TOWARDS A FORMAL THEORY OF DIGITAL PHYSICS: DIGITAL MULTIVERSES

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Abstract. We discuss the foundations of digital physics and its implications. The foundations of digital physics are expanded, and an analogue of the many-worlds interpretation of quantum mechanics under the digital physics formalism is presented, in addition to a more "economical" multiverse theory, which takes into account resource availability and discusses a naive account of universe likelihood. We also address some problems in the epistemology of physics along the way, which help to lay an epistemic groundwork and provide motivation for the feasibility of pursuing a digital theory of physics.

1. Introduction

To put it succinctly, the many-worlds interpretation of quantum mechanics, as proposed by Hugh Everett in his 1957 Ph.D. Thesis, denied the existence of the wavefunction collapse postulate that was included in other quantum theories. This led to a natural theory of infinitely-many orthogonal universes in Hilbert Space, which constitute Max Tegmark's Level III Multiverse [Tegmark, 2005]. This, however, works under the assumption that space is continuous over the real numbers. If we allow space to have a discrete topology, then we can give a quasi-synthetic a priori derivation that space is nothing more than a configuration of alphabet symbols of a Turing Machine, and our "universe" is exhaustively described by a Turing Machine (or cellular automaton, in the spirit of Stephen Wolfram [Wolfram, 2002], but it is at least as natural to use a Turing Machine). This is known as the Zuse Thesis [Zuse, 1967]. The derivation is intuitively depicted as follows:

Discrete Space Thesis → Kreisel Thesis → Church Thesis → Church-Turing Thesis → Zuse Thesis.

By assuming the Discrete Space Thesis, and accepting the (mostly non-controversial) theses of Kreisel, Church, and Church/Turing, we can derive the Zuse Thesis, which states that the universe is a digital computer. This statement leads to two epistemological and ontological discussion points:

(1) We take the Discrete Space Thesis as axiomatic, by virtue of self-evidence. The issue with this is that it may not be self-evident. There are several worthy refutations of discrete space, such as Zeno's Paradoxes of The Arrow and The Stadium, Hermann Weyl's Tile Argument [Weyl, 1949], the prevalence of continuous mathematical models of physics, e.g. General Relativity, and lack of empirical tools, for instance. If one can refute all known counterarguments, under some philosophy of mathematics/physics, to discrete physical space, then one could prove the validity of discrete space. This does not mean that one has exhaustively proven the validity of discrete space, as there could be other refutations out there that just haven't been thought of, but one could "epistemically" prove the validity at a certain time, so to speak. To "epistemically prove" not only the validity, but

the *absolute existence*, of discrete physical space, one would have to give an epistemic proof of the validity of discrete space and an epistemic disproof of the validity of continuous space. To give an *absolute proof* of discrete space would be to give an absolute proof of the validity of discrete space and an absolute disproof of continuous space, each under the same philosophy of mathematics/physics. Doing so under *every* philosophy of math/physics seems to be impossible. In order to not constrain ourselves and actually formulate a discrete (henceforth known as *digital* or *computational*, thanks to the several theses listed above) physical theory, we assume *digital philosophy* as our philosophy of mathematics/physics. That is, we assume a neo-Pythagorean stance in which "all is bits" or "all is computer"; this is precisely J.A. Wheeler's "it from bit" stance.

Indeed, these problems are more concerned with the epistemology of physics, and may not seem so troubling. In fact, my stance is that these "problems" are not so problematic after all, for the same reason that an analytic philosopher of mind or a proponent of computationalism in cognitive science would tend not to bother with arguments against the possibility of artificial intelligence given by an existential phenomenological philosopher: their views are inconsistent with one another, and often do not intend to address similar problems. Additionally, there are some philosophies of mathematics/physics that are inherently inconsistent with continuity (e.g. digital philosophy), and there are some philosophies that are inconsistent with computationalism (e.g. existential Heideggerian phenomenology, though I'd hesitate to peg this as a sufficient philosophy for that which we are discussing, since it has no formal foundation). So what we should take away from this is that we should not worry so much about exhaustively proving, in the sense that I have described above, digital physics or continuous physics or any other philosophy of physics; rather, we should develop as many well-defined theories of physics as we see fit. The major goal of science and philosophy is to provide interesting and reasonable ways of thinking about things, after all. Now we can describe the second issue...

(2) What does it mean for something to be a "universe", exactly? What does it mean to say that the universe is a digital computer? What is that thing to which we are referring when we say "the universe"? Is it our Hubble Volume? Is it the infinite ergodic space predicted by inflationary theory? Is it the Everettian space of orthogonal worlds? Is it something else? Does it include the laws that come with the space, or just the space itself? Is it isomorphic to or just ontologically a mathematical structure, or can it possibly be described by a mathematical structure (the many years of success of mathematics in physics suggests that it can be)? Obviously we need a preconceived intuitive depiction of what a universe is, before we can describe it as a digital computer, no? If by a digital computer we mean a Turing Machine, and by "the universe is a digital computer" we mean "the universe is isomorphic to a Turing Machine", then it is natural that our usual notion of a universe is fully characterized by some entities that are isomorphic to each of the 5 elements contained in the 5-tuple that is the usual definition of a Turing Machine. A main goal of ours is to figure out what these classical depictions of the universe are, and how they are isomorphic to their computational analogues.

2. Foundations of Digital Physics

This section is dedicated to an overview of the foundations of digital physics, mostly given by Baravalle and Beraldo-de-Araujo [Baravalle, 2016]. In fact, the only definitions presented that are not presented in the Baravalle paper are those after the Zuse Thesis. Let's list some fundamental definitions:

Definition 2.1. A mathematical structure D is *discrete* iff its domain contains only isolated points.

Definition 2.2. A discrete mathematical structure D is *digital* iff there exists a finite subset B of D's domain D_A such that all elements of D_A are composed of a finite concatenation of elements of B. That is, any finite mathematical structure with a discrete topology is digital.

Definition 2.3. *Digitalism* is the idea that physical reality is isomorphic to a digital mathematical structure.
Wolfram's theory of cellular automata as physics would be an example of digitalism, since cellular automata are digital mathematical structures. A Turing Machine (with a finite alphabet) analogue in physics would also be an example of digitalism.

Definition 2.4. *Pancomputationalism* is the idea that the processes performed by physical objects are computable, i.e. that a Turing Machine (or equivalent computational structure) can model that physical process isomorphically.
A main cornerstone of digital physics is the idea of Pancomputationalism, since it is the archetypal argument for the truth of the Zuse Thesis. It is also a cornerstone of digital philosophy, as it leads to computational theories of mind, especially.

Definition 2.5. The *Zuse Thesis* is the idea that physical reality is just a digital computer.

This is the fundamental thesis of digital physics. There is ambiguity in the phrase "is just a digital computer": Do we mean that physical reality can be mathematically modeled (i.e. an draw an isomorphism between the world and a computer) as a computer (i)? Do we mean that it can in principle be simulated on a computer, given enough resources (ii)? Or do we mean that it is ontologically a simulation carried out by some intelligent creature (iii)? To be conservative, we can say that (i) is the case. To be philosophically interesting, we can say that (ii) or (iii) is the case.

Baravalle's paper was mainly concerned with providing rigorous arguments for the Zuse Thesis and a logical foundation for the fundamental principles of digital physics, and not so concerned with coming up with any more ideas in digital physics, outside of the logical groundwork for the field. It is here that we diverge from their paper (all the definitions in this section so far come from their paper). We start by defining what it means to be a universe in a Turing machine (TM) physical reality.

Definition 2.6. A *universe* μ is characterized by the 4-tuple (q_n, q_k, f, t), where q_n is is the $n-th$ state of the TM in question, q_k is the n-tuple of previously observed states of μ, f is the transition function governing the TM, and t is the linearly-ordered time counter over the natural numbers.

We assume time is a discrete variable as well, of course. A digital universe may also be considered to just be a finite concatenation of TM states, but our definition takes care of the case when two universes of TM states up to a finite time are the same but the transition functions are different. For instance, a TM printing the first 100 natural numbers will have the same TM configuration as a TM printing the numbers $0-9$ looped over 10 times for the first 10 time-steps, but will differ afterwards. We wouldn't say that the TMs are the same, because they serve different functions. For this reason, we will give the following two definitions:

Definition 2.7. Two TMs $\mu_1 = (P_1, Q_1, R_1, S_1)$ and $\mu_2 = (P_2, Q_2, R_2, S_2)$ are said to be *teleologically equivalent* iff $(P, Q, R, S)_1 = (P, Q, R, S)_2$.
The "teleological" equivalance comes from the idea the TMs with the same transition function

Definition 2.8. Two TMs $\mu_1 = (P_1, Q_1, R_1, S_1)$ and $\mu_2 = (P_2, Q_2, R_2, S_2)$ are said to be *TM-state equivalent* if $P_1 = P_2, Q_1 = Q_2$, and $S_1 = S_2$. That is, they are TM-state equivalent if, at the same point in time, their spatial histories and current spatial configuration are exactly the same.
We see that all teleologically equivalent TMs are also TM-state equivalent, but not necessarily vice-versa: a counterexample is seen in the example given above, about printing natural numbers. However, probabilistic TMs may have the same transition function but not the same TM-state history, so they could be used to serve similar purposes but not have the same history. We use this reason to state the following definition:

Definition 2.9. Two TMs $\mu_1 = (P_1, Q_1, R_1, S_1)$ and $\mu_2 = (P_2, Q_2, R_2, S_2)$ are said to be *teleologically similar* iff $R_1 = R_2$, i.e. iff they have the same transition function.
We may infer that TMs that are teleologically equivalent but not teleologically similar must be non-deterministic, which can be given by a simple reductio-ad-absurdum argument: if they were deterministic and they were governed by the

same deterministic function, then they would each have one and only one history, which would be the history generated by that function, thus they would necessarily have the same histories (namely that history generated by the governing function). But we assumed that the histories are not necessarily similar, thus we have arrived at a contradiction, so each universe must be non-deterministic.

Definition 2.10. A *Level III Multiverse* M is a 3-tuple (μ, f, t) where μ is a set of disjoint universes generated by the transition function f, at the time t.

Keep in mind that the universe histories may be the same, under the condition that the state at time t is different. In fact, in the Everettian sense, there are necessarily some universes in μ that satisfy this property, since the Everettian multiverse admits a branching structure (See [Saunders et al, 2010]).

Definition 2.11. A *Level IV Multiverse* M is a 2-tuple (μ, t), where μ is, again, a set of universes, and t is some finite point in time (over $\mathbb{N}$).

This, as well as def. 2.10, is inspired by Max Tegmark's multiverse hierarchy [Tegmark, 2005]. The difference is that all the universes in the Level III Multiverse are generated by the same governing laws, whereas the universes in the Level IV Multiverse may be generated by different governing laws.

Definition 2.12. A *simulated universe* S is a 2-tuple (μ, w_1) where μ is a universe, and w_1 is a universe in which S is computed physically. We may then colloquially think of an observer in S as being in "w_0".

This is perhaps the least mathematically precise, but so far the most philosophically interesting. What is more interesting is a simulated universe with an intelligent "simulation creator", whatever that may mean.

Definition 2.13. An *intelligently simulated universe* S_i is a 2-tuple (S, C), where S is a canonical simulated universe, and $C \in w_1 \subset S$ is a "simulation creator" that lives in w_1 who "created the simulation", however vague that may be.
Of course, this is more philosophically interesting, as we now cross into something of a theological area. We can push further into a theologico-teleological (and of course an even murkier area) with the following definition:

Definition 2.14. A *teleologically simulated universe* P is a 2-tuple (S_i, r) where S_i is an intelligently simulated universe and r is a "reason" that $C \subset S_i$ had for creating the simulation.
These past three definitions are a formalism for the concept of a simulated universe, and may have applications in the philosophy of religion in particular.

Remark 2.15. We should suggest that each universe comes with an implicit finite alphabet of symbols, s, which are the analogues of elementary particles in the standard model of particle physics.
Now that we have covered the requisite definitions for a (lite) computational understanding of our cosmos, we can start talking about how to interpret modern physical theories in the language of digital physics.

3. The Finite Many-Worlds Thesis

The main philosophical consequence of the many-worlds interpretation of quantum mechanics is easily stated colloquially: "if some universe is possible according to the wavefunction, then it necessarily exists" (in a sense that is just as real as the "observed" universe relative to some observer). A computational analogue of this can be stated in the language of modal logic:

$$\Diamond\exists\mu_n[\mu_n = (q_n, q_k, f, t_n)] \rightarrow \Box\exists\mu_{\Diamond\sim n}[\mu_{\Diamond\sim n} = (q_n, q_{\Diamond\sim k}, f, t_n)]$$

This can be read as "if it's possible that we can compute some particular state of our observed universe at time t, then it's necessary that we can compute that particular state in some teleologically similar universe, which may or may not be a universe that is TM-state equivalent to our observed universe".

Theorem 3.1. *The finite many-worlds thesis admits finitely-many universes at any point in time.*

Proof. A universe at time $t = n$ may be seen as an $(n+2)$-tuple $(q_0, \cdots, q_n, n)$, where each q_j is a k-tuple $(s_0, \cdots, s_{k-1})$, where each s_j is an alphabet symbol. Then the upper bound on number of possible universes is expressed by $\#s^{n+1}$, where $\#s$ is the cardinality of the set of alphabet symbols and $n \in \mathbb{N}$ is the time-step. Since s is a finite set and n is finite, $\#s^{n+1}$ will always be finite. QED.

However unnatural the many-worlds interpretation of QM seems, this may seem even less natural, especially when we let the universe be a simulated universe. With the idea of a simulation comes an natural notion of economics, i.e. we don't necessarily want to simulate every possible universe in a Level III or Level IV multiverse; we may rather be concerned with simulating a certain class of universes, with particular initial conditions, governing laws, certain sets of alphabet symbols, etc. We may use the example of a game of chess to elucidate this:

Example 3.2. We may think of a chess match as a type of intelligently designed simulation, perhaps with a vacuous teleological reason (vacuous because we probably aren't very interested in why two people are playing chess, and thus it is philosophically vacuous, it doesn't lead to anything noteworthy). Given the heuristic that control of the center in the opening is important, we will see that the move 1. h3 is of very low frequency. Given the heuristic to avoid trades that put you down a minor piece, we will see very few universes in which this happens. Same with queen sacrifices, etc. Chess games in which many of these heuristics are violated are few and far between, if in existence in any chess database at all. The reasons for the unfeasibility of every possible simulation coming into existence are similar to this analogy.

It is of interest of us to formalize such an idea, but that will not be examined here.

4. Summary and Further Investigations

We've seen that it's possible to formalize several ideas of modern physics in the language of theoretical computer science. This leads us to some rather interesting thoughts in the philosophy of religion and metaphysics; specifically, it allows us to reasonably talk about the ontological nature of spacetime, and even topics as ambitious as theological teleology, from the well-understood perspective of computer science. In the future, we hope to investigate a digital theory of quantum mechanics and other specific theories of modern physics; another major aspect of future discussion should be on the philosophy associated with the theory of simulated universes. Even more ambitiously, it may be interesting to look at transfinite computation and its applications to physics, as done by people such as Hogarth et al.

5. Bibliography

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