Approximate Space-time Symmetries

The standard (FLRW) models of cosmology and their modifications, on which our confidence in the large-scale features of our universe's history rests, describe our universe as homogeneous and isotropic: at every instant of cosmological time, the distribution of matter is the same everyone and in every direction. Yet look around you! Our experience (thankfully) hardly confirms such monotony. This raises a puzzle, for the assumption of these symmetries has direct empirical significance (DES) that everyday experience seems to falsify grossly. How, then, can these models be successful?

The intuitive answer is that while these symmetries do not literally hold in the best models of our universe, they nevertheless hold approximately. Over the large distance scales relevant for cosmology, the universe is approximately homogeneous and isotropic. Hence—the intuitive answer continues—the idealization of exact symmetry does little harm for those purposes, even though much of the mathematical apparatus for drawing consequences from symmetries presumes that they are exact.

Approximate symmetry is widely invoked in contexts from space-time geometry to effective field theories, but it is rarely fully explicated. Such an explication, in the case of space-time symmetries, is the goal of this presentation, which reveals a surprising number of complexities. The mathematics involved can be subtle, and requires conceptual input regarding which properties (directly observable or not) are relevant in comparing approximate symmetry-related space-times, how different those properties can be while still preserving approximate symmetry, and what it means, in the case of local space-time symmetries, for two symmetry transformations to be similar to one another. This in turn sheds light on the DES of space-time symmetries, for being approximable is certainly sufficient for DES and—under the right conditions—it may be necessary, too.

The first step in this project is to develop the mathematical apparatus to represent approximate symmetry. Given, then, a space-time with a putative approximate group of symmetries, consider the image of the space-time under each element of the group. If the symmetry is exact, each space-time in this set will be automorphic to the one acted upon. But if the symmetry group is *merely approximate*, then some method is needed for comparison.

To this end I will deploy methods from topology, and uniform spaces in particular, which are a sort of intermediate between topological and metric spaces in terms of structure. I show how certain classes of observers in the space-time, when capable of measuring or determining various properties of space-time and matter fields thereon relevant to them, each determine a pseudometric on the set of space-times, whose values indicate how dissimilar two space-times are, according to that class of observers. (A pseudometric is a distance function that may also take on the value of zero for non-identical arguments.) Thus, for each element in the image of the merely approximate symmetry group, one can assign a measure of dissimilarity between it and the space-time acted upon by the group, relative to the class of observers and choice of contextually relevant properties.

At least two interesting types of approximate symmetry can then be defined: approximately symmetry *in the large*, and approximate symmetry *in the small*. For approximate symmetry in the large, each observer class determines that the putative approximate symmetry holds for them just when the supremum of their measures of dissimilarity across the image of the symmetry group falls below a certain maximum threshold value. Even after fixing a common threshold value and the properties to be compared, different classes of observers may come to different judgments as to whether approximate symmetry in the large holds. But this is as it should be, for the different circumstances in which an observer can find herself, with different degrees of measurement accuracy, ought to produce differing judgments on whether a symmetry holds due to its DES.

Approximate symmetry in the small is another, weaker, type of approximate symmetry available when one is concerned not with all elements of the symmetry group, but only ones *close to the identity*. Only over this restricted class would one take the supremum in the above construction. For instance, consider a homogeneous globally hyperbolic space-time with a foliation into space-like hypersurfaces on which there is a scalar field slowly changing from negative to positive infinity. One might want to consider it to be an approximately stationary space-time—one with an approximate time-like Killing vector field—since any observer whose four-velocity is close to being orthogonal to the hypersurfaces will not observe the scalar field change appreciably over small durations.

Making this idea precise requires putting topological structure (or something similar) on the symmetry transformations themselves, which in any case form a subset of the diffeomorphisms. This is trivial for discrete symmetry groups, and global continuous symmetries—ones that form a finite-dimensional Lie group—have a natural topology inherited from their manifold structure. But in the case of local continuous symmetries—ones that form an infinite-dimensional Lie group—there are different choices of topology available on the symmetry group, corresponding to different judgements about what it means for different transformations to be similar, just as in the case of topologies on the set of space-times. Thus, in this case, there is dependence on conceptual input twice over.

In general, a space-time will have lots of approximate symmetries in the small for certain classes of observers, but the ones that matter will depend on the context of investigation. This also raises the interesting question about which sorts of symmetries have merely approximate versions. Clearly, having this ought to be a sufficient condition for DES, once one describes how an approximate symmetry can be implemented as a physical transformation, i.e., on a subsystem of the world system. As time permits, I will discuss further consequences for DES, such as whether having a merely approximate version of a symmetry is a necessary condition for it to have DES, and whether this depends on what class of models is being considered.