Asymmetric exclusion: a way to anomalous scaling

Joint work with Timo Seppäläinen

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Erdős Centennial
July 1., 2013

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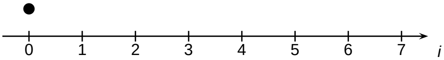

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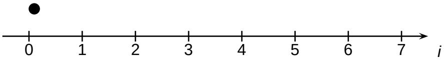

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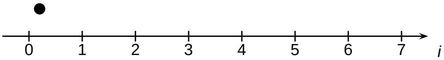

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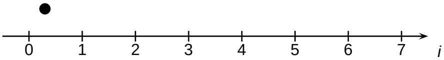

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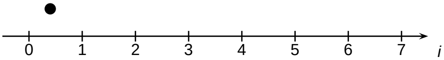

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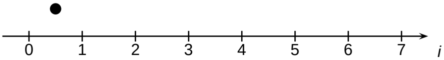

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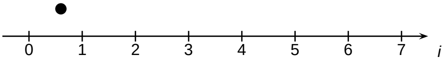

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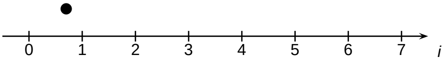

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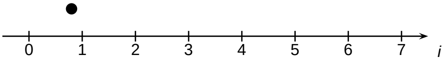

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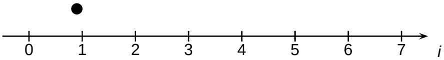

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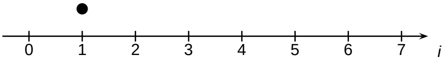

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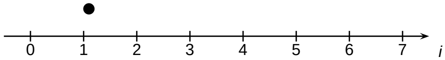

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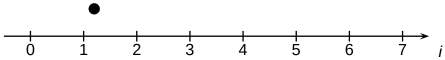

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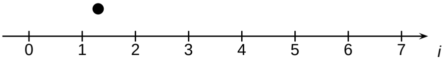

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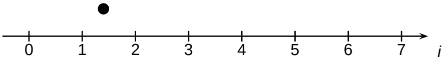

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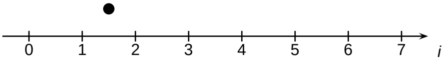

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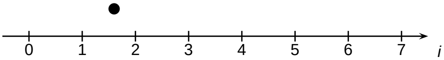

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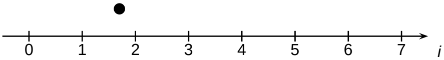

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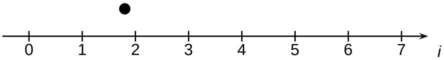

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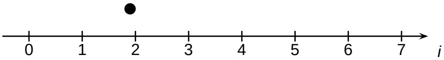

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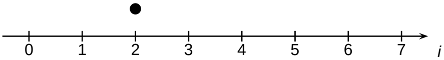

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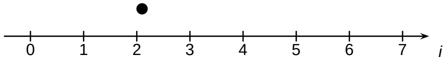

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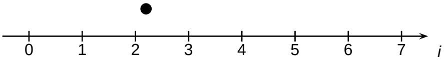

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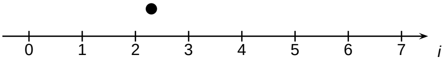

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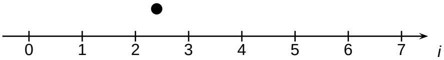

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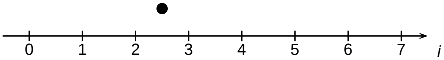

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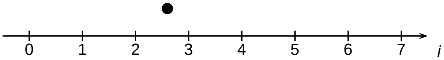

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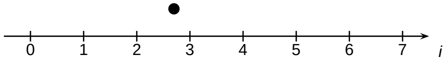

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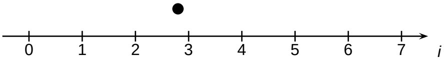

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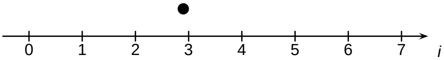

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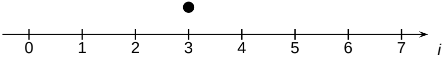

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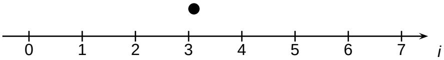

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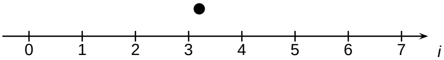

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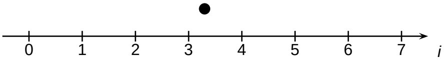

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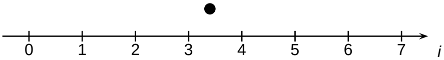

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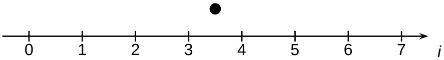

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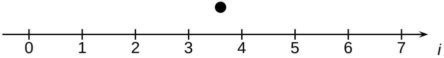

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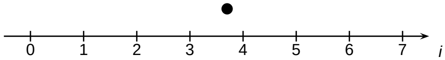

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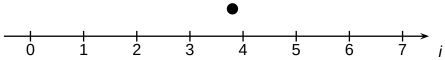

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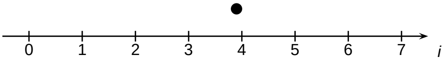

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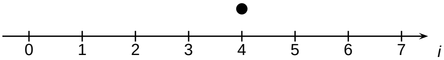

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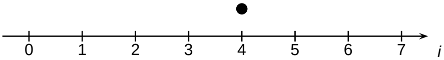

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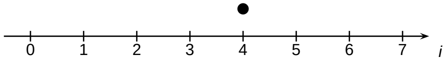

$\rightsquigarrow$ Continuous time Markov jump process with rate 1.

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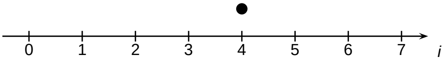

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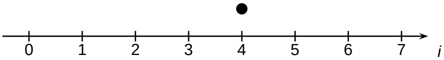

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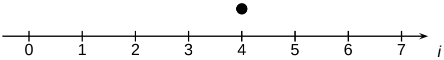

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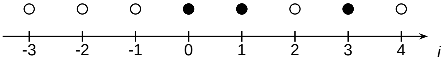
The totally asymmetric simple exclusion process



Bernoulli(ϱ) product distribution.

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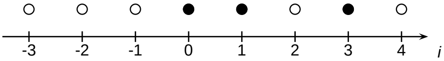

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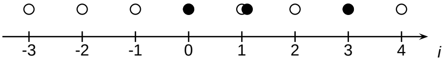

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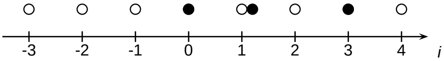

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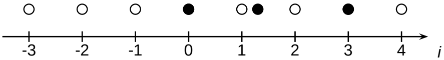

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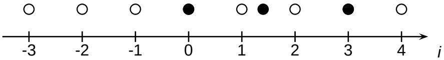

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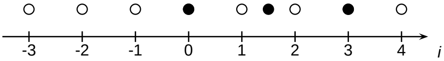

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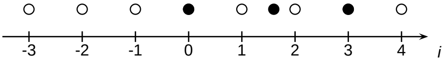

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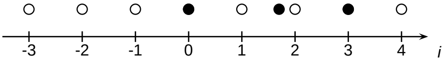

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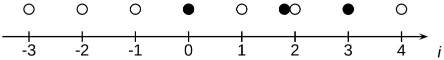

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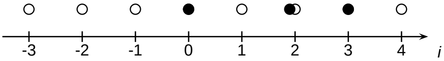

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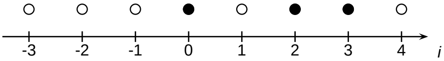

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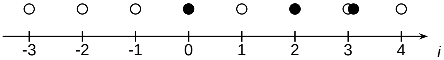

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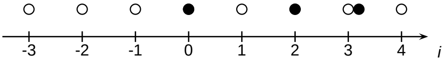

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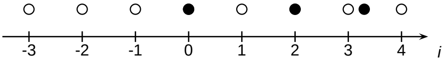

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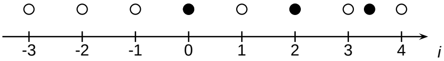

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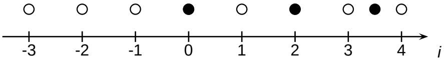

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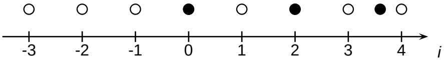

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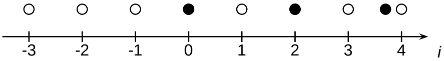

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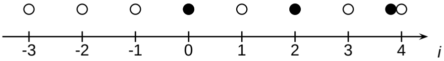

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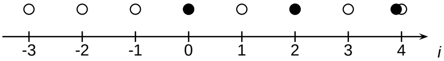

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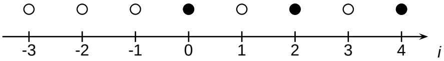

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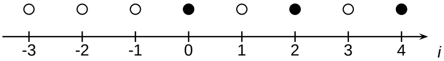

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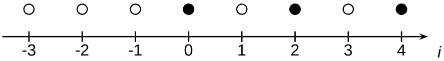

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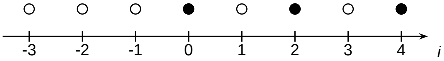

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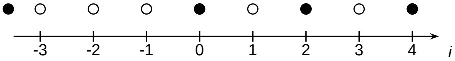

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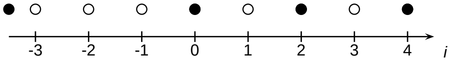

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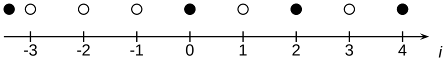

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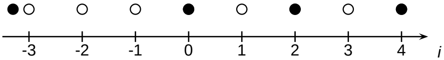

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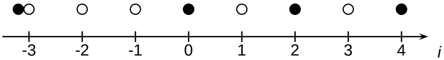

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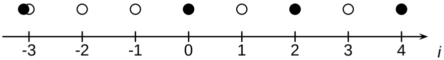

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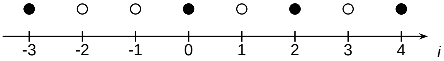

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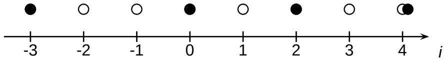

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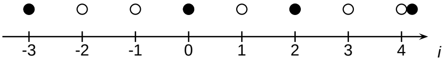

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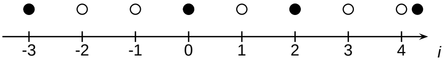

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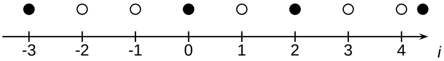

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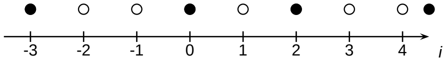

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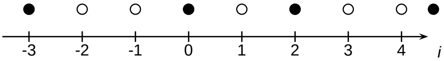

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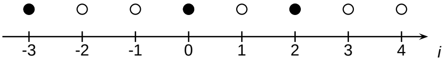

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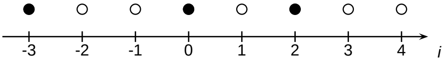

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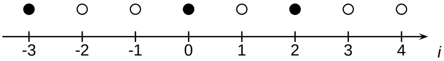

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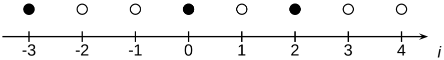

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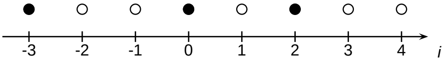

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The Bernoulli(ϱ) distribution is stationary (and non-reversible) for all $0 \leq \varrho \leq 1$.

These are the important ($=$ ergodic) stationary distributions.

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An observer starts from the origin, and moves with velocity V.

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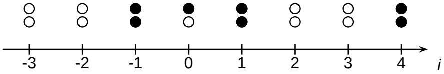

Stochastic coupling: evolution as close as possible

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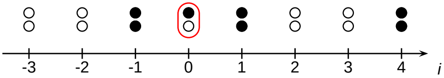

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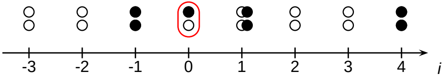

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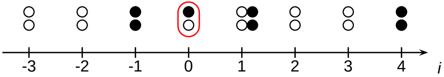

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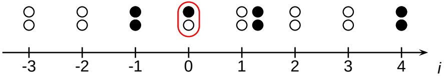

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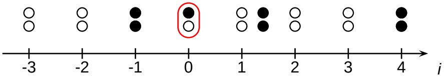

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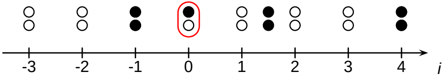

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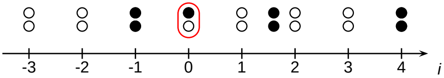

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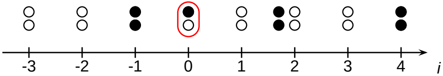

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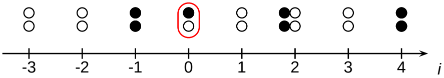

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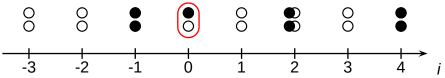

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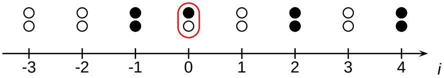

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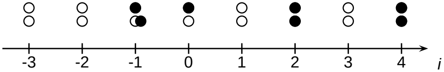

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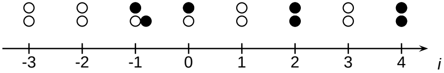

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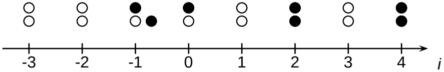

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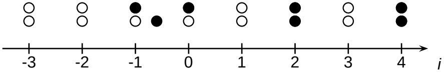

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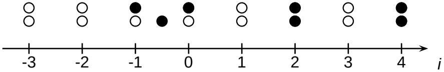

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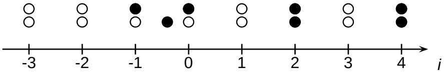

Stochastic coupling: evolution as close as possible

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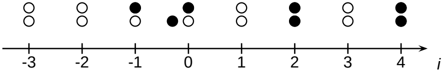

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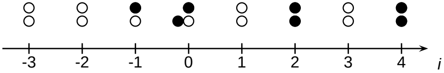

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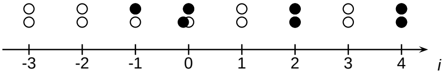

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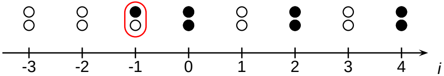

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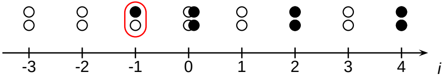

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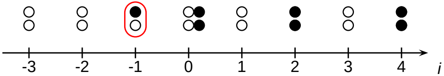

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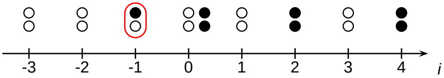

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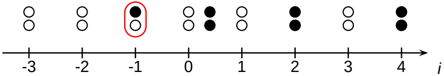

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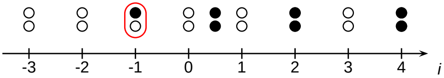

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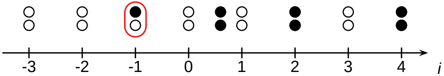

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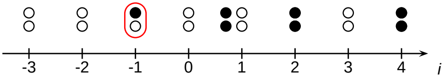

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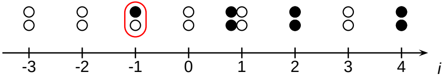

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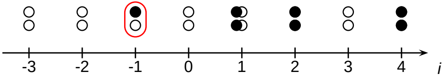

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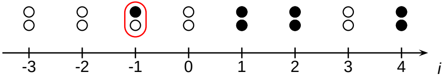

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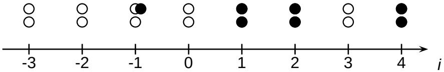

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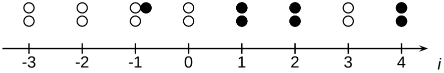

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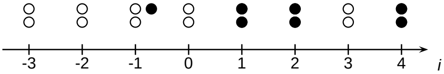

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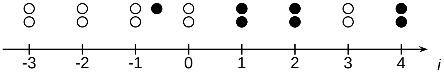

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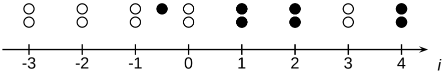

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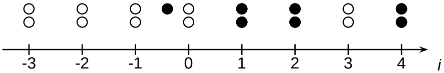

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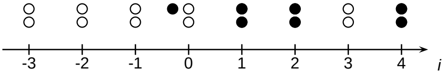

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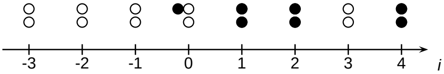

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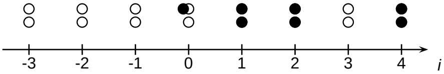

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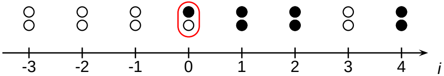

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Second class particle

Its position at time t: $Q(t)$.

$J_V(t) =$ net flux of particles

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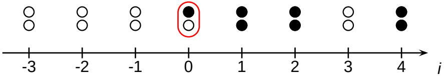

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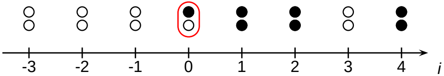

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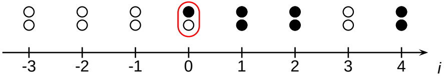

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This is the speed of information propagation.

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Exotic scaling

On the characteristics $V = 1 - 2\varrho$:

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Theorem (B. - Seppäläinen)

$$0 < \liminf_{t\to\infty} \frac{\mathbf{Var}(J_{1-2\varrho}(t))}{t^{2/3}} \le \limsup_{t\to\infty} \frac{\mathbf{Var}(J_{1-2\varrho}(t))}{t^{2/3}} < \infty.$$

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Func. analytic methods: Quastel, Valkó 2007. Combi.-analytic methods: Johansson, Tracy, Widom, Spohn, Prähofer, Ferrari, Borodin, Corwin and many others 1999-.

$J_V(t)$ = net flux of particles

1. An algebraic miracle

Miracle: exact identities.

Theorem (B. - Seppäläinen; ideas also from B. Tóth, H. Spohn, and M. Prähofer)

$$\mathbf{E}Q(t) = (1 - 2\varrho)t,$$

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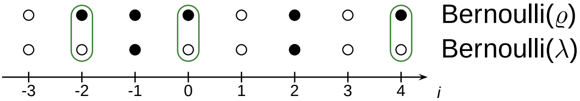

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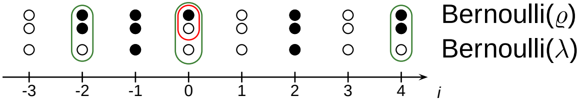
Coupling three processes:



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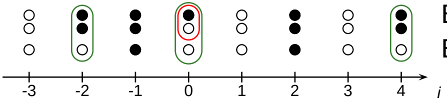

Bernoulli(ϱ)
Bernoulli(λ)

Push Q abnormally to the right: $\widetilde{Q}(t) \geq u$
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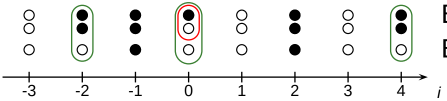

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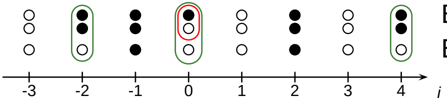

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$$\mathbf{P}\{\widetilde{Q}(t) \geq u\} \leq c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(J_{1-2\varrho}(t))$$

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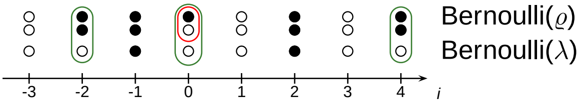

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that is: $E^3 \le c \cdot t^2$.

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\end{aligned}
$$

that is: $E^3 \le c \cdot t^2$.

$$
\mathbf{Var}(J_{1-2\varrho}(t)) \overset{\text{Miracle}}{=} \text{const.} \cdot E \le c \cdot t^{2/3}. \qquad \blacksquare
$$

$\mathbf{P}\{|\widetilde{Q}(t)| > u\} \le c \cdot \frac{t^2}{u^4} \cdot E$ $\qquad$ $\mathbf{Var}(J_{1-2\varrho}(t)) = c \cdot \mathbf{E}|\widetilde{Q}(t)|$

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works with similar arguments: compare models of which the densities differ by $t^{-1/3}$, and use connections between $Q(t)$, the green second class particles, and heights.

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Thank you.