Incorporating relativity in categorical models of abstract physical theories

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If we would release a body of antimatter at a height h above the earth's surface with an initial velocity parallel to the earth's surface, then modern physics predicts that the gravitational force exerted by the earth on that body of antimatter is directed *towards* earth, thus causing the body of antimatter to fall down: modern physics thus predicts that the height $h(t)$ of the body of antimatter as a function of time will be a *downward* curve. Currently it is a hot topic in experimental physics to confirm this prediction: there are three sizeable projects going on at CERN using antihydrogen—AEgIS [1], ALPHA [2], GBAR [3]—and one at the PSI using muonium [4]. However, thus far this prediction has **never** been confirmed: it is, thus, currently not the case that we already **know** that antimatter falls down on earth. There is, thus, nothing that withholds us from doing the experiment in our thoughts, and letting the experiment have the opposite outcome, which is that the height $h(t)$ of the body of antimatter as a function of time is an *upward* curve: in this thought experiment we consider a repulsive matter-antimatter gravity for the purpose of thinking through the consequences thereof for our understanding of the fundamental workings of the universe. Of course this is *speculative*, and of course one may *dislike* the idea of repulsive gravity, but the point is that consequently thinking through a thought experiment is a *perfectly valid technique c.q. method* in theoretical physics.

Proceeding from this thought experiment one quickly finds out that physical principles underlying repulsive gravity cannot be described consistently in the framework of any of the theories of modern physics. However, a clear and distinct idea has led to the development of the Elementary Process Theory (EPT): this is a collection of fundamentally new physical principles, expressed in a new language for physics, which support a matter-antimatter gravitational repulsion [5]. Now even apart from the question whether or not repulsive gravity exists, any set of new physical principles has to satisfy the condition that it must agree with existing knowledge of the physical world. The crux is then that on the one hand this existing knowledge is described in a mathematically concrete language, while on the other hand the EPT is an example of an *abstract physical theory*: this is a theory from the syntactic point of view with a physical interpretation, but such that constants of the theory that designate constituents of the physical world are abstract sets, i.e. sets whose elements are not specified—these contain, thus, no reference to a coordinate system of an observer. That means that it is **not possible** to prove that the EPT, or any other abstract physical theory, agrees with existing knowledge by proving that the EPT reduces to an existing theory when some limiting procedure is applied.

That said, the purpose of this talk is not to discuss the physical principles of the EPT at object level, but rather to present a new general method by which it can be proven that an abstract physical theory agrees with existing knowledge, in particular relativity.

First of all it must be understood that the one existing method, specifying a mathematically concrete set-theoretic model M of an abstract physical theory T, is *inadequate*. This is easy to see. Suppose a physical system described by T evolves from an initial state S_0 consisting of a single point-particle to a final state S_1 consisting of a single point-particle: then in the set-theoretic model M of T, the positions and momenta of the initial and final point-particles have concrete values in the coordinate system of an observer $\mathcal{O}$—we could say that T 'agrees' with existing knowledge if this model M reproduces a known experimental result. But the point is that this model M can only be associated with that coordinate system of that observer $\mathcal{O}$: the model M does <u>not</u> contain the values of positions and momenta in the coordinate systems of other observers, and in addition the model M is in itself <u>incapable</u> of predicting what the values of the positions and momenta of the point-particles in the coordinate system of another observer will be. Ergo, specifying a single set-theoretic model M of an abstract physical theory T is **insufficient** to prove agreement of T with relativity because it can <u>never</u> predict relativity of spatiotemporal characteristics of motion.

That is the motivation for introducing the notion of a categorical model $\mathscr{C}$ of an abstract physical theory T: a model of T is then identified with a (small) category $\mathscr{C}$, whose objects are mathematically concrete set-theoretic models of T, and whose arrows are model isomorphisms—note that the collection of objects forms a theory from the *semantic* point of view. Each object is then associated with the reference frame of an observer, and each arrow with a coordinate transformation. This incorporates relativity in a natural way: if we have the model M of a physical system in the coordinate system of an observer $\mathcal{O}$, and if we know which arrow A is associated with the transformation of the coordinate system of $\mathcal{O}$ to that of an observer $\mathcal{O}'$, then we can calculate the model M' of that physical system in the coordinate system of $\mathcal{O}'$ by applying A to M. That *does* predict relativity of spatiotemporal characteristics of motion.

Using Rosaler's elegant concept of empirical reduction [6], we are now in a position to formulate a notion of 'agreement' between an abstract physical theory and existing knowledge:

> an abstract physical theory T *agrees* with the knowledge of the physical world derived from the successful predictions of a scientific theory T' **if and only if** T has a categorical model $\mathscr{C}$ that reduces empirically to T', that is, a categorical model $\mathscr{C}$ such that for every experiment that has confirmed predictions of T', the experimentally successful predictions of T' can be reproduced by $\mathscr{C}$.

Recently it has been proven that the EPT agrees with the knowledge of the physical world derived from the successful predictions of Special Relativity (SR) by showing that the EPT has a categorical model in which SR is incorporated [7]. A negative result would thus have been that no such categorical model exists.

Once an initial categorical model $\mathscr{C}_0$ of an abstract physical theory T has been specified, a new (Lakatosian) research program arises in a natural way. Its hard core contains T: this is taken as fundamental. The negative heuristics are simple: developments that contradict T are not interesting. The positive heuristics are then to develop successors $\mathscr{C}_1$, $\mathscr{C}_2$, ... of $\mathscr{C}_0$ that are both theoretically and empirically progressive. The aim is then to develop a categorical model $\mathscr{C}_\infty$ that is *empirically adequate* as defined by Van Fraassen [8], which is when all observations in the realm of physics can be described as predictions of $\mathscr{C}_\infty$. The corresponding research program based on the EPT, outlined in [7], is then a new research program in theoretical physics.

References

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