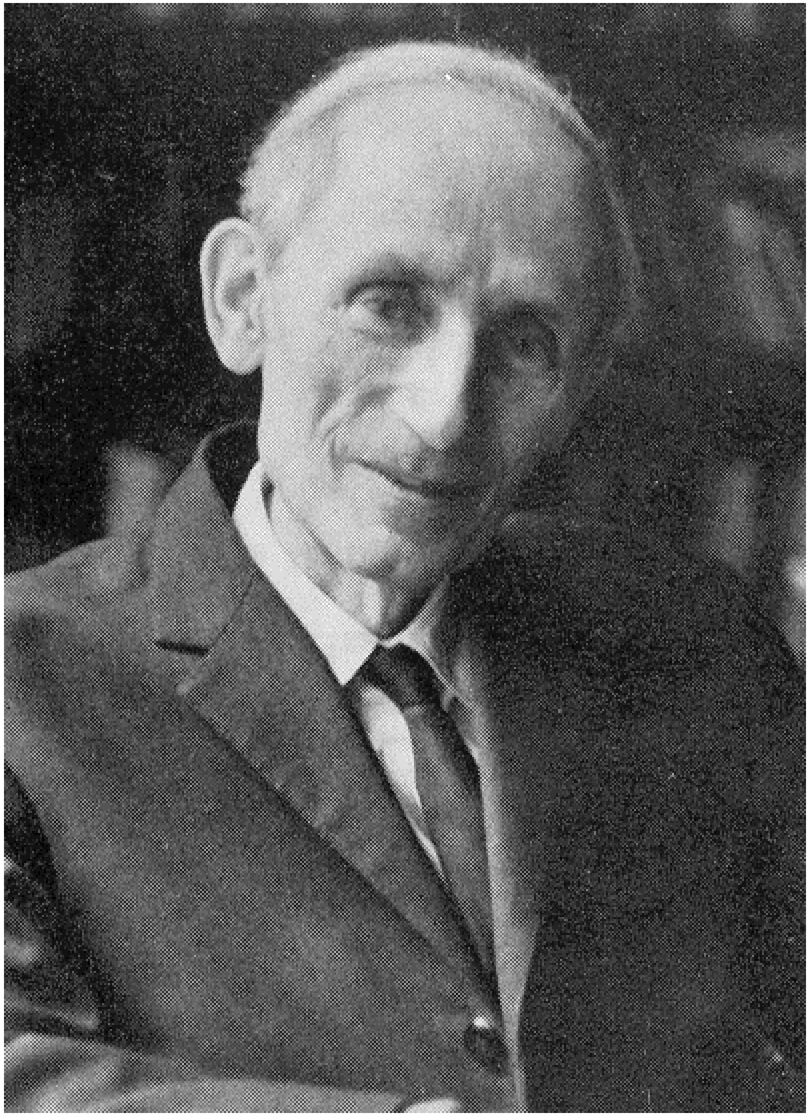
Tibor Gallai
and matching theory

László Lovász

Eötvös Loránd University, Budapest



Tibor Gallai
1912-1992

Tibor Gallai, Paul Erdős and Márta Svéd

Gallai's work in matching theory

Gallai Identities

Perfect matching in regular graphs

Realizable degree sequences (with Erdős)

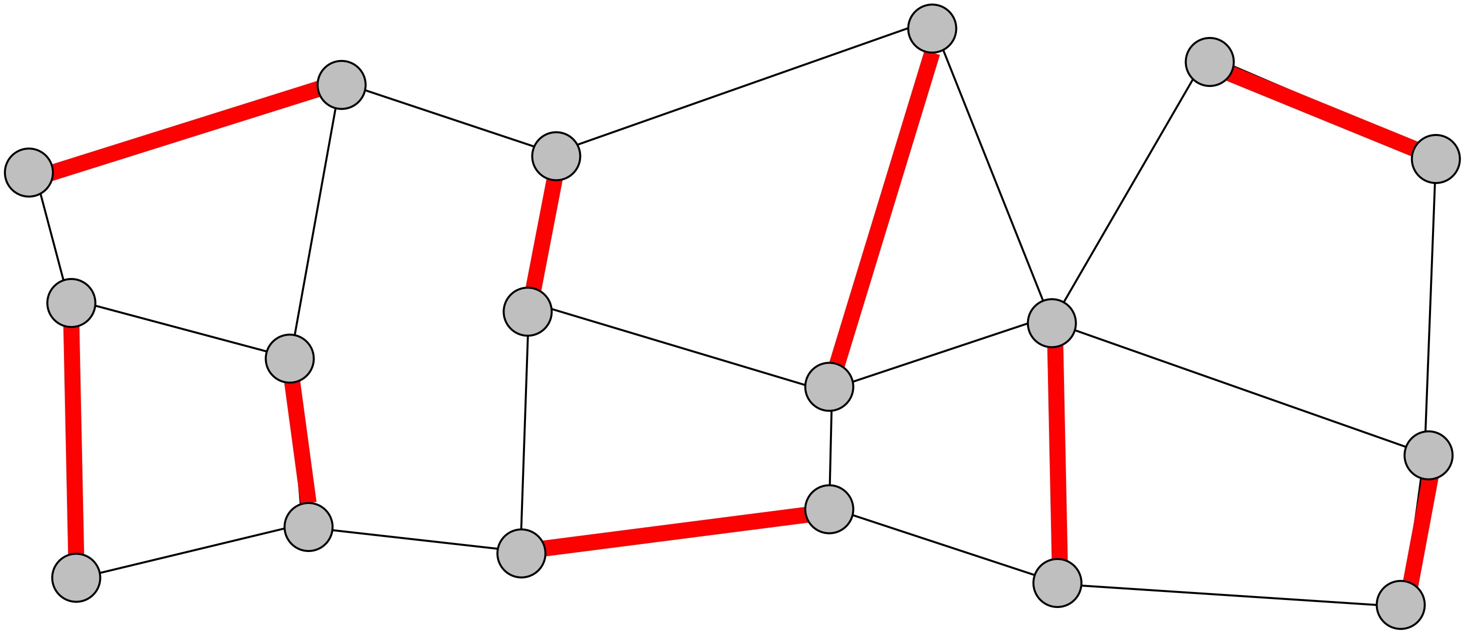
Factor-critical graphs

Edmonds-Gallai Structure Theorem

Point-disjoint *A*-paths

The perfect matching problem

Given a graph, is there a set of edges covering each node exactly once?



The perfect matching problem

Given a graph, is there a set of edges covering each node exactly once?

Answer for 3-regular graphs: Petersen 1891

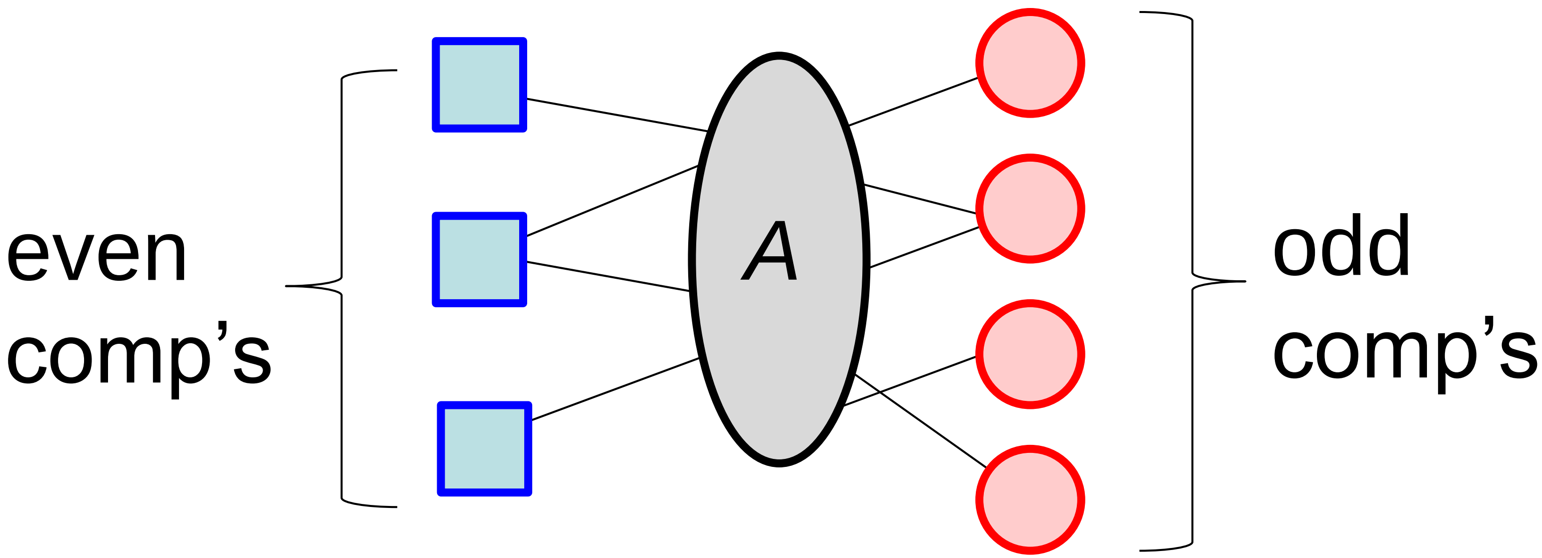
Answer for bipartite graphs: Frobenius 1912
Kőnig 1931

Maximum number of independent edges?

The perfect matching problem

For regular graphs: Bäbler, Belck, Gallai 1950

For all graphs: Tutte 1947



odd components ≤ |*A*|

The perfect matching problem

For regular graphs: Bäbler, Belck, Gallai 1950

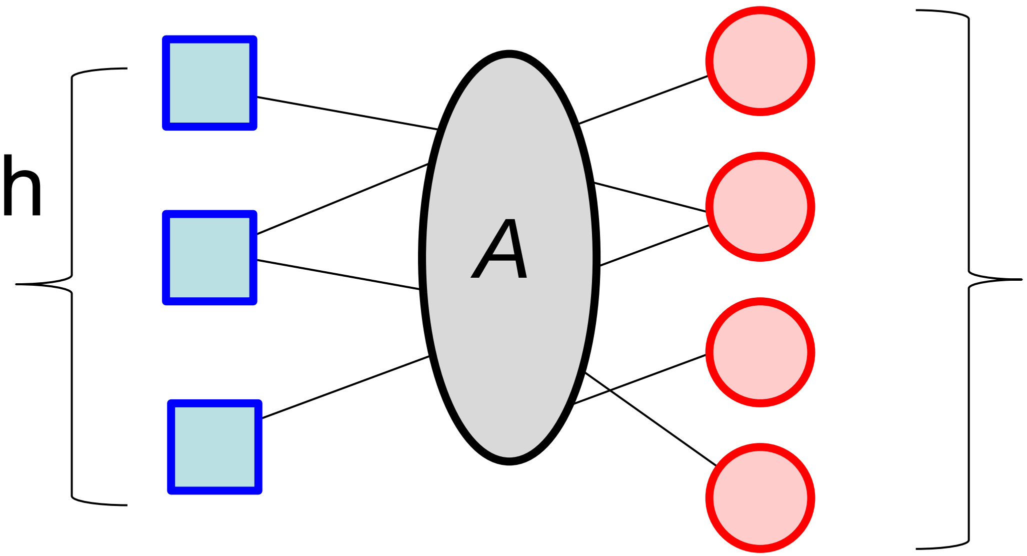
For all graphs: Tutte 1947

Formula for the maximum number of independent edges: Berge 1958

Polynomial time algorithm: Edmonds 1965

The Edmonds-Gallai Structure Theorem 1965

comp's with perfect matching



factor-critical comp's

odd components > |A|

characterizations by Gallai, L, Frank,...

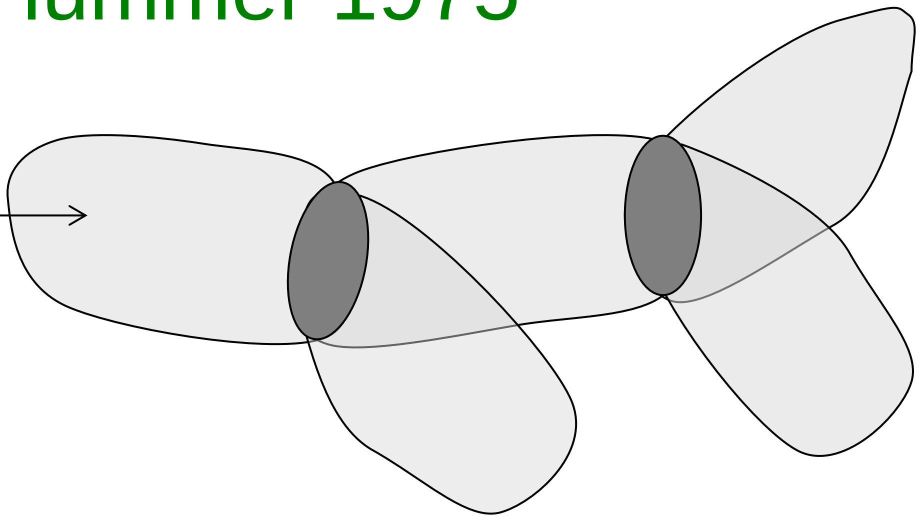
H-x has pm for all nodes x

Structure of graphs with perfect matchings

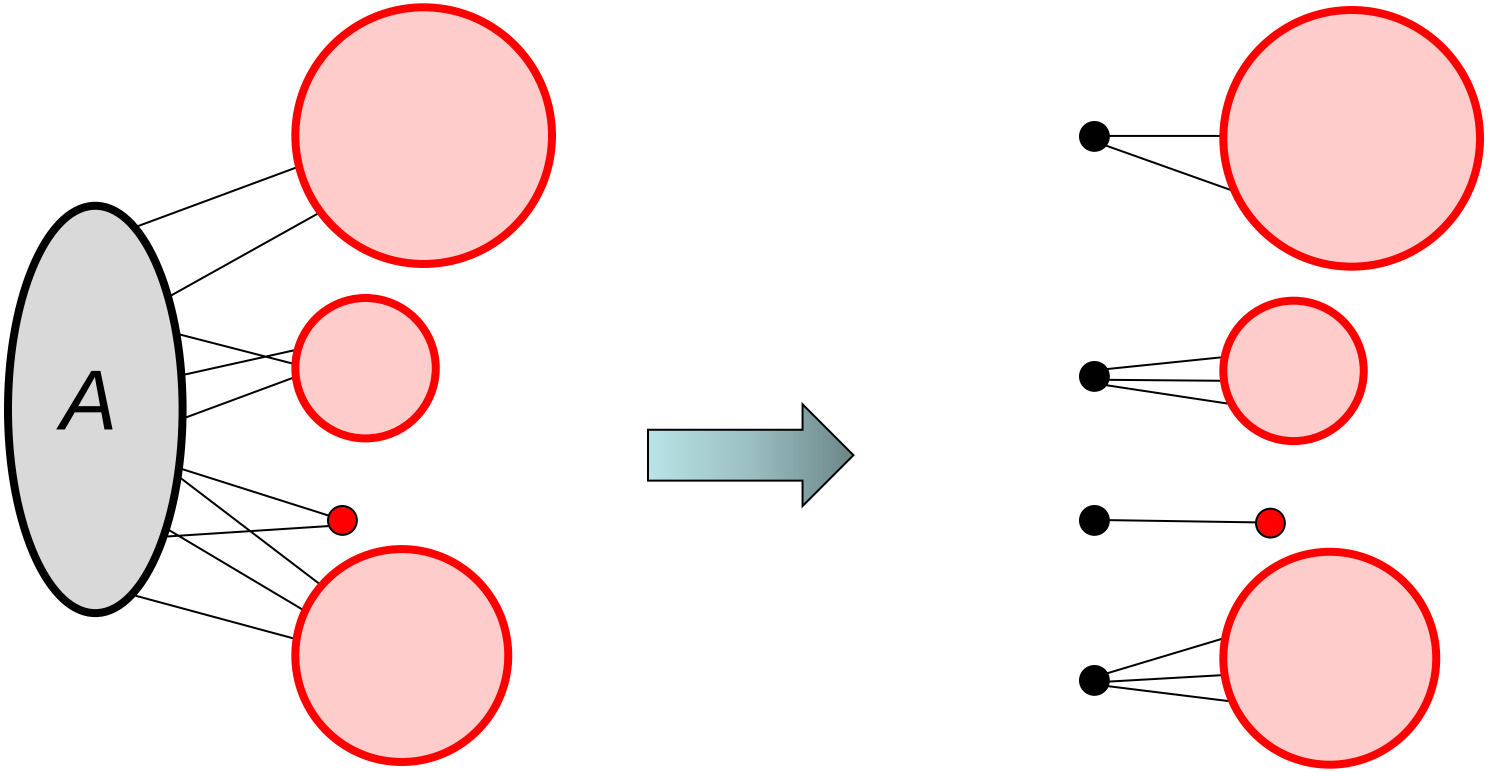
For bipartite: Kőnig 1915
Dulmage – Mendelsohn 1958-67

For all graphs: Kotzig 1959-60
L 1972
L – Plummer 1975

bicritical:
G-x-y has pm
for all nodes x,y

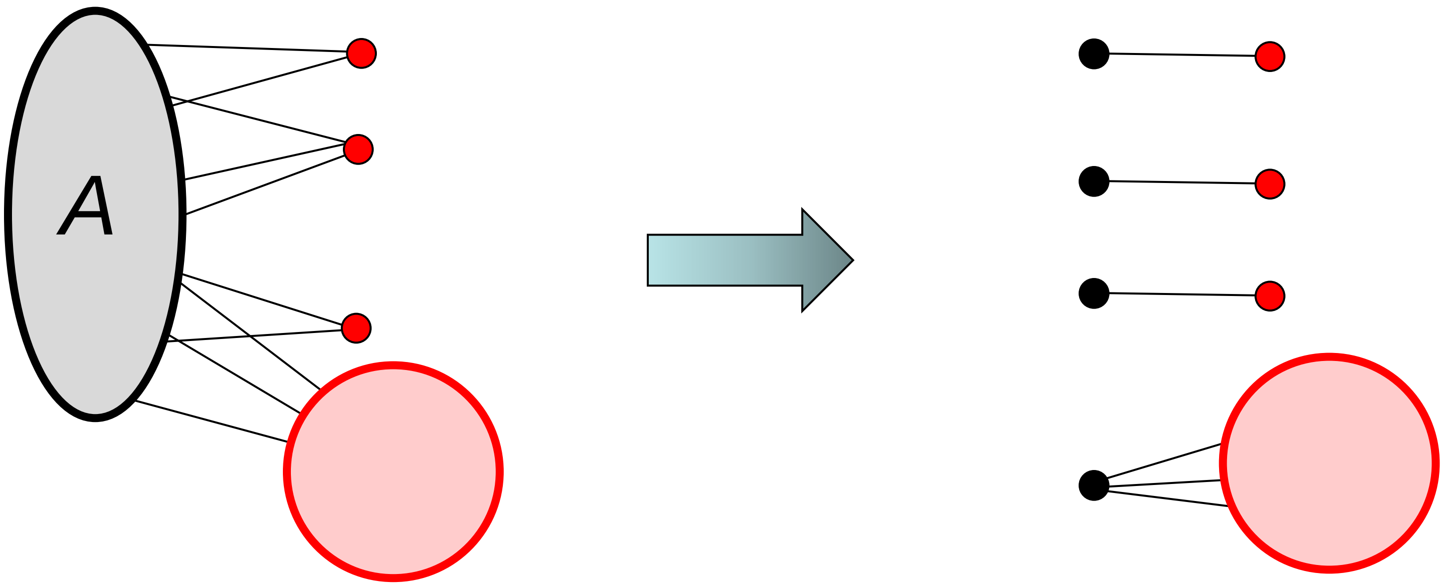


Structure of graphs with perfect matchings



odd components = |*A*|

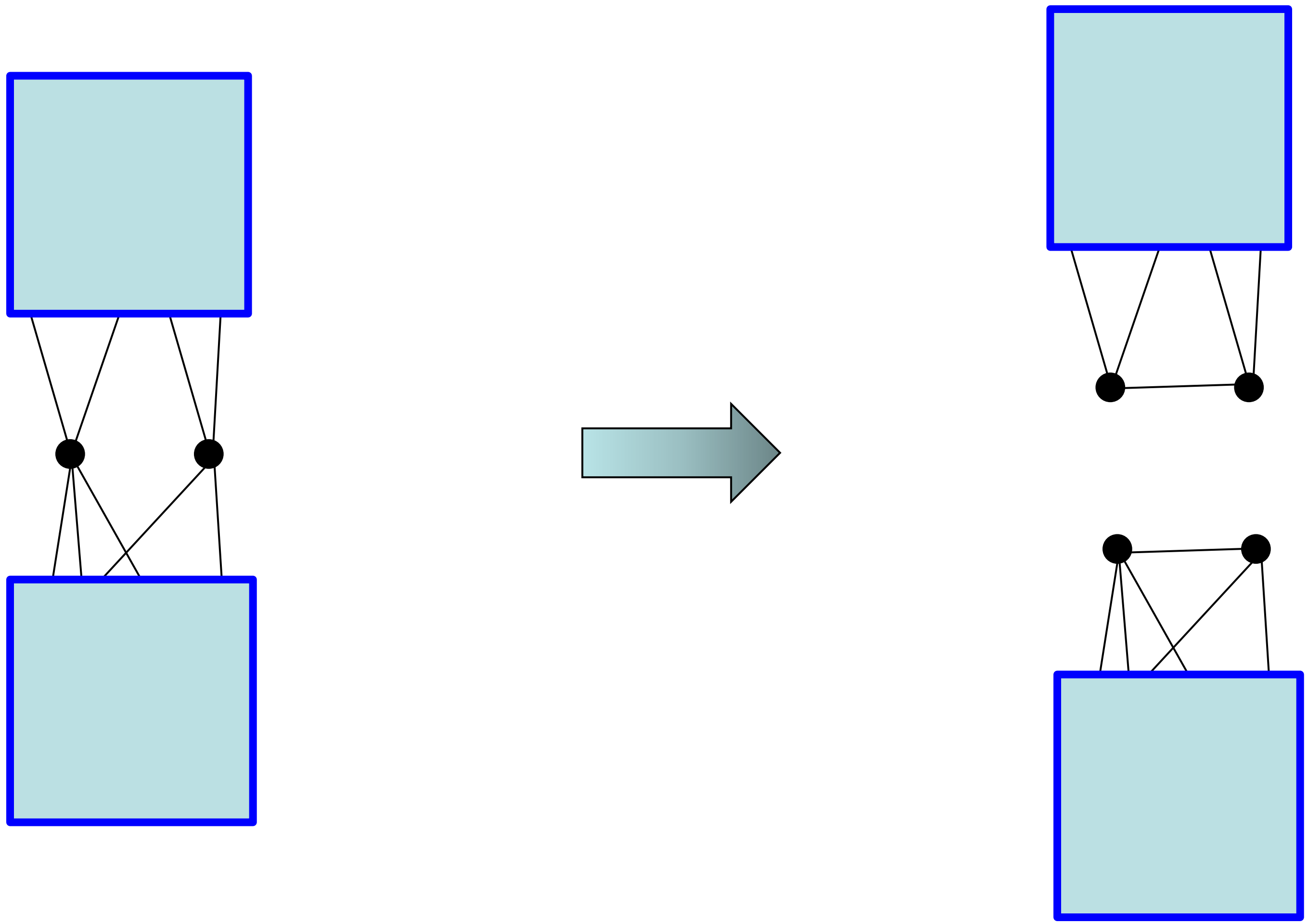
Structure of graphs with perfect matchings



odd components = |A|

bipartite reduction

Structure of graphs with perfect matchings



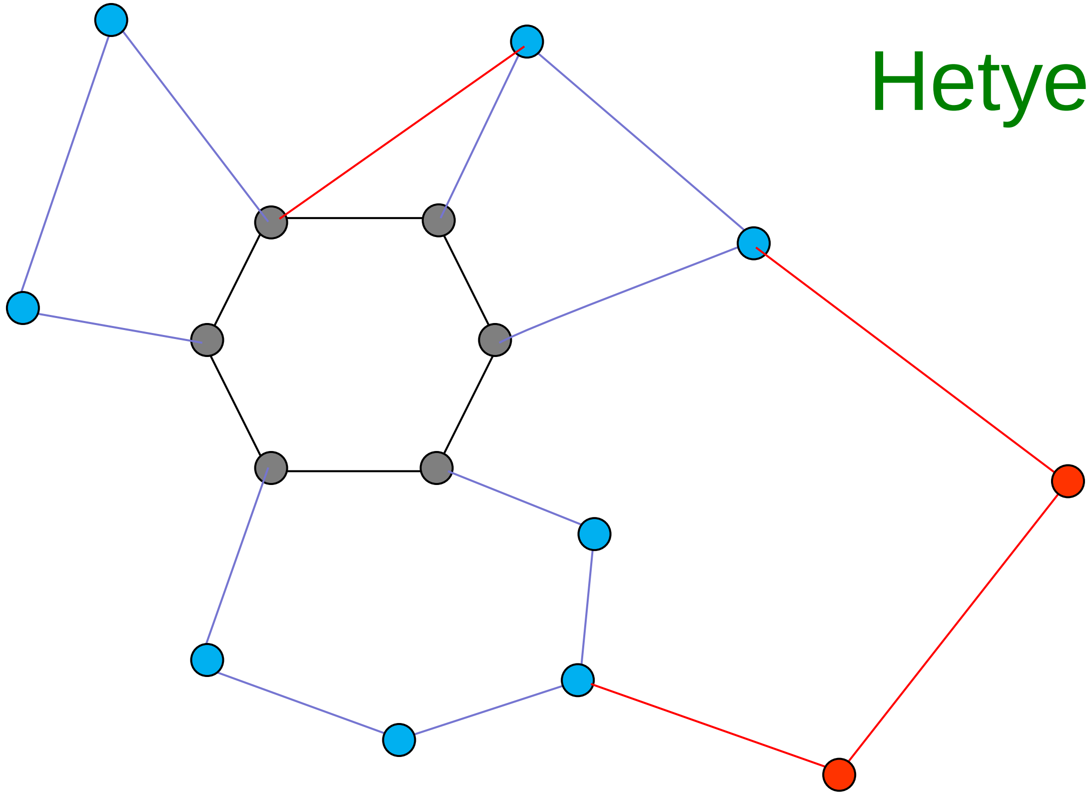
Ear-decomposition

Every edge of G is in a perfect matching

$\Rightarrow$ G has ear-decomposition starting with

an even cycle.

Hetyei 1976

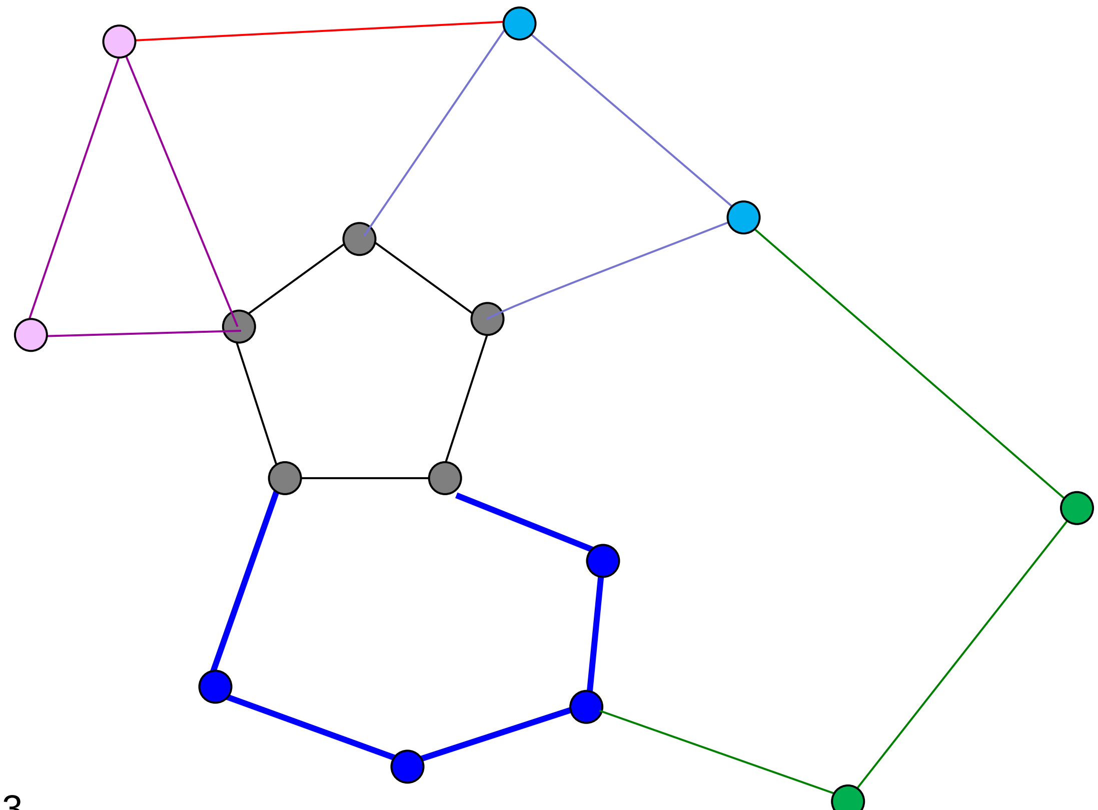


Ear-decomposition

G is critical ⇔ G has ear-decomposition starting

with an odd cycle, adding one ear at a time.

L 1972



Ear-decomposition

Every edge of G is in a perfect matching
$\Rightarrow$ G has ear-decomposition starting with
an even cycle, adding $\leq$2 ears at a time.

L-Plummer 1975

$\Rightarrow$ 2-edge-connected 3-regular graph
has at least n/2 perfect matchings.

$\forall$ 2-edge-connected 3-regular graph
has exponentially many perfect matchings.

Esperet-Kardos-King-Kral-Norine 2012

Ear-decomposition

G is 3-connected bicritical

$\Rightarrow$ G has ear-decomposition starting with one of the graphs below, allowing bipartite extension,

adding one ear at a time.

Carvalho-Lucchesi-Murty 2002