, \mathbb{T}]$, satisfying the following conditions:*

1. $\mathbb{T}\mathbf{m}(U[\mathcal{B}, \mathbb{T}]) = \mathbb{T}$;

2. $\mathbb{B}\mathfrak{s}(U[\mathcal{B}, \mathbb{T}]) = U(\mathbb{B}\mathfrak{s}(\mathcal{B})) = \{U(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}$;

3. Let $\widetilde{\omega}_1, \widetilde{\omega}_2 \in \mathbb{B}\mathfrak{s}(U[\mathcal{B}, \mathbb{T}])$ and $\mathsf{tm}(\widetilde{\omega}_1) \neq \mathsf{tm}(\widetilde{\omega}_2)$. Then $\widetilde{\omega}_1$ and $\widetilde{\omega}_2$ are united by fate in $U[\mathcal{B}, \mathbb{T}]$ if and only if, there exist united by fate in $\mathcal{B}$ elementary-time states $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ such, that $\widetilde{\omega}_1 = U(\omega_1)$, $\widetilde{\omega}_2 = U(\omega_2)$.

Remark 6. In the case, when $\mathbb{T} = \mathbb{T}\mathbf{m}(\mathcal{B})$ we use the denotation $U[\mathcal{B}]$ instead of the denotation $U[\mathcal{B}, \mathbb{T}]$:
$$U[\mathcal{B}] := U[\mathcal{B}, \mathbb{T}\mathbf{m}(\mathcal{B})].$$

Remark 7. Let $\mathcal{B}$ be any base changeable set and $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})} : \mathbb{B}\mathfrak{s}(\mathcal{B}) \mapsto \mathbf{Tm}(\mathcal{B}) \times \mathfrak{B}\mathfrak{s}(\mathcal{B})$ be the mapping, given by the formula: $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}(\omega) = \omega \quad (\omega \in \mathbb{B}\mathfrak{s}(\mathcal{B}))$. Then the triple $\left(\mathbb{T}\mathbf{m}(\mathcal{B}), \mathfrak{B}\mathfrak{s}(\mathcal{B}), \mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}\right)$, is, apparently, evolution projector for $\mathcal{B}$. Moreover, if we substitute $\mathbb{T}\mathbf{m}(\mathcal{B})$ and $\mathcal{B}$ into Theorem 1 instead of $\mathbb{T}$ and $U[\mathcal{B}, \mathbb{T}]$ (correspondingly), we can see, that all conditions of this Theorem are satisfied. Hence for the identity mapping $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}$ (on $\mathbb{B}\mathfrak{s}(\mathcal{B})$), we obtain:
$$\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}[\mathcal{B}] = \mathcal{B}.$$

Further $\mathfrak{R}(U)$ will mean the ***range*** of (arbitrary) mapping U.

Definition 7. *The evolution projector $(\mathbb{T}, \mathcal{X}, U)$ (where $\mathbb{T} = (\mathbf{T}, \le)$) for base changeable set $\mathcal{B}$ will be named **injective** if and only if the mapping U is injection from $\mathbb{B}\mathfrak{s}(\mathcal{B})$ to $\mathbf{T} \times \mathcal{X}$ (that is bijection from $\mathbb{B}\mathfrak{s}(\mathcal{B})$ onto the set $\mathfrak{R}(U) \subseteq \mathbf{T} \times \mathcal{X}$).*

Definition 8.

1. *The ordered composition of five sets $(\mathbb{T}, \mathcal{X}, U, \mathfrak{Q}, k)$ will be named by injective **kinematic vector projector** for base changeable set $\mathcal{B}$ if and only if:*

   1.1 $(\mathbb{T}, \mathcal{X}, U)$ *is injective evolution projector for $\mathcal{B}$.*

   1.2 $\mathfrak{Q} = (\mathfrak{X}, \|\cdot\|)$ *is a linear normed space.*

   1.3 k *is a mapping from $\mathcal{X}$ into $\mathfrak{X}$.*

2. *Any indexed family $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ (where $\mathcal{A} \ne \emptyset$) of injective kinematic vector projectors for base changeable set $\mathcal{B}$ we name by **kinematic vector multi-projector** for $\mathcal{B}$.*

Remark 8. Henceforward we will consider only injective kinematic vector projectors. That is why we will use the term “kinematic projector” instead of the term “injective kinematic vector projector”. Also we will use the term “kinematic multi-projector” instead of “kinematic vector multi-projector”.

Theorem 2 (on multi-image for kinematic sets [25, 28])**.** *Let $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ be a kinematic multi-projector for a base changeable set $\mathcal{B}$. Then:*

A) *Only one kinematic set $\mathfrak{C}$ exists, satisfying the following conditions:*

1. $\mathcal{L}k(\mathfrak{C}) = \{(\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \mid \alpha \in \mathcal{A}\}$.

2. *For any reference frames $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta]) \in \mathcal{L}k(\mathfrak{C})$ ($\alpha, \beta \in \mathcal{A}$) and any set $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l}) = U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B}))$ the following equality holds:*
$$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle A = U_\beta\left(U_\alpha^{[-1]}(A)\right) = \left\{U_\beta\left(U_\alpha^{[-1]}(\omega)\right) \mid \omega \in A\right\},$$
   *where $U_\alpha^{[-1]}$ is the mapping, **inverse** to U_α.*

3. *For any reference frame $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$ (where $\alpha \in \mathcal{A}$) the following equalities are performed:*
$$\textbf{2.1)}\ (\mathbf{Zk}(\mathfrak{l}), \|\cdot\|_{\mathfrak{l}}) = \mathfrak{Q}_\alpha; \qquad \textbf{2.2)}\ \mathfrak{q}_{\mathfrak{l}}(x) = k_\alpha(x) \quad (x \in \mathfrak{B}\mathfrak{s}(\mathfrak{l})).$$

B) *Kinematic set $\mathfrak{C}$, satisfying the conditions 1,2,3 is precisely visible.*

Remark 9. Suppose, that the kinematic set $\mathfrak{C}$ satisfies Condition 1 of Theorem 2. Then for any reference frame $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$, according to Remark 4 (item 4)), we have, $\mathtt{ind}(\mathfrak{l}) = \alpha$, $\mathfrak{l}^\wedge = U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]$, hence, $\mathbb{B}\mathfrak{s}(\mathfrak{l}) = \mathbb{B}\mathfrak{s}(\mathfrak{l}^\wedge) = \mathbb{B}\mathfrak{s}(U_\alpha[\mathcal{B}, \mathbb{T}_\alpha])$. Therefore, by Theorem 1, we obtain $\mathbb{B}\mathfrak{s}(\mathfrak{l}) = U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B}))$. Thus, Condition 2 of Theorem 2 is correctly formulated.

Definition 9. *Let, $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ be a kinematic multi-projector for a base changeable set $\mathcal{B}$. The kinematic set $\mathfrak{C}$, satisfying the conditions 1,2,3 of Theorem 2 will be named as **kinematic multi-image** of base changeable set $\mathcal{B}$ relatively the kinematic multi-projector $\mathfrak{P}$. This kinematic set will be denoted via $\mathfrak{Kim}[\mathfrak{P}, \mathcal{B}]$:*

$$\mathfrak{Kim}[\mathfrak{P}, \mathcal{B}] := \mathfrak{C}.$$

Example 1. Let, $(\mathfrak{X}, \|\cdot\|)$ be a linear normed space and $\mathcal{B}$ be a base changeable set such, that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{X}$ (such base changeable set $\mathcal{B}$ exists, because, for example, we may put $\mathcal{B} := \mathcal{A}t(\mathbb{T}, \mathcal{R})$, where $\mathcal{R}$ is a system of abstract trajectories from some linear ordered set $\mathbb{T}$ to a set $\mathbf{M} \subseteq \mathfrak{X}$, where the definition of $\mathcal{A}t(\mathbb{T}, \mathcal{R})$ can be found in [28]). Let $\mathbb{U}$ be any transforming set of bijections (in the sense of [25, formula (16)] or [28, Example I.11.2]) relatively the $\mathcal{B}$ on $\mathfrak{X}$, that is any mapping $\mathbf{U} \in \mathbb{U}$ is the bijection of kind, $\mathbf{U} : \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X} \longleftrightarrow \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X}$. Then, we have $\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{Bs}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X}$. Hence, the set of bijections $\mathbb{U}$ generates the kinematic multi-projector $\widehat{\mathbb{U}} := ((\mathbb{T}\mathbf{m}(\mathcal{B}), \mathfrak{X}, \mathbf{U}, (\mathfrak{X}, \|\cdot\|), \mathbb{I}_{\mathfrak{X}}) \mid \mathbf{U} \in \mathbb{U})$ for $\mathcal{B}$, where $\mathbb{I}_{\mathfrak{X}}$ is the identity mapping on $\mathfrak{X}$. Hence, in accordance with Theorem 2 and Definition 9, we can denote:

$$\mathfrak{Kim}(\mathbb{U}, \mathcal{B}; \mathfrak{X}) := \mathfrak{Kim}\left[\widehat{\mathbb{U}}, \mathcal{B}\right]. \tag{3}$$

3.3. Coordinate Transforms in Kinematic Sets Let, $\mathfrak{C}$ be any kinematic set. For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ we introduce the following denotations:

$$\mathbb{M}k(\mathfrak{l}; \mathfrak{C}) := \mathbf{Tm}(\mathfrak{l}) \times \mathbf{Zk}(\mathfrak{l}; \mathfrak{C}); \tag{4}$$

$$\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega; \mathfrak{C}) := (\mathsf{tm}(\omega), \mathsf{q}_{\mathfrak{l}}(\mathsf{bs}(\omega); \mathfrak{C})) \in \mathbb{M}k(\mathfrak{l}; \mathfrak{C}), \quad \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}). \tag{5}$$

The set $\mathbb{M}k(\mathfrak{l}; \mathfrak{C})$ we name by ***Minkowski set*** of the reference frame $\mathfrak{l}$ in the kinematic set $\mathfrak{C}$. The value $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega; \mathfrak{C})$ will be named by ***Minkowski coordinates*** of the elementary-time state $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ ***in the reference frame*** $\mathfrak{l}$.

In the cases, when the kinematic set $\mathfrak{C}$ is known in advance, we use the denotations $\mathbb{M}k(\mathfrak{l})$, $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)$ instead of the denotations $\mathbb{M}k(\mathfrak{l}; \mathfrak{C})$, $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega; \mathfrak{C})$ (correspondingly).

Definition 10. *Let $\mathfrak{C}$ be any precisely visible kinematic set and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ be arbitrary reference frames of $\mathfrak{C}$.*

1. *The mapping $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\,\cdot\,; \mathfrak{C}) : \mathbb{B}\mathfrak{s}(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$, represented by the formula:*

   $$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C}) = \mathbf{Q}^{\langle \mathfrak{m} \rangle}(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega), \quad \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$$

   *we name by **actual coordinate transform** from $\mathfrak{l}$ to $\mathfrak{m}$.*
   For any $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the value $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C})$ may be interpreted as Minkowski coordinates of the elementary-time state ω in the (another) reference frame $\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$.

2. *The mapping $\widetilde{Q} : \mathbb{M}k(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$ we will name by **universal coordinate transform** from $\mathfrak{l}$ to $\mathfrak{m}$ if and only if:*

   - *$\widetilde{Q}$ is bijection (one-to-one mapping) between $\mathbb{M}k(\mathfrak{l})$ and $\mathbb{M}k(\mathfrak{m})$.*
   - *For any elementary-time state $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the following equality is performed:*

   $$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C}) = \widetilde{Q}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)\right).$$

3. *We say, that reference frames* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\,(\mathfrak{C})$ ***allow universal coordinate transform****, if and only if at least one universal coordinate transform* $\widetilde{Q} : \mathbb{M}k(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$ *from* $\mathfrak{l}$ *to* $\mathfrak{m}$ *exists.*
   In the case, when reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\,(\mathfrak{C})$ *allow universal coordinate transform, we use the denotation:*
   $$\mathfrak{l} \underset{\mathfrak{C}}{\rightleftarrows} \mathfrak{m},$$
   In the case, when the kinematic set $\mathfrak{C}$ *is known in advance, we use the abbreviated denotation* $\mathfrak{l} \rightleftarrows \mathfrak{m}$.

4. *Indexed family of mappings* $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$ *will be named by* ***universal coordinate transform for the kinematic set*** $\mathfrak{C}$ *if and only if:*

   - *For arbitrary* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\,(\mathfrak{C})$ *the mapping* $\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}$ *is universal coordinate transform from* $\mathfrak{l}$ *to* $\mathfrak{m}$.
   - *For any* $\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k\,(\mathfrak{C})$ *and* $\mathrm{w} \in \mathbb{M}k(\mathfrak{l})$ *the following equalities are true:*

   $$\widetilde{Q}_{\mathfrak{l},\mathfrak{l}}(\mathrm{w}) = \mathrm{w}; \qquad \widetilde{Q}_{\mathfrak{p},\mathfrak{m}}\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w})\right) = \widetilde{Q}_{\mathfrak{p},\mathfrak{l}}(\mathrm{w}). \tag{6}$$

5. *We say, that the kinematic set* $\mathfrak{C}$ ***allows universal coordinate transform****, if and only if there exists at least one universal coordinate transform* $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$ *for* $\mathfrak{C}$.

Remark 10. In the cases, when the kinematic set $\mathfrak{C}$ is known in advance, we use the abbreviated denotation $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega)$ instead of the denotation $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C})$.

Assertion 2 (See at [26,28])**.** *For an arbitrary precisely visible kinematic set* $\mathfrak{C}$ *the following propositions are equivalent:*

1. $\mathfrak{C}$ *allows universal coordinate transform.*

2. *For arbitrary reference frames* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\,(\mathfrak{C})$ *it is true the correlation* $\mathfrak{l} \rightleftarrows \mathfrak{m}$ *(that is arbitrary two reference frames* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\,(\mathfrak{C})$ *allow universal coordinate transform).*

3. *There exists a reference frame* $\mathfrak{l} \in \mathcal{L}k\,(\mathfrak{C})$ *such, that for any reference frame* $\mathfrak{m} \in \mathcal{L}k\,(\mathfrak{C})$ *it is true the correlation* $\mathfrak{l} \rightleftarrows \mathfrak{m}$.

Theorem 3 (See at [28, 32])**.** *The kinematic set* $\mathfrak{C} = \mathfrak{Kim}\,(\mathbb{U}, \mathcal{B}; \mathfrak{X})$*, defined in (3), allows universal coordinate transform. Moreover,* $\mathcal{L}k\,(\mathfrak{C}) = ((\mathbf{U}, \mathbf{U}\,[\mathcal{B}]) \mid \mathbf{U} \in \mathbb{U})$*, and the system of mappings* $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$*, defined by:*

$$\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w}) = \mathbf{V}\left(\mathbf{U}^{[-1]}(\mathrm{w})\right), \quad \mathrm{w} \in \mathbb{M}k(\mathfrak{l}) = \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X} \tag{7}$$
$$(\,\mathfrak{l} = (\mathbf{U}, \mathbf{U}\,[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{C})\,, \quad \mathfrak{m} = (\mathbf{V}, \mathbf{V}\,[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{C})\,)$$

is universal coordinate transform for $\mathfrak{C}$.

Note that kinematic sets, which do not allow universal coordinate transform also exist. Nontrivial and interesting examples of such kinematic sets were investigated in the papers [28, 32].

4. Universal Kinematics.

4.1. Main Definitions. Universal Kinematics are defined as kinematic changeable sets with given universal coordinate transform.

Definition 11. *Let* $\overleftarrow{\mathcal{Q}} = \left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathfrak{C})}$ *be universal coordinate transform for the precisely visible kinematic set* $\mathfrak{C}$*. The pair*

$$\mathcal{F} = \left(\mathfrak{C}, \overleftarrow{\mathcal{Q}}\right)$$

is named by ***universal kinematic set*** *or, abbreviated, by* ***universal kinematics****.*

Remark 11. Let $\mathcal{F} = \left(\mathfrak{C}, \overleftarrow{\mathcal{Q}}\right)$, where $\mathfrak{C} = (\mathcal{Z}, \mathcal{G}) = \left(\mathcal{Z}, \left(\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}, k_{\mathfrak{l}}\right) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z})\right)\right)$ and $\overleftarrow{\mathcal{Q}} = \left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathfrak{C})=\mathcal{L}k(\mathcal{Z})}$ be any universal kinematics. In this case we use the following denotations:

1. We keep all denotations, introduced in Remark 5, formulas (4),(5) and Item 1 of Definition 10 for kinematic sets together with the abridged version of these denotations, where the symbol $\mathfrak{C}$ (in expressions "$\mathcal{L}k(\mathfrak{C})$", "$\mathcal{I}nd(\mathfrak{C})$", "$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C}\rangle$", "$\mathbb{M}k(\mathfrak{l}; \mathfrak{C})$", "$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle}(\omega; \mathfrak{C})$", etc) should be replaced by the symbol $\mathcal{F}$ and phrase "kinematic set" should be replaced by the phrase "universal kinematics".

2. For any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F}) = \mathcal{L}k(\mathcal{Z})$ we introduce the following denotation:

$$[\mathfrak{m} \leftarrow \mathfrak{l}; \mathcal{F}]\, \mathrm{w} = \widetilde{Q}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w}), \quad (\forall \mathrm{w} \in \mathbb{M}k(\mathfrak{l}))\,.$$

By Definition 10, the mapping $[\mathfrak{m} \leftarrow \mathfrak{l}; \mathcal{F}] : \mathbb{M}k(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$ is bijection between $\mathbb{M}k(\mathfrak{l})$ and $\mathbb{M}k(\mathfrak{m})$. In the cases, when the universal kinematics $\mathcal{F}$ is known in advance we use the abbreviated denotation $[\mathfrak{m} \leftarrow \mathfrak{l}]$ instead of the denotation $[\mathfrak{m} \leftarrow \mathfrak{l}; \mathcal{F}]$.

Using the introduced system of denotations and definition of universal coordinate transform (see Definition 10, item 4), we obtain the following assertion.

Assertion 3. *For arbitrary reference frames* $\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k(\mathcal{F})$ *of any universal kinematics* $\mathcal{F}$ *the following equalities are true:*

$$[\mathfrak{l} \leftarrow \mathfrak{l}]\, \mathrm{w} = \mathrm{w}; \quad (\forall \mathrm{w} \in \mathbb{M}k(\mathfrak{l}))\,; \tag{8}$$

$$[\mathfrak{p} \leftarrow \mathfrak{m}]\,[\mathfrak{m} \leftarrow \mathfrak{l}]\, \mathrm{w} = [\mathfrak{p} \leftarrow \mathfrak{l}]\, \mathrm{w} \quad (\forall \mathrm{w} \in \mathbb{M}k(\mathfrak{l}))\,. \tag{9}$$

4.2. Examples of Universal Kinematics.

Example 2. Let, $(\mathfrak{X}, \|\cdot\|)$ be a linear normed space, $\mathcal{B}$ be any base changeable set such, that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{X}$ and $\mathbb{U}$ be any transforming set of bijections relatively the $\mathcal{B}$ on $\mathfrak{X}$. Let $\mathfrak{C} = \mathfrak{Kim}(\mathbb{U}, \mathcal{B}; \mathfrak{X})$ be the kinematic set, defined in (3) and $\overleftarrow{\mathcal{Q}} = \left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathfrak{C})}$ be the system of mappings defined in (7). Then, according to Theorem 3, the ordered pair:

$$\mathfrak{Ku}(\mathbb{U}, \mathcal{B}; \mathfrak{X}) = \left(\mathfrak{Kim}(\mathbb{U}, \mathcal{B}; \mathfrak{X}),\, \overleftarrow{\mathcal{Q}}\right) \tag{10}$$

is an universal kinematics. Kinematics of type $\mathfrak{Ku}(\mathbb{U}, \mathcal{B}; \mathfrak{X})$ will be named by kinematics, generated by multi-image of the base changeable set $\mathcal{B}$.

Example 3. Let $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ be a Hilbert space over the real field, $\dim(\mathfrak{H}) \geq 1$ and $\mathcal{L}(\mathfrak{H})$ be the space of (homogeneous) linear continuous operators over the space $\mathfrak{H}$. Denote by $\mathcal{L}^{\times}(\mathfrak{H})$ the space of all operators of affine transformations over the space $\mathfrak{H}$, that is $\mathcal{L}^{\times}(\mathfrak{H}) = \left\{\mathbf{A}_{[\mathbf{a}]} \mid \mathbf{A} \in \mathcal{L}(\mathfrak{H}),\, \mathbf{a} \in \mathfrak{H}\right\}$, where $\mathbf{A}_{[\mathbf{a}]}x = \mathbf{A}x + \mathbf{a}$, $x \in \mathfrak{H}$. The ***Minkowski space*** over the Hilbert space $\mathfrak{H}$ is defined as the Hilbert space $\mathcal{M}(\mathfrak{H}) = \mathbb{R} \times \mathfrak{H} = \{(t, x) \mid t \in \mathbb{R},\, x \in \mathfrak{H}\}$, equipped by the inner product and norm: $\langle \mathrm{w}_1, \mathrm{w}_2\rangle = \langle \mathrm{w}_1, \mathrm{w}_2\rangle_{\mathcal{M}(\mathfrak{H})} = t_1 t_2 + \langle x_1, x_2\rangle$, $\|\mathrm{w}_1\| = \|\mathrm{w}_1\|_{\mathcal{M}(\mathfrak{H})} = \left(t_1^2 + \|x_1\|^2\right)^{1/2}$ (where $\mathrm{w}_i = (t_i, x_i) \in \mathcal{M}(\mathfrak{H})$, $i \in \{1, 2\}$) ([23, 28]).

Denote via $\mathbf{Pk}(\mathfrak{H})$ the set of all operators $\mathbf{S} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$, which have the continuous inverse operator $\mathbf{S}^{-1} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$. Operators $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H})$ will be named as ***coordinate transform operators***. Let, $\mathcal{B}$ be any base changeable set such, that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}$ and $\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R}, \leq)$, where $\leq$ is the standard order in the field of real numbers $\mathbb{R}$. Then $\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{Bs}(\mathcal{B}) \subseteq \mathbb{R} \times \mathfrak{H} = \mathcal{M}(\mathfrak{H})$. So, any set $\mathbb{S} \subseteq \mathbf{Pk}(\mathfrak{H})$ is a transforming set of bijections relatively the $\mathcal{B}$ on $\mathfrak{H}$. Therefore, according to (3) and (10), the universal kinematics $\mathfrak{Ku}(\mathbb{S}, \mathcal{B}; \mathfrak{H})$ exists.

Let $c \in (0, \infty]$ be a fixed positive constant, which has the physical content of the speed of light in vacuum. Let $\mathfrak{PT}(\mathfrak{H}, c)$, $\mathfrak{PT}_+(\mathfrak{H}, c)$, $\mathfrak{P}(\mathfrak{H}, c)$, $\mathfrak{P}_+(\mathfrak{H}, c)$ be the sets of operators, introduced in [24, 28]. According to [28, Corollary II.19.4], $\mathfrak{PT}(\mathfrak{H}, c), \mathfrak{PT}_+(\mathfrak{H}, c), \mathfrak{P}(\mathfrak{H}, c), \mathfrak{P}_+(\mathfrak{H}, c) \subseteq \mathbf{Pk}(\mathfrak{H})$. Substituting one of the sets $\mathfrak{PT}(\mathfrak{H}, c)$, $\mathfrak{PT}_+(\mathfrak{H}, c)$, $\mathfrak{P}(\mathfrak{H}, c)$ or $\mathfrak{P}_+(\mathfrak{H}, c)$ instead of $\mathbb{S}$, we deduce the following universal kinematics:

$$\begin{aligned}
\mathfrak{UPT}_0(\mathfrak{H}, \mathcal{B}, c) &:= \mathfrak{Ku}(\mathfrak{PT}(\mathfrak{H}, c), \mathcal{B}; \mathfrak{H}); \\
\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c) &:= \mathfrak{Ku}(\mathfrak{PT}_+(\mathfrak{H}, c), \mathcal{B}; \mathfrak{H}); \\
\mathfrak{UP}_0(\mathfrak{H}, \mathcal{B}, c) &:= \mathfrak{Ku}(\mathfrak{P}(\mathfrak{H}, c), \mathcal{B}; \mathfrak{H}); \\
\mathfrak{UP}(\mathfrak{H}, \mathcal{B}, c) &:= \mathfrak{Ku}(\mathfrak{P}_+(\mathfrak{H}, c), \mathcal{B}; \mathfrak{H}).
\end{aligned}$$

In the case $\dim(\mathfrak{H}) = 3$, $c < \infty$ the universal kinematics $\mathfrak{UP}(\mathfrak{H}, \mathcal{B}, c)$ represents the simplest mathematically strict model of the kinematics of special relativity theory in inertial frames of reference. Universal kinematics $\mathfrak{UP}_0(\mathfrak{H}, \mathcal{B}, c)$ is constructed on the basis of general Lorentz-Poincare group, and it includes apart from usual reference frames (with positive direction of time), which have understandable physical interpretation, also reference frames with negative direction of time. Universal kinematics $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$ and $\mathfrak{UPT}_0(\mathfrak{H}, \mathcal{B}, c)$ are based on generalized Lorentz-Poincare transforms over Hilbert space $\mathfrak{H}$ [23, 28], introduced for particular case $\dim(\mathfrak{H}) \leq 3$ in the papers of E. Recami, V. Olkhovsky and R. Goldoni (see [19–21]). These universal kinematics are including apart from standard ("tardyon") reference frames also "tachyon" reference frames, which are moving relatively the "tardyon" reference frames with velocity, greater than the velocity of light c. Universal kinematics $\mathfrak{UP}(\mathfrak{H}, \mathcal{B}, \infty) = \mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, \infty)$ in the case $\dim(\mathfrak{H}) = 3$, $c = \infty$ represents the mathematically strict model of the Galilean kinematics in the inertial frames of reference.

4.3. Short Overview of Some Results on Time Irreversibility of Universal Kinematics. Example 3 shows, that the theory of kinematic changeable sets may be used not only for mathematical foundations of classical Lorentz-Poincare and Galilean kinematics, but also for foundations of tachyon kinematics (that is kinematics under conditions of tachyon hypothesis). Among physicists it is widespread belief that, that the hypothesis of the existence of tachyons leads to temporal paradoxes, connected with the existence of theoretical possibility to change the own past. Nevertheless in the paper [18] it is shown, that the hypothesis of the existence of material objects, moving with superluminal velocities, does not result the violation of the principle of causality, that is the possibility of returning to the own past in the general case. Unfortunately, in [18] the superluminal reference frames are introduced only for the case of one dimensional space of geometric coordinates. Hence, the considered above tachyon kinematics of kind $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$ and $\mathfrak{UPT}_0(\mathfrak{H}, \mathcal{B}, c)$ are impossible to analyze on time irreversibility (IE on absence of possibility to return to the own past) using the results of [18]. Moreover, it can be proved, that the axiom "AxSameFuture" from [18, subsection 2.1] for these tachyon kinematics is not satisfied. In the papers [33, 34] it had been constructed the more general mathematical apparatus, suitable for investigation of these kinematics in the terms of time irreversibility. In these papers the strict definitions and sufficient conditions of time reversibility and time irreversibility for abstract universal kinematics had been established. Using these conditions it had been proved, that all tachyon kinematics, of kind $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$ and $\mathfrak{UPT}_0(\mathfrak{H}, \mathcal{B}, c)$ are (conditionally) time reversible. Also in the paper it was proved the existence of (certainly) time irreversible tachyon kinematics in the sense of E. Recami, V. Olkhovsky and R. Goldoni, which allows for inertial reference frames the motion with arbitrary velocity, different from the velocity of light.

5. Conclusions. The theory of kinematic changeable sets may be used not only for mathematical foundations of classical Lorentz-Poincare and Galilean kinematics, but also for foundations of different versions of tachyon kinematics. Investigations in this direction may be also interesting for astrophysics, because there exists the hypothesis, that in large scale of the Universe, physical laws (in particular, the laws of kinematics) may be different from the laws, acting in the neighborhood of our solar System.

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