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ABSTRACTS

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Plenary lectures

Noga Alon: Tel Aviv University

Paul Erdős and Probabilistic Reasoning

Abstract: One of the major contributions of Paul Erdős is the development of the Probabilistic Method and its applications in Combinatorics, Graph Theory, Additive Number Theory and Combinatorial Geometry. I will describe some of the beautiful applications of the method, focusing on the long-term impact of the work, questions and results of Erdős and their influence on recent work.

Béla Bollobás: University of Cambridge and University of Memphis

The Phase Transition in the Erdős–Rényi Random Graph Process

Abstract: The theory of random graphs that Paul Erdős founded with Alfréd Rényi around 1960 is one of the brightest jewels in his legacy: by now thousands of papers have been published on it. For me personally it has been one of the main themes of my research for forty years. Perhaps the most striking Erdős–Rényi result is that the component structure of the random graph process $(G_{n,m})$ undergoes a phase transition when the ratio between the number m of edges and the number n of vertices passes through 1/2: if $m \sim cn/2$ for a fixed $c < 1$ then with high probability every component of $G_{n,m}$ has $O(\log n)$ vertices, but if $m \sim cn/2$ with $c > 1$ then with high probability there is a linear-sized component, and all other components have $O(\log n)$ vertices.

For close to a quarter century this result stood in splendid isolation, but then the study of the finer nature of this phase transition started in earnest, with emphasis on what happens close to 'time' $n/2$: within the 'window' of the phase transition and near to it. Since then, a host of beautiful and difficult results have been proved on this by Aldous, Coja-Oghlan, Ding, Frieze, Janson, Kang, Karonski, Kim, Krivelevich, Lubetzky, Łuczak, Peres, Pittel, Riordan, Sudakov, Wierman, Wormald, and many others.

Around the turn of the millennium several new kinds of graph process were introduced and studied by Albert, Barabási, Janson, Riordan, and others, often with the aim of modelling large-scale real-world networks.

The study of phase transitions started by Edrős and Rényi over fifty years

ago will continue for many more years. In my talk, based on my review article with Oliver Riordan, I shall give a brief account of 'the story so far', including some very recent results of ours.

'For me personally it has been one of the main themes of my research for X years'.

Timothy Gowers: University of Cambridge

Erdős and arithmetic progressions

Abstract: In this talk I shall discuss two well-known conjectures of Erdős concerning arithmetic progressions: his famous sum-of-reciprocals conjecture and his discrepancy problem. Both problems are still very much open, but there have been extremely interesting partial results and some promising ideas for how to tackle the problems. These will form the main topic of the talk.

Ben Green: University of Cambridge

The sum-free set constant is $\frac{1}{3}$

Abstract: Erdős showed in 1965 that any set of n integers contains a sum-free subset of size at least $\frac{1}{3}n$, that is to say a set containing no elements x, y, z with $x + y = z$. In joint work with Sean Eberhard and Freddie Manners, we have shown that for every $\varepsilon > 0$ there is a set of n integers not containing any sum-free subset of size greater than $\left(\frac{1}{3} + \varepsilon\right) n$. In my talk I will discuss some aspects of the proof.

Péter Komjáth: Eötvös Loránd University

Finite substructures of uncountable graphs and hypergraphs

Abstract: We survey some old and some recent results on finite substructures of uncountable graphs and hypergraphs which either have large chromatic number or have no large independent set. Most of the classic results and problems are due to Paul Erdős and András Hajnal.

Elon Lindenstrauss: The Hebrew University of Jerusalem

Spectral gap and self-similar measures

Abstract: Around 70 years ago, Erdős wrote two important papers on Bernoulli convolutions — the distribution of the sum $\sum_{i=1}^{\infty} \pm\lambda^i$ where the signs are unbiased independent random variables and $\lambda \in (0,1)$. In modern language what is highlighted in these papers is the intricacy of the study of self similar measures when the Open Set Condition (or some similar such condition) does not hold.

Perhaps the most fundamental problem regarding these Bernoulli convolutions is whether there is some interval $(\lambda_0, 1)$ so that for any λ in this range the corresponding Bernoulli convolution is absolutely continuous with respect to Lebesgue measure.

Despite some very recent and remarkable advances on this classical problem, it remains open. However, we show that such an interval holds for higher dimensional self similar measures provided the rotations involved define an averaging operator with a spectral gap. Our proof goes via establishing a new spectral gap for random walks on the isometry group of R^n which is of independent interest.

Joint work with Péter Varjú.

Tomasz Łuczak: Adam Mickiewicz University and Emory University

Threshold functions: a historical overview

Abstract: Since the seminal paper of Erdős and Rényi on the evolution of random graphs properties of random structures are typically characterized by their threshold functions. In the talk we overview some results and ideas related to this fundamental notion of probabilistic combinatorics.

Daniel Mauldin: University of North Texas

Steinhaus' problem on simultaneous tilings of the plane

Abstract: In the 1950's Steinhaus asked whether there is a subset S of $\mathbb{R}^2$ which meets each isometric copy of $\mathbb{Z}^2$ in exactly one point. We will discuss a positive solution of this problem. We will also discuss a some related unsolved problems.

János Pach: Rényi Institute and EPFL

Paul Erdős and the beginnings of geometric graph theory

Abstract: Geometric graphs (topological graphs) are graphs drawn in the plane with possibly crossing straight-line edges (resp., curvilinear edges). Starting with a problem of Heinz Hopf and Erika Pannwitz from 1934 and a seminal paper of Paul Erdős from 1946, we give a biased survey of Turán-type questions in the theory of geometric and topological graphs. What is the maximum number of edges that a geometric or topological graph of n vertices can have if it contains no forbidden subconfiguration of a certain type? We put special emphasis on open problems raised by Erdős or directly motivated by his work.

Yuval Peres: Microsoft Research

Coloring a graph arising from a lacunary sequence, Diophantine approximation, and constructing a Kakeya set: Applications of the probabilistic method.

Abstract: Let $\{n_k\}$ be a lacunary sequence, i.e., the ratio of successive elements of the sequence is at least some $q > 1$. In 1987, Erdős asked for the chromatic number of a graph on the integers, where two integers are connected by an edge iff their difference is in the sequence $\{n_k\}$. Y. Katznelson found a connection to a Diophantine approximation problem considered by Khinchin in 1926 and by Erdős in 1975: finding irrationals x such that n_k times x is at least $r > 0$ away from the integers for all k. In joint work with W. Schlag, we improved Khinchine's and Katznelson's bounds for both problems using the Lovász local lemma. It is still an open problem to obtain matching upper and lower bounds. In the second part of the lecture (joint work with Babichenko, Peretz, Sousi and Winkler) I will describe a new probabilistic construction of a planar Kakeya set (a set of zero area containing line segments in all directions) that is based on finding optimal strategies for a search game between a hunter and a rabbit on a graph.

János Pintz: Rényi Institute

Paul Erdős and the difference of primes

Abstract: We discuss several problems concerning the difference of primes, primarily regarding the difference of consecutive primes. Most of them were either initiated by Paul Erdős (sometimes with coauthors), or were raised ahead of Erdős; nevertheless he was among those who reached very important results in them (like the problem of the large and small gaps between consecutive primes).

Carl Pomerance: Dartmouth College

Paul Erdős and the rise of statistical thinking in elementary number theory

Abstract: In ancient times mathematicians were fascinated with prime numbers, perfect numbers, amicable numbers, abundant numbers, and the like. They would marvel at new examples but only make the barest attempts to systematically study their distribution. By the nineteenth century we saw great interest and success in studying the distribution of prime numbers. In the twentieth century, led principally by Paul Erdős, we began in earnest to study elementary number theory from this statistical viewpoint. One can see a direct progression for example from the ancient concepts of abundant and deficient numbers to distribution functions, the celebrated Erdős–Kac theorem, and probabilistic number theory. In this talk we shall see some of the triumphs of the Erdős way of thinking about elementary number theory, and we shall see that this statistical viewpoint is flourishing.

Vojtěch Rödl: Emory University

On two Ramsey type problems for K_{t+1}-free graphs

Abstract: In 1970 Erdős and Hajnal asked if for any r and t there is a K_{t+1}-free graph H with the property that any r-coloring of edges of H yields a monochromatic K_t. This conjecture was resolved positively by Folkman for $r = 2$ and by Nešetřil and speaker for r arbitrary.
In this talk we will discuss some old and new results related to this conjecture.
A related question was raised by Hajnal. For a graph H the t-independence

number $\alpha_t(H)$ is the size of the largest subset of vertices containing no K_t. Hajnal asked what is the minimum value $h_t(n)$ of $\alpha_t(H)$, where minimum is taken over all graphs H on n vertices containing no K_{t+1}. The question was addressed first by Erdős and Rogers who proved that $h_t(n)$ is at most $n^{1-\varepsilon}$, where $\varepsilon = \Theta(1/t^4 \log t)$. Recently, jointly with Dudek and Retter, we extended an earlier result of Wolfovitz proving a new upper bound on $h_t(n)$. Together with the earlier known lower bounds this implies that $h_t(n) = n^{1/2+o(1)}$.

Terence Tao: UCLA

Sets with few ordinary lines

Abstract: Given n points in the plane, an *ordinary line* is a line that contains exactly two of these points, and a *3-rich line* is a line that contains exactly three of these points. An old problem of Dirac and Motzkin seeks to determine the minimum number of ordinary lines spanned by n non-collinear points, and an even older problem of Sylvester (the "orchard planting problem") seeks to determine the maximum number of 3-rich lines. In recent work with Ben Green, both these problems were solved for sufficiently large n, by combining tools from topology (Euler's formula), algebraic geometry (the Cayley-Bacharach theorem, and the classification of cubic curves), additive combinatorics (via the group structure of said cubic curves), and even some Galois theory (through the theorem of Poonen and Rubinstein that a non-central interior point in the unit disk can pass through at most seven chords connecting roots of unity). We will discuss how these ingredients enter into the solution to these problems in this talk.

Vilmos Totik: University of Szeged and University of South Florida

Erdős on polynomials

Abstract: Some results of Erdős on polynomials and some later developments are reviewed. The topics that this survey covers are: discrepancy estimates for zero distribution, orthogonal polynomials, distribution and spacing of their zeros and critical points of polynomials.

Algebra

Miklós Abért: Rényi Institute

A p-adic analogue of the Erdős-Turán statistical group theory

Abstract: Between 1965 and 1972, Erdős and Turán published a series of seven papers titled 'On some problems of a statistical group theory I-VII'. In these papers, among other things, they analyzed the asymptotic behavior of uniform random elements of large finite symmetric groups. These influential papers had manifold continuations. I will talk about a p-adic analogue of the Erdős-Turán project. From one point of view, this means analyzing the asymptotic behavior of random elements and subgroups of the Sylow p-subgroups of large symmetric groups, for some fixed prime p. From another point of view, this is about building a theory of residually p-groups and pro-p groups acting on rooted trees. Part of the results are joint work with Bálint Virág.

Alexander Ivanov: Imperial College London

Majorana representation of the Monster

Abstract: The Majorana representations of groups were introduced in [A.A. Ivanov, The Monster Group and Majorana Involutions, Cambridge Univ. Press, Cambridge 2009]. by axiomatising some properties of the $2A$-axial vectors of the 196 884-dimensional Monster algebra, inspired by the sensational classification of such representations for the dihedral groups achieved by S. Sakuma. This classification took place in the heart of the theory of Vertex Operator Algebras and expanded earlier results by M. Miyamoto. Every subgroup G of the Monster which is generated by its intersection with the conjugacy class of $2A$-involutions possesses (possibly unfaithful) Majorana representation obtained by restricting to G the action of the Monster on its algebra. This representation of G is said to be *based on an embedding of G in the Monster.*

The principal ingredient of a Majorana representation of a group G is an algebra generated by idempotents (called *Majorana axes*), which are indexed by a generating set of involutions in G. The spectrum of a Majorana axes is a subset of $\{0, 1, \frac{1}{4}, \frac{1}{32}\}$ and the corresponding eigenspaces satisfy the fusion rules of the Majorana Fermion. For quite a few groups G, consid-

ered so far, the number of Majorana representations of G turned out to be equal to the number of embeddings of G in the Monster as a $2A$-generated subgroup. In this situation every Majorana representation of G is based on an embedding of G in the Monster, and this gives a highly valuable information on the subalgebras of the Monster algebra unaccessible in the original 196 884-dimensional setting.

What are the Majorana representations of the Monster itself?

This, most natural question, was for a while overlooked. Of course the Monster algebra is closed on the linear span of all the Majorana axes. It will be explained in my lecture how to deduce this conclusion directly from the Majorana axioms, making use of a result by Ákos Seress left unpublished.

László Pyber: Rényi Institute

Random generation of finite and profinite groups

Abstract: By a classical result of Dixon two random elements generate the symmetric group $\mathrm{Sym}(n)$ with probability close to $3/4$ when n is large enough. With Jaikin-Zapirain in 2011 we gave a characterisation of the class of finite groups for which a similar result holds. We will describe this theorem and various applications.

Jan-Christoph Schlage-Puchta: Ghent University

Origami and the product replacement algorithm

Abstract: An origami is a marked covering of the flat torus. Origamis have been used to study curves in certain Teichmüller spaces. The product replacement algorithm is a commonly used method to produce random elements in finite groups. In this talk we describe how these concepts are related and use this relation to obtain statistical information on Origami.

Aner Shalev: The Hebrew University of Jerusalem

Words and Groups

Abstract: In recent years there has been much interest in word maps on groups, with various motivations and applications. Substantial progress has been made and many fundamental questions were solved, using a wide spectrum of tools, including representation theory and geometry. The talk will describe past and current developments in this field, with emphasis on finite (often simple) groups and p-adic groups. I will also suggest open problems, conjectures and directions for further research.

Analysis

Zoltán Buczolich: Eötvös Loránd University

Divergent square averages and related topics

Abstract: Research related to almost everywhere convergence of ergodic averages along the squares was initiated by questions of A. Bellow and of H. Furstenberg. J. Bourgain proved that if $f \in L^p(\mu)$, for some $p > 1$ then the ergodic averages along the squares

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(T^{n^2}(x)) \tag{1}$$

converge almost everywhere. Bourgain also asked whether this result is true for $p = 1$, that is, for L^1 functions. With D. Mauldin we managed to show that this is not true:

Theorem. *The sequence $\{n^2\}_{n=1}^{\infty}$ is L^1-universally bad.*

In this talk I would also like to survey some other results related to this question. For example:

The divergent squares result was later generalized by P. LaVictoire. He showed that the sequence of powers $\{n^m\}_{n=1}^{\infty}$ is L^1-universally bad for $m > 1$ he also proved a similar result for the sequence of prime numbers. The general problem, that is to determine which polynomials with integer coefficients provide L^1-bad/good sequences seems to be still open.

I verified the following theorem which disproved a conjecture of J. Rosenblatt and M. Wierdl.

Theorem. *There exists a sequence (n_k) satisfying $n_{k+1} - n_k \to \infty$ (and hence of zero Banach density) which is universally L^1-good, that is, for any invertible aperiodic ergodic dynamical system (X, Σ, μ, T) and $f \in L^1(\mu)$ we have*

$$\lim_{N\to\infty} A(f, x, N) = \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} f(T^{n_k} x) = \int_X f d\mu, \tag{2}$$

for μ almost every $x \in X$.

This result was later sharpened by R. Urban and J. Zienkiewicz. They showed the following:

Theorem. *The sequence* $\lfloor n^\alpha \rfloor$, $1 < \alpha < 1.001$ *is universally* L^1 *good.*

P. LaVictoire also continued this line of research for example he studied the averages along $n^2 + \lfloor \rho(n) \rfloor$ for a slowly growing function ρ. Under some monotonicity assumptions, the rate of growth of $\rho'(x)$ determines the L^1-goodness of these sequences.

Methods developed for the solution of Bourgain's question about divergent square averages turned out to be useful in other areas, for example in a joint paper with I. Assani and D. Mauldin we have solved I. Assani's counting problem:

Suppose that $(X, \mathcal{B}, \mu)$ is a probability measure space, T is an invertible measure preserving transformation and f belongs to $L^1_+(\mu)$, that is, f is nonnegative and belongs to $L^1(\mu)$. By Birkhoff's Ergodic Theorem $\frac{f(T^n x)}{n}$ converges to 0, μ almost everywhere. Hence,

$$\mathbf{N}_n(f)(x) = \# \left\{ k : \frac{f(T^k x)}{k} > \frac{1}{n} \right\}$$

is finite μ almost everywhere. If $f \in L^p_+$ for $p > 1$, or $f \in L^+ \log L^+$ and the transformation T is ergodic, then $\frac{\mathbf{N}_n(f)(x)}{n}$ converges almost everywhere to $\int f d\mu$.

Assani's counting problem was about the μ almost everywhere finiteness of $\sup_n \frac{\mathbf{N}_n(f)(x)}{n}$ for $f \in L^1_+(\mu)$.

Theorem. *In any nonatomic, invertible ergodic system* $(X, \mathcal{B}, \mu, T)$ *there exists* $f \in L^1_+$ *such that* $\sup_n \frac{\mathbf{N}_n(f)(x)}{n} = \infty$ *almost everywhere.*

This of course implies that $\frac{\mathbf{N}_n(f)(x)}{n}$ for these functions f does not converge almost everywhere.

Marianna Csörnyei: University of Chicago

Geometry of null sets

Abstract: We will show how elementary product decompositions of measures can detect directionality in sets.
In order to prove this we will need to prove results about the geometry of sets of small Lebesgue measure: we show that sets of small measure are always contained in a "small" collection of Lipschitz surfaces.
The talk is based on a joint work with G. Alberti, P. Jones and D. Preiss.

Kenneth Falconer: University of St Andrews

Some Problems in Measure Combinatorial Geometry

Abstract: This talk will survey some geometrical problems in the Euclidean plane which have a measure-theoretic flavor and which featured in discussions and correspondence with Paul Erdős in the 1980s. Such problems include questions on the realization of certain geometric configurations within plane measurable sets.

Paul Humke: St. Olaf College

Differentiation properties related to the Keleti perimeter to area conjecture

Abstract: We examine finite unions of unit squares in same plane and consider the ratio of perimeter to area of these unions. In 1998, T. Keleti published the conjecture that this ratio never exceeds 4. Here we study the continuity and differentiability of functions derived from the geometry of the union of those squares. Specifically we show that if there is a counterexample to Keleti's conjecture, there is also one where the associated ratio function is differentiable.
This is joint work with former students Cameron Marcott, Bjorn Mellem, and Cole Stiegler.

Pertti Mattila: University of Helsinki

Singular integrals on subsets of metric groups

Abstract: Suppose μ is an Ahlfors-David regular measure on a metric space X; $\mu(B(x,r))$
$\approx r^s$ for $x \in \mathrm{spt}\mu$ and for small radii r, and K is a kernel such that $|K(x,y)| \approx d(x,y)^{-s}$ for small $d(x,y)$. The basic question is: when is the maximal operator T^*:

$$T^*f(x) = \sup_{\epsilon>0} \Big| \int_{d(x,y)>\epsilon} K(x,y)f(y)d\mu y\Big|,$$

bounded in $L^2(\mu)$? A rectifiability characterization is known when $X = \mathbb{R}^n$, K is the Riesz kernel $K(x) = |x|^{-s-1}x$ and $s = 1$ (by Melnikov, Verdera and myself) or $s = n - 1$ (by Nazarov, Tolsa and Volberg). I shall discuss much weaker partial results in general metric groups with dilations and,

as the main particular case, in Heisenberg groups. This is joint work with Vasilis Chousionis.

Assaf Naor: Courant Institute

Super-expanders

Abstract: A bounded degree n-vertex graph $G = (V, E)$ is an expander if and only if for every choice of n vectors $\{x_v\}_{v \in V}$ in $\mathbb{R}^k$ the average of the Euclidean distance between x_u and x_v is within a constant factor of the average of the same terms over those pairs $\{u, v\}$ that form an edge in E. The fact that this property is equivalent to the usual combinatorial notion of graph expansion is very simple, and once stated, it is obvious to ask what happens when $\mathbb{R}^k$ is replaced by other metric spaces. It turns out that this is a subtle question that relates to a long line of investigations in analysis and geometry. Graphs that are expanders in the classical sense (including random graphs) may or may not be expanders with respect to certain non-Euclidean geometries of interest. Existence of such "metric" expanders becomes a delicate question due in part to the insufficiency of the existing combinatorial, probabilistic and spectral methods. Indeed, constructing metric expanders is in essence a problem in analysis rather than combinatorics or spectral theory. In this talk we will formulate some of the basic questions in this direction, and explain some of the ideas and methods that were introduced in order to address them.

Benjamin Weiss: MIT

Paul Erdős as an ergodic theory pioneer

Abstract: In a joint paper with Yael Dowker that appeared in 1959 Paul Erdős gave some of the first examples that demonstrated the importance of rank one transformations in ergodic theory. I will describe some of this work and some of the later developments in the theory of these transformations. In particular I will explain what these transformations are and sketch a recent result that shows how the classical isomorphism problem has a simpler solution when restricted to this class.

Approximation Theory

Len Bos: University of Verona

Fekete Points as Norming Sets

Abstract: Suppose that $K \subset \mathbb{R}^d$ is a compact set. The Fekete points for K of degree n are those points in K which maximize the Vandermonde determinant associated to the polynomials of degree at most n, restricted to K. A sequence of finite sets $\Omega_n \subset K$ is said to be a norming set for K if there exists a constant C such that

$$\|p\|_K \leq C\|p\|_{\Omega_n}$$

for all polynomials p of degree at most n. It is said to be of optimal order if $\#(\Omega_n) = O(n^d)$. There is a conjecture that for any constant $A > 1$, the Fekete points of degree An form an optimal order norming set for K. We discuss this conjecture and give some cases when it holds

Tamás Erdélyi: Texas A&M University

The Mahler measure of the Rudin-Shapiro polynomials

Abstract: Littlewood polynomials are polynomials with each of their coefficients in $\{-1, 1\}$. A sequence of Littlewood polynomials that satisfies a remarkable flatness property on the unit circle of the complex plane is given by the Rudin-Shapiro polynomials. The Rudin-Shapiro polynomials appear in Harold Shapiro's 1951 thesis at MIT and are sometimes called just Shapiro polynomials. They also arise independently in a paper by Golay in 1951. They are remarkably simple to construct and are a rich source of counterexamples to possible conjectures. Despite the simplicity of their definition not much is known about the Rudin-Shapiro polynomials. It is outlined in this talk that the Mahler measure and the maximum modulus of the Rudin-Shapiro polynomials on the unit circle of the complex plane have the same size. This settles a longstanding conjecture of a number of experts. An analogous result, stating that the Mahler measure and the maximum norm of the Rudin-Shapiro polynomials have the same size even on not too small subarcs of the unit circle of the complex plane, is also announced. Not even nontrivial lower bounds for the Mahler measure of the Rudin Shapiro polynomials have been known before.

Manfred Golitschek: University of Würzburg

On the L_∞-norm of the L_2-spline projector

Abstract: In 2001, A. Shadrin published a proof of a famous problem in univariate spline theory, well-known as de Boor's conjecture (1973) :
Let $\Delta : a = t_0 < t_1 < \cdots < t_n = b$ be a finite sequence of knots. Let $\mathcal{S}_m(\Delta)$ be the linear space of polynomial splines of degree m with simple knots Δ. The L_∞-norm of the L_2-spline projector $\mathcal{P} := \mathcal{P}_m(\Delta) : C[a,b] \to \mathcal{S}_m(\Delta)$ is defined by

$$\|\mathcal{P}\|_\infty := \sup\Big\{\|\mathcal{P}f\|_\infty : \; f \in C[a,b], \; \|f\|_\infty = 1\Big\},$$

with respect to the uniform norm $\|\cdot\|_\infty$ on $[a,b]$.

Theorem A. (Shadrin)
There exists a positive number K_m depending only on m such that $\|\mathcal{P}\|_\infty \leq K_m$.

It is the purpose of my talk to provide an alternative proof of Theorem A and to discuss the size of the local supremum $|\mathcal{P}|(\tau) := |\mathcal{P}_m(\Delta)|(\tau)$ at points $\tau \in [a,b]$, where

$$|\mathcal{P}|(\tau) := \sup\Big\{|\mathcal{P}f(\tau)| : \; f \in C[a,b], \; \|f\|_\infty = 1\Big\}.$$

Vilmos Komornik: University of Strasbourg

Expansions in noninteger bases and Pisot numbers

Abstract: The familiar integer base expansions were generalized to noninteger bases by Rényi in 1956. Around 1990 Erdős et al. discovered several surprising phenomena concerning the number of expansions in such bases. Since then unexpected connections have been uncovered to probability and ergodic theory, combinatorics, symbolic dynamics, measure theory, topology, Diophantine approximation and number theory. We present some aspects of this field, including several recent results obtained in collaboration with P. Loreti, M. de Vries and S. Akiyama.

Dany Leviatan: Tel Aviv University

Weighted D-T moduli revisited and applied

Abstract: We introduce weighted moduli of smoothness for functions $f \in L_p[-1,1] \cap C^{r-1}(-1,1)$, $r \geq 1$, that have an $(r-1)$st absolutely continuous derivative in $(-1,1)$ and such that $\varphi^r f^{(r)}$ is in $L_p[-1,1]$, where $\varphi(x) = (1-x^2)^{1/2}$. These moduli are equivalent to certain weighted D-T moduli, but our definition is more transparent and simpler. In addition, instead of applying these weighted moduli to weighted approximation, which was the purpose of the original D-T moduli, we apply these moduli to obtain Jackson-type estimates on the approximation of functions in $L_p[-1,1]$ (no weight), by means of algebraic polynomials. We also have some inverse theorems that yield characterization of the behavior of the derivatives of the function by means of its degree of approximation.

We conclude with some direct and inverse results on weighted approximation. Joint work with K. Kopotun and I. A. Shevchuk.

Giuseppe Mastroianni: University of Basilicata

L^p-convergence of Lagrange and Hermite interpolation

Abstract: Let w be an arbitrary weight function on $(-1,1)$ and $\{p_m(w)\}_m$ be the corresponding sequence of orthonormal polynomials. Then, for any continuous function f, the Lagrange polynomial based on the zeros of $p_m(w)$ satisfies the following estimate:

$$\left\|L_m(w,f)\sqrt{w}\right\|_2 \leq \left\|\sqrt{w}\right\|_2 \|f\|_\infty . \tag{1}$$

Naturally, to obtain an analogous inequality for the L^p-norm $\|L_m(w,f)\,u\|_p$, with w, u arbitrary weights and f a continuous function, is still an open problem and its solution seems to be far.

After the result due to Erdős and Turán, numerous Authors (which I will not mention for the sake of brevity), stimulated also by a problem posed separately by Turán and Freud, have characterized special weight functions such that the estimate

$$\|L_m(w,f)\,u\|_p \leq \mathcal{C}\|f\|_\infty , \qquad \mathcal{C} \neq \mathcal{C}(m,f), \tag{2}$$

holds.

More recently, letting w, u be Jacobi weights and f be continuous on $(-1,1)$, inequality (2) has been replaced by

$$\|L_m(w,f)\,u\|_p \le \mathcal{C}\left(\sum_{k=1}^{m} \Delta x_k\, |fu|^p\,(x_k)\right)^{1/p}, \qquad \mathcal{C} \neq \mathcal{C}(m,f)\,, \tag{3}$$

where $\Delta x_k = x_{k+1} - x_k$ and x_k are the zeros of $p_m(w)$.

In this talk I am going to discuss some interesting consequences of (3), showing, in particular, the close connection among the L^p-convergence of Lagrange, Hermite and Hermite–Fejer polynomials.

Paul Nevai: King Abdulaziz University and Ohio State University

The pathbreaking Erdős–Turán papers on interpolation

Abstract: Although the three Erdős-Turán papers on interpolation published in Annals of Mathematics in 1937, 1938, and 1940, respectively, received plenty of attention and recognition, they have nevertheless been under-recognized and under-utilized as a valuable trove of ideas that led to the re-emergence of orthogonal polynomials in the 1970s as a modern subject occupying a central position in approximation theory and related areas. Also, maybe Shohat should get more credit for the work he has done on interpolation and orthogonal polynomials.

Allan Pinkus: Technion Haifa

On Ridge Functions

Abstract: In this lecture we will review various problems and properties associated with "Ridge Functions".

Boris Shekhtman: University of South Florida

Some problems and results in multivariate interpolation

Abstract: In the first part of the talk I will present a resolution of a conjecture of Ron-Qing Jia and A. Sharma regarding regularity of certain multivariate Birkhoff interpolation schemes. While originally conjectured for polynomials over reals, the conjecture turns out to be true over the complex field and false over the real field. In the second part I will discuss

some results and combinatorial problems regarding the minimal number of subspaces needed for regularity of Lagrange interpolation scheme.

Péter Vértesi: Rényi Institute

Paul Erdős and interpolation: Problems, results and new developments

Abstract: The present lecture tries to give a short summary of some significant results proved by Erdős (and his coauthors). Moreover, we mention several new developments of the quoted theorems.

Songping Zhou: Zhejiang Sci-Tech University

A New Important Development to Uniform Convergence of Trigonometric Series

Abstract: Generally speaking, nonnegativity and monotonicity are two most important factors setting on coefficients of a trigonometric series to guarantee the necessary and sufficient condition for its uniform convergence. The Mean Value Bounded Variation (MVBV) condition is proved to be the ultimate generalization to monotonicity in general sense (under nonnegativity of the coefficients). Now we find the MVBV condition in real sense can be also applied to guarantee the necessary and sufficient condition for uniform convergence. This is a new important development. The technique for proof is nonstandard and elegant.

This is a joint work with Lei Feng and Vilmos Totik.

Combinatorics

József Balogh: University of Illinois at Urbana-Champaign

Phase transitions in Ramsey-Turán Theory

Abstract: Let $f(n)$ be a function and L be a graph. Denote by

$$RT(n, L, f(n))$$

the maximum number of the edges of an L-free graph on n vertices with indepen- dence number less than $f(n)$.
This was introduced by by Erdős and Sós in 1970. In this talk I will survey some of the recent progress of the area. The newer results are partially joint with P. Hu, J. Lenz and M. Simonovits.

Fan Chung: University of California, San Diego

Recent results and problems in spectral graph theory

Abstract: We will discuss some recent developments in spectral graph theory and mention a number of results and problems on random walks on directed graphs, vertex and edge ranking, interlacing theorems, partition and clustering, among others.

Ralph Faudree: University of Memphis

Saturation Numbers for Graphs

Abstract: For a fixed graph F, a graph G is F*-saturated* if there is no copy of F in G, but for any edge $e \notin G$, there is a copy of F in $G+e$. The minimum number of edges in an F-saturated graph of order n will be denoted by $sat(n, F)$. A graph G is weakly F-saturated if there is an ordering of the missing edges of G so that if they are added one at a time in the given order, each edge added creates a new copy of F. The minimum size of a weakly F-saturated graph G of order n will be denoted by $wsat(n, F)$. General results and properties of $sat(n, F)$ and $wsat(n, F)$ will be presented, constructions that place bounds on $sat(n, f)$ and $wsat(n, F)$ will be described, and the precise values of $sat(n, F)$ and $wsat(n, F)$ for some families of graphs will

be determined. Bad behavior properties of both $sat(n, f)$ and $wsat(n, F)$ as compared to $ex(n, F)$ will be discussed.

Péter Frankl: Japan

Erdős on set intersections

Abstract: Intersection theorems on finite sets, which is a rich and developing topic now, started with the Erdős-Ko-Rado Theorem and some extensions of it by Erdős (sometimes with co-authors). His problems paved the way for deeper results and some of his conjectures present a great challenge even today. In the lecture we review results old and new.

Ehud Friedgut: Weizmann Institute

A sharp threshold for Ramsey properties of random sets of integers

Abstract: An extremely rich theme in combinatorics is the study of Ramsey-like properties in a random ground set which is much sparser than the complete ground set on which the original theorem is proven. For example (one out of many many such), van der Waerden's theorem states that for any r and k any r-coloring of the integers contains a monochromatic k-term arithmetic progression; Rödl and Ruciński showed that for any r and k there exist constants c and C, such that a random subset of $\{1, 2, \ldots, n\}$ of size $cn^{(k-2)/(k-1)}$ has this property with probability that tends to 0 as n tends to infinity, and a random subset of size $Cn^{(k-2)/(k-1)}$ has this property with probability that tends to 1. We show (at least for the case of 2 colors) that this can be sharpened, and that there exists $t(n)$, so that for any $\epsilon > 0$, c and C can be replaced by $(1 - \epsilon)t(n)$ and $(1 + \epsilon)t(n)$.

Our main tool is a theorem of Bourgain, which characterizes non-sharp thresholds. An additional result, that turns out to be crucial in the proof, is a recent powerful theorem of Balogh-Morris-Samotij which offers a better understanding of the independent sets in hypergraphs.

The talk will focus on the tools rather than the details of the proof.

Joint work with Hiệp Hàn, Yury Person and Mathias Schacht.

Zoltán Füredi: Rényi Institute

Turán type hypergraph problems: partial trees and linear cycles

Abstract: A linear cycle $\mathbf{C}_\ell^{(k)}$ is a family of k-sets $\{F_1, \dots, F_\ell\}$ such that $|F_i \cap F_{i+1}| = 1$ (for $1 \le i < \ell$), $|F_\ell \cap F_1| = 1$ and there are no other intersections. We can represent the hyperedges by intervals along a cycle. With an intensive use of the delta-system method we prove that for $t > 0$, $k \ge 5$ and sufficiently large n, ($n > n_0(k,t)$), if $\mathcal{F}$ is an n-vertex k-uniform family with

$$|\mathcal{F}| > \binom{n-1}{k-1} + \binom{n-2}{k-1} + \cdots + \binom{n-t}{k-1},$$

then it contains a linear cycle of length $2t+1$. The only extremal family consists of all edges meeting a given t-set. We also determine the even case, $\mathbf{ex}_k(n, \mathbf{C}_{2t}^{(k)})$, exactly. This is a joint work with Tao Jiang. The cases $k \le 4$ remain open.

András Gyárfás: Rényi Institute

Problems and memories

Abstract: I shall recall some combinatorial problems with associated memories during my encounters with Paul Erdős through the years 1962-1996

Ervin Győri: Rényi Institute

Hypergraph generalizations of the extremal graph theorem of Erdős and Gallai on paths

Abstract: Several theorems were proved recently about hypergraphs not containing cyles of given length. In these proofs several (graph and hypergraph) versions of Erdős-Gallai theorem on paths were used. In this paper, the systematic discussion of hypergraph versions will be presented. Many theorems are proved but some other conjectures are still open. We plan to cite all of them. Joint work with Gyula Y. Katona and Nathan Lemons.

Svante Janson: Uppsala University

Graph properties, graph limits and entropy

Abstract: Consider a graph property P. Many authors have studied the rate of growth of the number of (labelled) graphs of order n with this property, in particular for hereditary properties. We relate this to the entropy of the graph limits that arise as limits of sequences of graphs having the property P. In the case when there is a unique such graph limit with maximal entropy, we further show that a uniformly random graph of with property P and order n converges in probability to this maximizing graph limit as n tends to infinity. (Joint work with Hamed Hatami and Balázs Szegedy.)

Gil Kalai: The Hebrew University of Jerusalem

Some old and new problems in combinatorics and geometry

Abstract: Using this familiar title by Erdős, I will describe some old and new problems in combinatorics and geometry.

I) Remaining geometric questions around Borsuk's problem.

II) Geometric and purely combinatorial questions around Tverberg theorem.

III) Exotic combinatorial questions arising from Helly type theorems: The following conjecture of Meshulam and myself is an example: There is an absolute upper bound for the chromatic number of graphs with no induced cycles of length divisible by 3.

IV) Embedding of 2-dimensional complexes in four-dimensional space.

V) Three questions related to influences and Sauer-Shelah theorem.

VI) A question about random graphs.

Gyula O.H. Katona: Rényi Institute

Results on largest families of sets, following theorems of Sperner and Erdős

Abstract: Let $[n] = \{1, 2, \ldots, n\}$ and let $\mathcal{F} \subset 2^{[n]}$ be a family of its subsets. Sperner proved in 1928 that if $\mathcal{F}$ contains no pair of members in inclusion that is $F_1, F_2 \in \mathcal{F}$ implies $F_1 \not\subset F_2$ then $|\mathcal{F}| \leq \binom{n}{\lfloor n/2 \rfloor}$. Inspired by an application, Erdős generalized this theorem in the following way. Suppose that $\mathcal{F}$ contains no $k+1$ distinct members $F_1 \subset F_2 \subset \ldots \subset F_{k+1}$ then $|\mathcal{F}|$

is at most the sum of the k largest binomial coefficients of order n. This bound is, of course, tight. We will survey results of this type: determine the largest family of subsets of $[n]$ under a certain condition forbidding a given configuration described solely by inclusion among the members. This maximum is denoted by $\mathrm{La}(n, P)$ where P is the forbidden configuration. The final (hopelessly difficult) conjecture is that $\mathrm{La}(n, P)$ is asymptotically equal to the sum of the k largest binomial coefficients where the k largest levels contain no configuration P, but $k+1$ levels do. The most annoying unsolved case at the moment is when P consist of four sets A, B, C, D satisfying $A \subset B, A \subset C, B \subset D, C \subset D$. The trivial lower bound is the sum of the two largest binomial coefficients, but the best know upper bound is 2.25 times $\binom{n}{\lfloor n/2 \rfloor}$ proved by Kramer, Martin and Young.

János Körner: University of Rome

From intersection theorems to capacity results

Abstract: Intersection theorems for set families, graphs and other finite structures represent but one of the many research areas initiated by a seminal paper of Erdős, this time in [3]. This is still a very active research area, with a major recent breakthrough [2]. We introduce a general framework [5] to comprise intersection problems along with capacity–like problems [6] from Shannon's information theory. Let $\mathcal{F}$ and $\mathcal{G}$ be two not necessarily different families of finite structures such as graphs on a fixed set of n vertices and let $M(\mathcal{F}, \mathcal{G})$ be the cardinality of the largest subfamily of $\mathcal{F}$ with the property that the union of any two of its distinct elements belongs to $\mathcal{G}$. If $\mathcal{F} = \mathcal{G}$, this is just a reformulation of intersection problems in terms of complementary graphs. If however, the families $\mathcal{F}$ and $\mathcal{G}$ are disjoint and even "very different", we obtain an entire range of new problems reminiscent of Shannon's graph capacity and its generalizations [4]. In this talk we will survey some of these new problems, both open and solved. Here is an example.

Let $1 < k < n$ be natural numbers. Let $\mathcal{F} = \mathcal{F}_k$ be the family of k–disconnected graphs on the fixed vertex set $[n]$, where a graph is called k–disconnected if the number of its connected components is at least k. Let $\mathcal{G}$ consist of all the connected graphs with vertex set $[n]$. We have

Theorem CFK[1]

$$\lim_{n\to\infty} \sqrt[n]{M(\mathcal{F}_n, \mathcal{G})} = 2^{h(1/k)}$$

where $h(\cdot)$ is the binary entropy function.

The construction part of this result uses the interweaving technique introduced in [4] to solve problems about the capacity of families of graphs. Most of our problems remain wide open.

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Alexandr Kostochka: University of Illinois at Urbana-Champaign

On a conjecture by Gallai and a question by Erdős

Abstract: The goal of the talk is twofold. First, we prove the conjecture by Gallai from 1963 that for $n \equiv 1 \pmod{k-1}$, the minimum number of edges in a k-critical n-vertex graph is $\frac{(k+1)(k-2)n-k(k-3)}{2(k-1)}$ and describe the extremal graphs.

The second part uses the above result to study 3-coloring of planar graphs. The Grünbaum-Axenov Theorem states that every planar graph with at most three triangles is 3-colorable. It turns out that the family $\mathcal{P}_4$ of 4-critical plane graphs with exactly four triangles is infinite. Axenov and Erdős asked for a description of graphs in $\mathcal{P}_4$. Using the first result, we desribe the smaller family $\mathcal{P}_4'$ of graphs in $\mathcal{P}_4$ with no 4-faces, which is also infinite, as shown by Thomas and Walls. It turns out that the graphs in $\mathcal{P}_4'$ are exactly the extremal graphs for the first problem for $k = 4$ that have exactly four triangles. This is joint with M. Yancey, O. Borodin and B. Lidický.

Michael Krivelevich: Tel Aviv University

Biased positional games and the Erdős paradigm

Abstract: We discuss briefly recent and not so recent results about biased positional (mostly Maker-Breaker) games, stressing the role of probabilistic intuition allowing one quite frequently to guess correctly the asymptotics of the critical bias of these games upon looking at their random graph counterparts. This phenomenon, frequently addressed as the Erdős paradigm, is one of the truly amazing foresights of the Great Master.

Dhruv Mubayi: University of Illinois at Chicago

Intersection Theorems for Finite Sets

Abstract: Finite extremal set theory is concerned with the following general problem: Suppose we have a collection F of subsets of an n-element set and we have some restriction on the possible intersection sizes of pairs of sets in F. What is the maximum number of subsets that F can contain? Surprisingly, solutions to various special cases of this problem have deep implications in many other areas, including coding theory, geometry, and computer science. A particular famous example is due to Frankl and Rödl, who solved a 250-dollar problem of Erdős by proving that if n is a multiple of 4 and $n/4$ is excluded as an intersection size, then $|F| < (1.99)^n$. We extend this result by showing that if some additional (rather mild) restrictions are placed on the possible intersection sizes, then $|F| < (1.63)^n$. This is joint work with Vojtěch Rödl.

Deryk Osthus: University of Birmingham

Hamilton decompositions of regular expanders: a proof of Kelly's conjecture for large tournaments

Abstract: A conjecture of Kelly from 1968 states that every regular tournament on n vertices can be decomposed into $(n-1)/2$ edge-disjoint Hamilton cycles. We prove this conjecture for large n. In fact, we prove a far more general result, based on our recent concept of robust expansion and a new method for decomposing graphs: we show that every sufficiently large regular digraph G on n vertices whose degree is linear in n and which is a robust outexpander has a decomposition into edge-disjoint Hamilton cy-

cles. (Roughly speaking, a digraph is a robust outexpander if its expansion is resilient to the deletion of a small fraction of vertices or edges.)

This enables us to obtain numerous further results, e.g. as a special case we confirm a conjecture of Erdős on packing Hamilton cycles in random tournaments. We also apply our result to solve a problem on the domination ratio of the Asymmetric Travelling Salesman problem, which was raised e.g. by Glover and Punnen as well as Alon, Gutin and Krivelevich. As a final example, our result is an ingredient in the proof the long-standing 1-factorization conjecture on edge-colourings of dense regular graphs (the latter is joint work with B. Csaba, D. Kühn, A. Lo and A. Treglown).

Oleg Pikhurko: University of Warwick

Asymptotic Structure of Graphs with the Minimum Number of Triangles

Abstract: We describe the structure of graphs (modulo changing $o(1)$-fraction of edges) that have the minimum number of triangles given their order and size. Joint work with Alexander Razborov.

Andrzej Ruciński: A. Mickiewicz University and Emory University

On a Hamiltonian Problem For Triple Systems

Abstract: In 1952 Dirac proved that if every vertex in an n-vertex graph has degree at least $n/2$ then the graph is Hamiltonian. That result as well as those of Turán, Zarankiewicz, and Gallai, influenced the subsequent work of Paul Erdős in the extremal theory of graphs and hypergraphs. However, after more than 60 years the analog of Dirac's theorem for k-uniform hypergraphs, $k \geq 3$, is still unknown. In other words, it is an open problem to determine how large the *minimum* vertex degree in a k-uniform hypergraph must be in order to guarantee the existence of a tight Hamiltonian cycle. I will outline an approach leading, for $k = 3$, to a new bound on this extremal parameter (joint work with Vojtěch Rödl, Mathias Schacht and Endre Szemerédi).

Mathias Schacht: University of Hamburg

Extremal results in random graphs

Abstract: According to Paul Erdős it was Paul Turán who "created the area of extremal problems in graph theory". However, without a doubt, Paul Erdős popularized *extremal combinatorics*, by his many contributions to the field, his numerous questions and conjectures, and his influence on discrete mathematicians in Hungary and all over the world. Paul Erdős also established the *probabilistic method* in discrete mathematics, and in collaboration with Alfréd Rényi, he started the systematic study of *random graphs*. We shall survey recent developments at the interface of extremal combinatorics and random graph theory.

Asaf Shapira: Tel Aviv University

Deterministic vs Non-deterministic Graph Property Testing

Abstract: A graph property P is said to be testable if one can check whether a graph is close or far from satisfying P using few random local inspections. Property P is said to be non-deterministically testable if one can supply a "certificate" to the fact that a graph satisfies P so that once the certificate is given its correctness can be tested. The notion of non-deterministic testing of graph properties was recently introduced by Lovász and Vesztergombi, who proved that (somewhat surprisingly) a graph property is testable if and only if it is non-deterministically testable. Their proof used graph limits, and so it did not supply any explicit bounds. They thus asked if one can obtain a proof of their result which will supply such bounds. We answer their question positively by proving their result using Szemerédi's regularity lemma.

An interesting aspect of our proof is that it highlights the fact that the regularity lemma can be interpreted as saying that all graphs can be approximated by finitely many "template" graphs.

Joint work with L. Gishboliner

Miklós Simonovits: Rényi Institute

The exact solution of the Erdős-T. Sós conjecture I

Abstract: This is the first part of two lectures on the proof of

Conjecture (Erdős-T. Sós conjecture)**.** *If T_k is a fixed tree of k vertices, then every graph G_n of n vertices and*

$$e(G_n) > \frac{1}{2}(k-2)n \tag{1}$$

edges contains T_k.

Our main result is that if k is sufficiently large, then this conjecture holds.

In the second part (given by Endre Szemerédi) we sketch a proof of the weakened Erdős-T. Sós conjecture, according to which for every $\eta > 0$ there exists an integer $n_0(\eta)$ such that if $n, k > n_0(\eta)$ and a graph G on n vertices contains no T_k then

$$e(G_n) \leq \frac{1}{2}(k-2)n + \eta n.$$

That proof, combined with some stability methods shows that in most cases either we know that $T_k \subseteq G_n$ even under the weaker condition (1) or we can prove that the structure of G_n is very near to the conjectured extremal graphs: it is the union of small complete blocks or some complete bipartite graphs. Then, for $k > k_0$, applying some elementary arguments, we can embed T_k into G_n using only (1).

This lecture will describe the lemmas needed to use stability arguments, and also some further lemmas related to the fact that for sparse graphs the Regularity Lemma cannot be applied directly.

This is a joint work with Miklós Ajtai, János Komlós, and Endre Szemerédi.

Benny Sudakov: UCLA and ETHZ

Paul Erdős and Graph Ramsey Theory

Abstract: Graph Ramsey Theory, which was introduced by Erdős and his coauthors about 40 years ago, has quickly become one of the most active areas of Extremal Combinatorics. This area is not only a source of many fascinating problems but also serves as a testing ground for a variety of important combinatorial techniques. In this talk we discuss several old problems which have played an important role in the development of Graph Ramsey Theory, and report on the progress achieved so far.

Endre Szemerédi: Rényi Institute and Rutgers University

The exact solution of the Erdős-T. Sós conjecture II

Abstract: This is a joint work with Miklós Ajtai, János Komlós, and Miklós Simonovits and the second part of two lectures on the proof of the

Conjecture (Erdős-T. Sós)**.** *If T_k is a fixed tree of k vertices, then every graph G_n of n vertices and $e(G_n) > \frac{1}{2}(k-2)n$ edges contains T_k.*

Our main result is that if k is sufficiently large, then this conjecture holds.

One of the difficulties in proving this conjecture is that it would be natural to apply the Regularity Lemma, however, in the sparse case, i.e., when $n \gg k$, one has to use some more involved approach.

In this part we shall sketch, how the sparse case of the Erdős-Sós conjecture can be handled, how to decompose the vertex set of G_n so that the most difficult case becomes very similar to the case when we have $n = O(k)$, i.e., in the dense case.

Our basic approach is first to prove a weakening of the Erdős-T. Sós conjecture, according to which for every $\eta > 0$ there exists an integer $n_0(\eta)$ such that if $n, k > n_0(\eta)$ and a graph G_n on n vertices contains no T_k then

$$e(G_n) \leq \frac{1}{2}(k-2)n + \eta n.$$

The proof is rather involved. In the dense case it uses the regularity lemma.

The first lecture contains information on how to use stability arguments and some elementary methods, to prove the sharp version ($\eta = 0$) as well, for $k > k_0$. The first part is given by Miklós Simonovits.

Gábor Tardos: Rényi Institute

On the local chromatic number and its variants

Abstract: A proper vertex coloring of a graph is called a *local k-coloring* if one finds less than k distinct colors in the neighborhood of any vertex. This notion was introduced by Erdős, Füredi, Hajnal, Komjáth, Rödl and Seress in 1986 and the systematic study of the *local chromatic number* of a graph (the smallest k such that a local k-coloring exists) and its directed and/or fractional variants have not started till the 2005 paper of Körner, Pilotto and Simonyi. In this talk we survey old and new results on these nice chromatic parameters.

Prasad Tetali: Georgia Institute of Technology

Sidon sets with small gaps in sums

Abstract: The talk will contain an outline of some (now old) joint work with Joel Spencer on a (probabilistic) construction of an infinite Sidon set S with "small" gaps in the sumset $S + S$. This work was inspired by the work of Erdős-Sárközy-Sós from 1993. The talk will also include an update (based on recent results of Javier Cilleruelo) on a related favorite problem of Erdős concerning Sidon sets and asymptotic bases of order 3.

Andrew Thomason: University of Cambridge

List colouring of graphs and hypergraphs

Abstract: It seems that the now well-known notion of list colouring was first brought out in the 1970s by Vizing and by Erdős, Rubin and Taylor. The *list chromatic number* $\chi_\ell(G)$ (also called the *choice number*) of a graph G is the smallest k such that, whenever we allocate to each vertex $v \in V(G)$ a list L_v of k colours, then it is possible for each v to choose a colour from its list L_v so that no edge of G is monochromatic (i.e., for every edge uv, u and v choose different colours). This parameter is very natural, both from a theoretical and from a practical standpoint, and as such it has been widely studied since its introduction.

Of course, if all the lists L_v are the same, then a list colouring becomes just an ordinary k-colouring and so $\chi_\ell(G)$ is at least $\chi(G)$, the ordinary chromatic number of G. One of the main discoveries of Erdős et

al. is that $\chi_\ell(G)$ can be much larger than $\chi(G)$; they showed $\chi_\ell(K_{d,d}) = (1+o(1))\log_2 d$, whereas $\chi(K_{d,d}) = 2$. (The proof is closely connected to Erdős's study of "property B".) In fact, unlike $\chi(G)$, $\chi_\ell(G)$ must grow with the minimum degree of the graph G. Alon in the 1990s showed that $\chi_\ell(G) \geq (1/2 + o(1))\log_2 d$ holds for any graph G of minimum degree d.

The notion of list colouring applies just as well to r-uniform hypergraphs: how do these behave? Following work of Haxell, Pei and Verstraëte in the case $r = 3$, Alon and Kostochka showed that $\chi_\ell(G) \to \infty$ as $d \to \infty$ for simple r-uniform hypergraphs, where d is the average degree of G (the condition of simplicity is needed here).

We show that $\chi_\ell(G)) = \Omega(\log d)$. The constants involved are not too bad: indeed, in the case $r = 2$ we obtain $\chi_\ell(G) \geq (1+o(1))\log_2 d$, which closes the gap above and shows that, for graphs, the example of Erdős, Rubin and Taylor is best possible. It is likely that the natural hypergraph analogue of their example is best possible for all $r \geq 2$.

Joint work with David Saxton.

William Trotter: Georgia Institute of Technology

Tree-width and dimension

Abstract: Over the last 40 years, researchers have investigated connections between dimension for posets and planarity for graphs. In 2012, this line of research was extended to the structural graph theory parameter tree-width by proving that the dimension of a finite poset is bounded in terms of its height and the tree-width of its cover graph. This talk will focus on background and motivation, as well as the tell-tale signs which might have prompted a full-scale assault on the problem some 35 years ago, if only "our brains were open". This is joint work with Gwenaël Joret, Piotr Micek, Kevin G. Milans, Bartosz Walczak and Ruidong Wang.

Zsolt Tuza: Rényi Institute and University of Pannonia

Graph coloring and covering

Abstract: In the talk I recall some stories and open problems from the time of my collaboration with Uncle Paul.

Diophantine Number Theory

Mike Bennett: University of British Columbia

Shifted powers in binary recurrence sequences

Abstract: We consider the problem of explicitly determining, for given integers a and b, all terms of the form $ax^n + b$ in a fixed binary recurrence sequence. Such curious equations arise in the classification of congruent number curves with non-trivial integer points and bad reduction at two primes.

Yann Bugeaud: University of Strasbourg

On the continued fraction expansion of algebraic numbers

Abstract: We survey recent results on the continued fraction expansion of algebraic numbers of degree at least three. We show that, if the sequence of partial quotients of a real number enjoys certain combinatorial properties, then this number must be transcendental. As a consequence, the infinite word composed of the partial quotients of an algebraic number of degree at least three cannot have sublinear block complexity and it cannot be generated by a finite automaton.

Jan-Hendrik Evertse: University of Leiden

P-adic decomposable form inequalities

Abstract: Let $F \in \mathbb{Z}[X_1, \dots, X_n]$ be a decomposable form, that is a homogeneous polynomial that factors into linear forms over $\mathbb{C}$. For a positive integer m, denote by $N(F, m)$ the number of solutions of the inequality $|F(\mathbf{x})| \leq m$ in $\mathbf{x} \in \mathbb{Z}^n$, and by $V(F)$ the volume of the set $\{\mathbf{x} \in \mathbb{R}^n : |F(\mathbf{x})| \leq 1\}$. Under suitable conditions imposed on the decomposable form F, Thunder obtained an asymptotic formula $N(F, m) = V(F)m^{n/d} + o(m^{n/d})$ as $m \to \infty$, where $d = \deg F$.

Recently, my PhD-student Junjiang Liu obtained a similar formula for the p-adic decomposable form inequality

$$|F(\mathbf{x})| \cdot \prod_{i=1}^{t} |F(\mathbf{x})|_{p_i} \leq m$$

in integer vectors $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{Z}^n$ with $\gcd(x_1, \ldots, x_n, p_1 \cdots p_t) = 1$, where $p_1, \ldots, p_t$ are distinct primes. I will discuss this, and related results.

Kálmán Győry: University of Debrecen

Perfect powers in products with terms from arithmetic progression – A survey

Abstract: By a celebrated theorem of Erdős and Selfridge the product of consecutive positive integers is never a power. It is an old conjecture that more generally the equation

$$m(m+d)\ldots(m+(k-1)d) = y^n$$

has no solution in positive integers m, d, k, y, n with $\gcd(m, d) = 1, k \geq 3, n \geq 2$ and $(k, n) \neq (3, 2)$. This equation has been investigated by many people. In the last fifteen years the conjecture was confirmed for $k < 35$ (Gy, $k = 3$; Gy, Hajdu, Saradha, $k = 4, 5$; Bennett, Bruin, Gy, Hajdu, $6 \leq k \leq 11$; Gy, Hajdu, Pintér, $12 \leq k \leq 34$). In our talk we give a survey of these results and the methods utilized in the proofs.

Lajos Hajdu: University of Debrecen

A Hasse-type principle for exponential diophantine equations and its applications

Abstract: Let $a_1, \ldots, a_k$, $b_{11}, \ldots, b_{1\ell}, \ldots, b_{k1}, \ldots, b_{k\ell}$ be non-zero integers, c be an integer, and consider the exponential diophantine equation

$$a_1 b_{11}^{\alpha_{11}} \ldots b_{1\ell}^{\alpha_{1\ell}} + \cdots + a_k b_{k1}^{\alpha_{k1}} \ldots b_{k\ell}^{\alpha_{k\ell}} = c \tag{1}$$

in non-negative integers $\alpha_{11}, \ldots, \alpha_{1\ell}, \ldots, \alpha_{k1}, \ldots, \alpha_{k\ell}$.

The effective and ineffective theory of (1) has a long history. In case of $k = 2$, one can apply Baker's method to give explicit bounds for the exponents α_{11}, α_{12}, α_{21}, α_{22}; see results of Győry and others. Note that by results of Vojta and Bennett, the solutions to (1) can still be "effectively determined" for $k = 3, 4$, under some further restrictive assumptions. On the other hand, by results of Evertse and others, it is also known that for any k, the number of those solutions to equation (1) for which the left hand side of (1) has no vanishing subsum is finite, and it can be bounded explicitly in terms of the parameters involved.

In our talk, our starting point is the following

Conjecture. Suppose that equation (1) has no solutions. Then there exists an integer m with $m \geq 2$ such that the congruence

$$a_1 b_{11}^{\alpha_{11}} \dots b_{1\ell}^{\alpha_{1\ell}} + \dots + a_k b_{k1}^{\alpha_{k1}} \dots b_{k\ell}^{\alpha_{k\ell}} \equiv c \pmod{m} \qquad (2)$$

has no solutions in non-negative integers $\alpha_{11}, \dots, \alpha_{1\ell}, \dots, \alpha_{k1}, \dots, \alpha_{k\ell}$. The conjecture is a variant of a classical conjecture of Skolem, where the right hand side c is taken as zero. Note that if true, then the conjecture can be considered as a Hasse-type principle for exponential diophantine equations. Schinzel proved that in case of $k = 1$ the conjecture is true.

In our talk, we present a new result showing that the conjecture is true for "almost all" cases. Further, we also check its validity for a relatively large set of the parameters involved. The main tool behind our results is a variant of a nice theorem of Erdős, Pomerance and Schmutz concerning small values of Carmichael's λ-function. As an application, we present a method for the solution of equations of the type (1), under certain assumptions. We give concrete examples, as well. We mention that in the literature one can find several sparse results of this type, e.g. due to Alex, Brenner, Foster and others. However, in our method the appropriate moduli as in (2) can be constructed systematically, based upon arguments of Erdős, Pomerance and Schmutz.

The new results presented are joint with Cs. Bertók.

Ákos Pintér: University of Debrecen

Variations on a theme: the power values of power sums

Abstract: In this talk we present some recent results and new directions concerning the Schäffer conjecture which states that the equation

$$1^k + 2^k + \ldots + x^k = y^n$$

in positive integers $k \geq 1, x \geq 2, y \geq 2, n \geq 2$ with $(k, n) \notin \{(1, 2), (3, 2), (3, 4), (5, 2)\}$ possesses only one anomalous solution $(k, x, y, n) = (2, 24, 70, 2)$.

Gallai Memorial Session

László Lovász: Eötvös Loránd University

Tibor Gallai and matching theory

Abstract: The theory of matchings has played a key role in the development of graph theory: repeatedly it provided problems that were solvable, but their solution needed new ideas, which then found further important applications. Tibor Gallai contributed much to the development of matching theory in the 1950's and 60's, through his work on matchings in regular graphs, the Edmonds-Gallai Structure Theorem, and applications of matching theory. This talk will survey some of his results and later developments based on them.

Alexander Schrijver: CWI and University of Amsterdam

Tibor Gallai's basic work in combinatorial optimization

Abstract: We discuss the basic work of Tibor Gallai in combinatorial optimization that formed the bridgehead for several further fundamental developments in this area.

Gábor Simonyi: Rényi Institute

Applications and extensions of a theorem of Gallai

Abstract: In honour of Gallai's work on comparability graphs an edge coloring of a graph that creates no 3-colored triangles is called a Gallai coloring. Gallai proved a structural theorem about monochromatic subgraphs of Gallai colored complete graphs. This theorem turned out to be relevant and useful for several problems in graph theory. Extensions concern properties of Gallai colored but not necessarily complete graphs. In the talk I try to show some of these applications and extensions.

Éva Tardos: Cornell University

Games, Auctions, Learning, and the Price of Anarchy

Abstract: Selfish behavior can often lead to suboptimal outcome for all participants, a phenomenon illustrated by many classical examples in game theory. Over the last decade, computer scientists and game theorists have developed good understanding how to quantify the impact of strategic user behavior on overall performance in environments that include selfish traffic routing, service location, and bandwidth sharing. In this talk, we will consider E-commerce applications from this perspective.

The Internet provides an environment running millions of auctions, an environment where simplicity is more important than perfect efficiency, and where the systems used do not satisfy the usual standards of mechanism design. We'll consider such auctions as games, and we discuss how to analyze such games providing robust guarantees for their performance even when players participate in multiple auctions, have valuations that are complex functions of multiple outcomes, and are using learning strategies to deal with an uncertain environment.

Bjarne Toft: University of Southern Denmark

Gallai, colourings and critical graphs – a 50 year anniversary

Abstract: Exactly 50 years ago, in 1963, Tibor Gallai published two papers on colour-critical graphs (Kritische Graphen I & II, Publ. Math. Inst. Hungar. Acad. Sci. 8, 1963, 165-192 & 373-395). The same year he lectured about critical graphs at the graph theory meeting in Smolenice. Gallai's results marked a breakthrough in colouring theory. In my talk I shall review the papers in context and point out their importance for the development of graph colouring theory up to this day. Some of my personal reminiscences about this most extraordinary mathematician will be part of the talk too.

Graph Homomorphisms

Pavol Hell: Simon Fraser University

Combinatorial Dichotomy Classifications

Abstract: In classifying the complexity of certain homomorphism problems it sometimes turns out that it is the presence of a combinatorial obstruction in the target structure that causes a problem to become intractable. I will discuss several recent results of this type.

Jaroslav Nešetřil: Charles University, Prague

Orderings of sparse graphs

Abstract: Ordered graphs (rather than just graph orientations) play an important role in Ramsey theory and applications (such as in model theory). We survey the recent development, particularly related to the statistics of orderings of sparse graphs.

Patrice Ossona de Mendez: CNRS Paris and Charles University

Low Tree-depth Decompositions

Abstract: The *tree-depth* of a graph G is the minimum height of a forest F such that every edge of G connects vertices that have an ancestor-descendant relationship in F. This parameter is minor-monotone and it is closely related to other measures in combinatorics such as vertex ranking number, ordered chromatic number, and minimum elimination tree height; it is also closely related to the cycle rank of directed graphs, the star height of regular languages, and the quantifier rank of formulas.

For non-negative integer p, a *low tree-depth decomposition* with parameter p of a graph G is a partition $V_1, \ldots, V_k$ of its vertex set such that every $i \leq p$ parts induce a subgraph with tree-depth at most i. The minimum number of parts k for which a low tree-depth decomposition with parameter p of G exists is $\chi_p(G)$. This minor-monotone graph invariant is related to the densities of the shallow minors (topological minors, or immersions) of the graph G.

In this talk, we survey several theoretical and algorithmic applications of low-tree depth decompositions, like distance coloring, taxonomy of graph

classes, restricted homomorphism dualities, and first-order model-checking in classes of sparse graphs. We also discuss perspectives opened by the recent construction of explicit limit objects for graphs with bounded tree-depth.

Joint work with Jaroslav Nešetřil.

Alex Scott: University of Oxford

Discrepancy of graphs and hypergraphs

Abstract: How uniformly is it possible to distribute edges in a graph or hypergraph? This question was asked in 1971 by Erdős and Spencer, who defined the *discrepancy* of a hypergraph to be the maximum difference between the number of edges and the number of nonedges in any induced subgraph.

Erdős and Spencer proved that every graph on n vertices has an induced subgraph in which the numbers of edges and nonedges differ by at least $cn^{3/2}$ (which is optimal up to the constant), and gave an extension to hypergraphs. Erdős, Goldberg, Pach and Spencer subsequently proved a weighted version for graphs with density p.

We shall discuss generalizations of these results and related questions involving intersections of pairs of graphs or hypergraphs. Joint work with Béla Bollobás.

Balázs Szegedy: University of Toronto

On the Erdős–Simonovits, Sidorenko Conjecture

Abstract: The so-called Erdős–Simonovits, Sidorenko conjecture says that the density of a fixed bipartite graph in another graph is minimized by the random graph if the edge density is fixed. It can be equivalently formulated as an integral inequality related to statistical physics and quantum mechanics. We provide an information theoretical approach which gives a unified treatment for all the known cases and proves the conjecture for many new graphs. We also show related results for hypergraphs.

Xuding Zhu: Zhejiang Normal University, China

Circular flow of signed graphs

Abstract: A signed graph is a pair (G, σ), where G is a graph and σ is a signature which assigns to each edge e a sign $\sigma(e) \in \{1, -1\}$. If $\sigma(e) = 1$, then e is a positive edge. Otherwise e is a negative edge. In an orientation of a signed graph (G, σ), a positive edge $e = xy$ is oriented either from x to y, or from y to x, and a negative edge $e = xy$ is oriented either away from both x and y, or oriented towards both x and y. Let $E^+(x)$ and $E^-(x)$ be the set of edges oriented from x and towards x, respectively. A circular r-flow in (G, σ) is an orientation of (G, σ) together with a mapping $f : E \to [1, r-1)$ such that for each vertex x, $\sum_{e \in E^+(x)} f(e) = \sum_{e \in E^-(x)} f(e)$. Signed graphs is a generalization of graphs, namely, a graph is a signed graph with no negative edges. It was proved recently by Lovász, Thomassen, Wu and Zhang that a $6k$-edge connected graph admits a circular $(2 + \frac{1}{k})$-flow. We prove an analog result for signed graph: if (G, σ) is essentially $(2k + 1)$-unbalanced and $(12k - 1)$-edge connected, then (G, σ) admits a circular $(2 + \frac{1}{k})$-flow.

Number Theory

Krishnaswami Alladi: University of Florida

Multiplicative functions and small divisors

Abstract: Motivated by applications to the distribution of additive functions in probabilistic number theory, I showed in 1983 that if g is a multiplicative function satisfying $0 \leq g \leq 1$, then for all square-free n,

$$\sum_{d|n} g(d) \leq 2 \sum_{d|n, d \leq \sqrt{n}} g(d).$$

Soon after, in collaboration with Erdős and Vaaler, I showed that there is a one to one correspondence between the set of divisors $D_<$ of a square-free n which are $< \sqrt{n}$ and the set of divisors $D_>$ of n which are $> \sqrt{n}$, such that each divisor in $D_>$ can be realized as a multiple of the corresponding divisor in $D_<$. This immediately implies the above inequality. Motivated by this unusual correspondence between the small and large divisors, and with applications to probabilistic number theory in mind, Alladi-Erdős-Vaaler extended the above inequality as follows: Let k be an integer ≥ 2. If g is multiplicative, and if $0 \leq g \leq \frac{1}{k-1}$, then for all square-free n,

$$\sum_{d|n} g(d) << _k \sum_{d|n, d \leq n^{1/k}} g(d).$$

Our proof of this general inequality used a powerful theorem of Baranyai on hypergraphs. Subsequently, several authors have simplified our proof, or made improvements on our results. After recalling my joint work with Erdős on this problem, I will report on recent work on this topic including the results of Srinivasan, Ritabrata Munshi and Surya Mohan.

Antal Balog: Rényi Institute

The 'Decomposition Theorem'

Abstract: Let A be a finite set of real numbers, the sum set and the product set are defined by

$$A + A = \{a + b;\ a, b \in A\}, \quad A \cdot A = \{ab;\ a, b \in A\}.$$

$A + A$ can be small if A has some 'additive structure', similarly $A \cdot A$ can be small if A has some 'multiplicative structure'. The Erdős–Szemerédi sum–product conjecture claims that these structures cannot come together, namely for any $\varepsilon > 0$ and for any sufficiently big finite set A

$$\max\{|A + A|, |A \cdot A|\} \geq |A|^{2-\varepsilon}.$$

Another notion to measure the 'additivity' (or 'multiplicativity') of a set is the additive energy (or multiplicative energy). They are defined by

$$E_{+}(A) = \#\{a_1 + a_2 = a_3 + a_4;\ a_1, a_2, a_3, a_4 \in A\},$$

$$E_{\times}(A) = \#\{a_1 a_2 = a_3 a_4;\ a_1, a_2, a_3, a_4 \in A\}.$$

When A has some additive structure then $E_{+}(A)$ is big, and similarly the multiplicative structure implies that $E_{\times}(A)$ is big. One can believe that the sum–product conjecture appears as one of the two energies should be small. However, this is very far from true, for example if half of A has some additive structure, while the other half has some multiplicative structure then both energies are big.

We will show that this is the only example (in a weak sense). Every finite set of real numbers A can be split into two disjoint parts $A = B \cup C$ with both $E_{+}(B)$ and $E_{\times}(C)$ are small. The sum–product conjecture would follow if these energies are as small as $|A|^{2+\varepsilon}$. However, this is not true either. The right exponent is between $2 + 1/3$ and $3 - 2/33$, but what it is exactly, we do not know.

András Biró: Rényi Institute

A Poisson-type summation formula with automorphic weights

Abstract: We show a summation formula which has a strong formal analogy to the classical Poisson Summation Formula well-known in Fourier analysis. The new formula contains certain triple product integrals of automorphic forms as weights. Such triple product integrals are investigated very intensively in the theory of automorphic forms. The integral transform involved in the new identity is the so-called Wilson function transform of type II, whose kernel function is an ${}_7F_6$ hypergeometric function. This transform (which was introduced only recently by Groenevelt) plays the role of the classical Fourier transform in our formula, and it shares some nice properties of the Fourier transform.

Javier Cilleruelo: University of Madrid

Infinite Sidon sequences

Abstract: In 1932 Simon Sidon asked to a young Paul Erdős about the slowest possible growth of a sequence of positive integers having the property that no two pairs of its terms have the same sum. Erdős named them Sidon sequences and they became one of his favorit topics. The interest of Erdős for Sidon sequences describes very well his taste for additive problems with a combinatorial flavor.

Erdős observed that the greedy algorithm (start with 1 and add successively the smallest allowed term) provides a Sidon sequence A with counting function $A(x) > x^{1/3}$. Erdős also proved that $A(x) \gg x^{1/2}$ cannot hold when A is an infinite Sidon sequences but he conjectured the existence, for each $\epsilon > 0$, of an infinite Sidon sequence with $A(x) \gg x^{1/2-\epsilon}$.

The greedy Sidon sequence was the densest known one for almost 50 years until Ajtai, Komlós and Szemerédi [1] proved, in 1981, the existence of an infinite Sidon sequence A with counting function $A(x) \gg (x \log x)^{1/3}$.

In 1998 Ruzsa [4] made a breakthrough by proving the existence of a Sidon sequence with $A(x) = x^{\sqrt{2}-1+o(1)}$. Both Ruzsa's construction and the one obtained by Ajtai, Komlós and Szemerédi were probabilistic and it was an open problem to give an explicit construction of a Sidon sequence with $A(x) \gg x^{\delta}$ for some $\delta > 1/3$.

We use a new method, based in the discrete logarithm, to make the first explicit construction of this kind [2]. Indeed, our Sidon sequence is as dense as the one by Ruzsa:

$$A(x) = x^{\sqrt{2}-1+o(1)}.$$

Our method generalizes to B_h sequences, those ones having the property that all the h-fold sums are distinct. We prove, for each $h \geq 3$, the existence of a B_h sequence with counting function

$$A(x) = x^{\sqrt{(h-1)^2+1}-(h-1)+o(1)}.$$

Unfortunately, our method does not give an explicit construction when $h \geq 3$. The cases $h = 3$ and $h = 4$ had been obtained before [3] using a variant of Ruzsa's construction.

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Cécile Dartyge: University of Nancy

The Chebychev's problem for the twelfth cyclotomic polynomial

Abstract: Let $P^+(n)$ denote the greatest prime factor of the integer n and

$$\Phi_{12}(X) = X^4 - X^2 + 1$$

the twelfth cyclotomic polynomial. We prove that there exists $c > 0$ such that for x large enough we have

$$P^+\Big(\prod_{n \leq x} \Phi_{12}(n)\Big) \geq x^{1+c}.$$

Jean-Marc Deshouillers: Institut Mathématique de Bordeaux

On the coefficients of the powers of a Laurent series algebraic over $\mathbb{F}_q(X)$

Abstract: Let $\mathbb{F}_q$ be the Galois field with q elements. For a Laurent series $\boldsymbol{\theta} \in \mathbb{F}_q((X))$, algebraic over $\mathbb{F}_q(X)$, we shall discuss the distribution of the coefficients of the powers of $\boldsymbol{\theta}$, say $\boldsymbol{\theta}^n = \sum_m \theta(n,m)X^m$. We shall mention some old and new results and mainly concentrate on the *distribution modulo one* of $(\boldsymbol{\theta}^n)_n$, *i. e.* the study of the density of the sequence $\{n \ : \ (\theta(n,1),\ldots,\theta(n,s)) = (\eta_1,\ldots,\eta_s)\}$ for a given block $(\eta_1,\ldots,\eta_s) \in \mathbb{F}_q^s$, a topic where some fundamental questions are still open.

Harold G. Diamond: University of Illinois at Urbana-Champaign

Optimal Chebyshev Bounds for Beurling Generalized Numbers

Abstract: Let $N(x)$ denote the integer counting function of a system of Beurling generalized numbers. It was conjectured by the first author that if

$$\int_1^\infty |N(x) - Ax|\, x^{-2} dx < \infty \tag{1}$$

for some positive number A, then the analogue of Chebyshev's famous prime number bounds holds. However, this conjecture was shown false by J.-P. Kahane. With the addition of the pointwise condition

$$x^{-1}\log x\,(N(x) - Ax) = o(1),$$

the Chebyshev bounds were established by J. Vindas. We establish Chebyshev-type bounds under (1) and the weaker hypothesis

$$x^{-1}\log x\,(N(x) - Ax) = O(1).$$

Also, we show by example that this result is optimal. Joint work with Wen-Bin Zhang.

Michael Drmota: TU Wien

Gelfond problems on the sum-of-digits function and subsequences of automatic sequences

Abstract: In 1969 Gelfond has formulated three problems on the distribution of q-ary the sum-of-digits function $s_q(n)$ in residue classes. The first problem on the joint distribution $(s_{q_1}(n), s_{q_2}(n))$ with coprime bases q_1, q_2 was solved by Besineau (in a weak) and by Kim (in a strong form). However, the second problem on the distribution of $s_q(p)$ of primes p was just proved recently by Mauduit and Rivat (2010). The third problem on the distrbution of $s_q(P(n))$ for polynomial values $P(n)$ was proved for squares by Mauduit and Rivat (2010) and for general polynomials, provided that the basis q is sufficently large, by Drmota, Mauduit and Rivat (2011).

The purpose of this talk is give first an overview of the methods that have been developed to prove these results and second to survey recent developments for extension and variations of such results that have been obtained in collaboration with Mauduit, Morgenbesser, and Rivat. In particular we will discuss more precise (and "local") results for q-additive functions of primes and squares and several results on subsequences of the form $[n^c]$, where $c > 1$ is not an integer.

Kevin Ford: University of Illinois at Urbana-Champaign

Multiplicative structure of integers, shifted primes and arithmetic functions

Abstract: This talk concerns a number of interrelated topics connected with the distribution of prime factors and of divisors, of integers taken as a whole and also of special subsets such as shifted primes and the values of certain arithmetic functions such as Euler's totient function. We will discuss the history of these problems, often beginning with pioneering work of Paul Erdős, and describe the current state of the art. Throughout, we will emphasize connections with other parts of number theory and other areas of mathematics, especially the influence of probabilistic ideas on the subject. In the spirit of Erdős, we will also present many open problems and directions for future research.

Katalin Gyarmati: Eötvös Loránd University

On the reducibility of large sets of residues modulo p

Abstract: Ostmann introduced the notion of reducibility of infinite sequences of nonnegative integers. Sárközy extended this definition to subsets of $\mathbb{F}_p$: A set $\mathcal{C} \subseteq \mathbb{F}_p$ is said to be reducible if it can be represented in the form $\mathcal{C} = \mathcal{A} + \mathcal{B}$ with $|\mathcal{A}|, |\mathcal{B}| \geq 2$. If $\mathcal{C}$ is not reducible, then it is called primitive. Let $f(p)$ denote the cardinality of the largest primitive subset of $\mathbb{F}_p$. With my coauthors S. Konyagin and A. Sárközy we proved

$$p - c_1 \tfrac{p}{\log p} < f(p) < p - c_2 \tfrac{\log \log p}{(\log p)^{1/2}}.$$

The proofs use Weil's theorem and methods from combinatorial number theory. (Note that a slightly related problem, the cardinality of the largest subset of $\mathbb{F}_p$ which can not be represented in the form $\mathcal{A} + \mathcal{A}$ was studied by Gowers, Green and Alon.)

Bob Hough: University of Cambridge

The least modulus of a covering set is uniformly bounded

Abstract: A covering system of congruences is a collection

$$a_i \bmod n_i, \qquad 1 < n_1 < n_2 < \ldots < n_k$$

such that every integer satisfies at least one of them. Erdős asked whether there are covering systems for which the least modulus n_1 is arbitrarily large. Building on earlier work of Filaseta, Ford, Konyagin, Pomerance and Yu, I have recently found a negative answer to this question. I will discuss some aspects of the proof.

Aleksandar Ivić: Serbian Academy of Sciences, Belgrade

The divisor function and divisor problem

Abstract: We first analyze some of the work of Erdős on the classical divisor function $d(n)$, the number of divisors of n. Then we present some new results involving

$$\Delta(x) := \sum_{n \leq x} d(n) - x(\log x + 2\gamma - 1),$$

the classical error term in the Dirichlet divisor problem
($\gamma = -\Gamma'(1) = 0.57721566\ldots$ is Euler's constant).

In particular, W. Zhai and the speaker (2012) have proved the following.

Suppose $\log^2 T \ll U \le T^{1/2}/2, T^{1/2} \ll H \le T$. Then we have

$$\int_T^{T+H} \max_{0\le u\le U} \left|\Delta(x+u) - \Delta(x)\right|^2 dx \ll HU\mathcal{L}^5 + T\mathcal{L}^4 \log\mathcal{L} + H^{\frac{1}{3}} T^{\frac{2}{3}} U^{\frac{2}{3}} \mathcal{L}^{\frac{10}{3}+\varepsilon},$$

where $\mathcal{L} := \log T$. This generalizes a result of Heath-Brown & Tsang (1994). It implies the folllowing: Suppose T, U, H are large parameters and $C > 1$ is a large constant such that

$$T^{131/416+\varepsilon} \ll U \le C^{-1} T^{1/2} \mathcal{L}^{-5}, \qquad CT^{1/4} U \mathcal{L}^5 \log\mathcal{L} \le H \le T.$$

Then in the interval $[T, T+H]$ there are $\gg HU^{-1}$ subintervals of length $\gg U$ such that on each subinterval one has $\pm\Delta(x) \ge c_\pm T^{1/4}$ for some $c_\pm > 0$.

Gyula Károlyi: Eötvös Loránd University & University of Queensland

Restricted set addition in finite groups

Abstract: We survey direct and inverse results related to the Erdős–Heilbronn problem, from the group of integers modulo a prime through abelian groups to finite groups in general, with an emphasis on various successful algebraic approaches. We study these problems locally, that is, when cardinalities are smaller than the size of the smallest nontrivial subgroup. A recent surprise is the emergence of unexpected critical pairs in non-nilpotent groups.

Imre Kátai: Eötvös Loránd University

Arithmetical functions with regular behaviour

Abstract: Erdős proved in 1946 that if f is an additive function and

$$f(n+1) - f(n) \to 0 \quad \text{as} \quad n \to \infty$$

or f is monotonic, then $f(n) = c\log n$. These assertions have plenty of generalizations. Analogue questions and assertions can be formulated for multiplicative functions and for continuous homomorphisms in compact groups.

Sergei Konyagin: Moscow

Numbers that become composite after changing one or two digits

Abstract: M. Filaseta, M. Kozek, Ch. Nicol and J. Selfridge [1] proved that there are infinitely many composite natural numbers N, coprime to

10, with the property that if we replace any digit in the decimal expansion of N with $d \in \{0, \ldots, 9\}$, then the number created by this replacement is composite. The aim of the talk is to give answers to two questions posed in [1].

Theorem 1 For any base $b > 1$ the set of composite natural numbers N that remain composite when any one or two digits base b are changed has a positive lower density.

Theorem 2 For any base $b > 1$ there are infinitely many primes p that are composite for every replacement of a digit.

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Vsevolod Lev: University of Haifa

Flat-containing and shift-blocking sets in finite vector spaces

Abstract: Suppose we want to color, say green, some of the points of a finite vector space V, so that every $v \in V$ lies in a d-dimensional affine subspace which is entirely green with the possible exception of v itself. What is the smallest possible number of points to color? Equivalently, how many points one can color black so that every translate of the set of black points avoids some punctured d-dimensional linear subspace? We establish a number of upper and lower bounds in the characteristic-2 case.

Helmut Maier: Universität Ulm

Smooth numbers and zeros of Dirichlet- L- functions

Abstract: We report on joint work with Hans- Peter Reck:
Let χ be a non- principal Dirichlet- character. We investigate relations between the size of sums of the form

$$\sum \chi(p)p^{it}$$

and zero- free regions of the L- function $L(s, \chi)$.
A basic ingredient of the proof is an upper bound for the number of smooth integers, i.e. integers without large prime factors.

Christian Mauduit: CNRS, Marseille

Prime Number Theorems for deterministic sequences

Abstract: The difficulty of the transition from the representation of an integer in a number system to its multiplicative representation (as a product of prime factors) is at the source of many important open problems in mathematics and computer science.

We will present recent results and some open problems concerning the obtention of Prime Number Theorems for "deterministic sequences", i. e. sequences produced by a dynamical system of zero entropy or defined using a simple algorithm.

Attila Pethő: University of Debrecen

Primes and the Internet

Abstract: The late Paul Erdős regularly visited Debrecen. He always gave talks and reported on recent results and problems on primes. Primes were his favorite topic. The elementary proof of the prime number theorem, which he found independently with A. Selberg is one of his most famous results. The authors are not prime number theorists, but directly or indirectly inspired by Erdős are interested for the applications of primes in computer science.

The Internet influences considerably our everyday life. In the talk we show that Internet could hardly work without primes and, on the other hand, mathematical tools developed for the investigation to Internet lead to interesting observations about primes.

To be more informative, the first part will be devoted to the security of Internet. We present examples to convince that security is vital for Internet and it is impossible to guarantee security without primes. In the second half we turn to Google, more precisely to PageRank. Frahm, Chepelianskii and Shepelyansky (2012) defined a network on integers and showed that the ranking of the integers induced by the PageRank of the nodes is related to the number and the size of the prime factors of the nodes. Slight modification of the definition of their network lead to a ranking of integers, which depends on the number of prime factors, i.e. primes have the largest PageRank. These results are experimental.

Primes are essential building blocks of the human knowledge, because new mathematical inventions were often illustrated by their application

to the primes and properties of primes were often used in the proof of theorems. Joint work wirh Norbert Bátfai.

Joël Rivat: University of Marseille

On the digits of prime numbers

Abstract: We will present a recent joint work with Christian Mauduit giving general sufficient conditions for a digital function taken along the sequence of prime numbers to be equidistributed in arithmetic progressions.

András Sárközy: Eötvös Loránd University

On additive and multiplicative decompositions of subsets of $\mathbb{F}_p$

Abstract: In 1956 Ostmann introduced and studied the notion of reducibility, primitivity and totalprimitivity of finite or infinite sets of non-negative integers. Such a set is called reducible if it has an additive 2-decomposition of the form $\mathcal{A}+\mathcal{B}$ which is non-trivial in the sense that both $\mathcal{A}$ and $\mathcal{B}$ have at least two elements, and if the set is not reducible, then it is said to be primitive. Ostmann's work was followed by several papers written on related questions by different authors, including a paper of mine and another one written jointly by E. Szemerédi and me in which two related problems of Erdős have been settled.

A few years ago I proposed to introduce and study the analogues of these notions in $\mathbb{F}_p$. First I conjectured that if p is a prime large enough then the set of the modulo p quadratic residues is primitive and I proved partial results in this direction. Next C. Dartyge and I proved similar results on the set of the primitive roots modulo p. In a third paper I studied multiplicative decompositions of shifted sets of the quadratic residues modulo p. In another paper K. Gyarmati, S. Konyagin and I showed that large subsets of $\mathbb{F}_p$ are reducible. Finally, in a recent paper K. Gyarmati and I presented criteria for primitivity of subsets of $\mathbb{F}_p$, and we introduced and studied the notion of k-primitivity in $\mathbb{F}_p$.

Andrzej Schinzel: Polish Academy of Sciences

On sum of powers of the positive integers

Abstract: The following problem is considered. Given k, for which exponents l the divisibility $1^k + \cdots + n^k | 1^l + \cdots + n^l$ holds for all positive integers n.

Igor Shparlinski: Macquarie University

Distribution of Points on Varieties over Finite Fields

Abstract: We give a survey of recent results on the distribution of rational points on algebraic varieties that belong to a given box. Generally, these results fall into two categories:

- large boxes, where asymptotic results are possible;
- small boxes, where only upper bounds are possible.

We discuss results of both types and also indicate the underlying methods. Some of these work for very general varieties, some apply only to very specific varieties such as hypersurfaces $x_1^{-1} + \ldots + x_n^{-1} = a$ that appear in the Erdős-Graham problem or "hyperbolas" $x_1 \ldots x_n = a$.

Besides intrinsic interest, the aforementioned results have a wide scope of applications in number theory and beyond. These include: bounds of Kloosterman sums, shifted power identity testing, distribution of elliptic curves in isogeny and isomorphism classes, polynomial dynamical systems in finite fields and several others.

We conclude with several open problems.

Cameron Stewart: University of Waterloo

On the greatest prime factor of $2^n - 1$

Abstract: For any integer m with $m > 1$ let $P(m)$ denote the greatest prime factor of m. In 1965 Erdős conjectured that

$$\frac{P(2^n - 1)}{n} \to \infty \text{ as } n \to \infty.$$

We shall discuss our proof of this conjecture and related work.

Mihály Szalay: Eötvös Loránd University

Paul Erdős's results and influence in the theory of integer partitions

Abstract: Let Π be a generic "unrestricted" partition of the positive integer n, i.e., a representation of n as the sum of any number of positive integral parts arranged in descending order of magnitude:

$$\Pi : \quad \lambda_1 + \ldots + \lambda_m = n, \quad \lambda_1 \geq \ldots \geq \lambda_m (\geq 1), \quad \lambda_j\text{'s integers}, \quad m = m(\Pi).$$

[the number of summands can depend on the partition]

Their number $p(n)$ is

$$\sim \frac{1}{4n\sqrt{3}} \exp(\frac{2\pi}{\sqrt{6}}\sqrt{n})$$

according to (the simplest form of) a theorem of G. H. Hardy and S. Ramanujan from 1918.

By the words of P. Turán [The fiftieth anniversary of Pál Erdős, *Mat. Lapok* **14** (1963), 1–28 (in Hungarian); Coll. Papers of P. Turán, Vol. **2**, 1493–1516], *"Erdős, consequently carrying through his program, here also gauged the power of "elementary" methods: in a paper published in the Annals of Math. in 1942 he showed that this formula — except the factor* $\frac{1}{4\sqrt{3}}$ *— lies within the range of "elementary" methods. ... Erdős added two further interesting contributions to the partition problem. With J. Lehner in 1941 in the Duke Journal he proved that — like Hardy and Ramanujan found for the distribution of the prime factors of integers — "almost all" additive representations of a positive integer* n *contain "approximately"* $\frac{1}{\pi}\sqrt{\frac{3}{2}}\sqrt{n}\log n \stackrel{\text{def}}{=} A(n)$ *summands."*

Thinking of the associate (or conjugate) partitions, the same holds for the maximal summand: If $\omega(n) \nearrow \infty$ arbitrarily slowly then

$$m = \frac{\sqrt{6}}{2\pi}\sqrt{n}\log n + O(\sqrt{n}\,\omega(n))$$

and

$$\lambda_1 = \frac{\sqrt{6}}{2\pi}\sqrt{n}\log n + O(\sqrt{n}\,\omega(n))$$

for almost all unrestricted Π's, i.e., with the exception of $o(p(n))$ partitions of n at most. Erdős and Lehner also proved that, with any real constant c,

$$\lambda_1 \leq \frac{\sqrt{6}}{2\pi}\sqrt{n}\log n + \frac{\sqrt{6}}{\pi}\sqrt{n} \cdot c$$

for

$$(\exp(-\frac{\sqrt{6}}{\pi}e^{-c}) + o(1))p(n)$$

unrestricted Π's of n. [This is a doubly exponential or extreme-value distribution.]

As to the other contribution mentioned by Turán, cite again.

Let $p_k(n)$ be the number of partitions containing exactly k summands and for a given n define $k_0(n)$ by

$$p_{k_0}(n) = \max_k p_k(n).$$

In 1946 Erdős showed that for $n \to \infty$ we have

$$k_0(n) = A(n) + \frac{2}{\pi}\sqrt{\frac{3}{2}} \log \frac{\sqrt{6}}{\pi}\sqrt{n} + o(\sqrt{n}).$$

Later Szekeres proved that this $k_0(n)$ is unique, namely for a fixed n, $p_k(n)$ is increasiong for $k \le k_0$ and decreasing later.

Thus, Erdős and Szekeres proved two conjectures of Auluck, Chowla and Gupta.

Let $p_A(n)$ be the number of partitions of n into parts taken from the set $A \subseteq N^*$, repetitions being allowed. With P. T. Bateman in 1956 in Mathematika Erdős obtained conditions for A which imply that $p_A(n)$ is non-decreasing for large n and in Publ. Math. Debrecen they proved the monotonicity for $n \ge 1$ when A is the set of primes.

In 1962 in Acta Arithmetica Erdős investigated the representation of large integers as sums of *distinct* summands taken from a fixed set. The result is weaker than Cassels's one, but the proof is elementary.

In the talk we consider the next 50 years.

Gérald Tenenbaum: University of Nancy

On the core of an integer

Abstract: The core, or squarefree kernel, of an integer is defined as its largest squarefree divisor. The underlying arithmetic function is linked to many problems and conjectures in number theory, including the *abc* conjecture. This talk will be devoted to presenting new, uniform asymptotic formulae for the number of integers not exceeding x with core at most y. These estimates have been obtained recently in collaboration with O.

Robert. A number of applications to questions studied by Erdős, de Bruijn and others will also be described. Finally, an account on how these bounds are employed, in a joint paper with O. Robert and C.L. Stewart, to derive a refined form of the *abc* conjecture will be provided.

Robert Tichy: Technische Universität Graz

Metric Discrepancy Theory

Abstract: During the 1950ť Erdős, Gál and Koksma systematically investigated limit theorems for trigonometric sums as well as for summations $\sum_{n=1}^{N} f(a_n x)$, where f is a suitably regular $1-$ periodic function and (a_n) a lacunary sequence of integers, i.e. satisfying a Hadamard gap condition $a_{n+1}/a_n \geq q > 1$ for all $n \geq 1$. W. Philipp (1975) proved for such sequences that the corresponding discrepancy function fulfills a law of the iterated logarithm (LIL)

$$0 < c_1 \leq \limsup_{N\to\infty} \frac{N D_N(a_n x)}{\sqrt{2N \log\log N}} \leq c_2 \quad \text{a.e.},$$

where c_1, c_2 are constants (only depending on q). Note that for the discrepancy of an i.i.d. sequence X_n the corresponding $\limsup$ equals to $1/2$ by the Chung-Smirnov LIL. Recently, Fukuyama determined this $\limsup$ for sequences $a_n = \theta^n$ and it is shown that it is different from $1/2$ for any integer > 2. If θ^r is irrational for $r = 1, 2, 3, \ldots$, it is shown that the value of the $\limsup$ equals $1/2$ as in the i.i.d. case. In the present talk we will focus on recent work of C. Aistleitner, I. Berkes and R. F. Tichy, where permutation invariant limit laws for lacunary sums and discrepancy functions have been established. The results depend on the number of solutions of diophantine equations of the type

$$(*) \quad A a_n + B a_m = C \qquad , 1 \leq n, m \leq N,$$

with non-zero integer constants A, B, C.
Theorem Let f be a $1-$periodic function with zero mean and of bounded variation on $[0,1]$. Assume that the number of solutions of the diophantine equation (*) is bounded by a constant $K(A,B)$ independent of C. Then for any permutation $\sigma : \mathbb{N} \to \mathbb{N}$ we have

$$\limsup_{N\to\infty} \frac{\sum_{k=1}^{N} f(a_{\sigma(n)} x)}{\sqrt{2N \log\log N}} = \| f \|_2 \quad \text{a.e.}$$

$$\limsup_{N\to\infty} \frac{ND_N(a_{\sigma(n)}x)}{\sqrt{2N\log\log N}} = \frac{1}{2} \quad \text{a.e.}$$

The proofs depend on probabilistic and diophantine tools.

Robert Tijdeman: Leiden University

Representations as sums of products of fixed primes

Abstract: Let P be a given finite set of primes. Let Q be the set of integers which are products of primes from P. How many terms may you need to represent a large integer as sum of elements of Q? And how many terms may you need to represent a large integer as sum of elements of Q and $-Q$? The lecture deals with these and related questions, thereby answering some questions of Nathanson. In the proofs a theorem of Erdős, Pomerance and Schmutz (1991) is applied. The research was done jointly with Lajos Hajdu.

Van Vu: Rutgers University

The optimal density of square sum-free sets

Abstract: A subset A of $\{1, \dots, n\}$ is square sum-free if no subset of A sums up to a square. In the 80s, Erdős posed the question of estimating the largest size of a square sum-free set. It is easy to see that if p is a prime around $n^{2/3}$ and $k = n^{1/3}$, then $\{p, 2p, \dots, kp\}$ is square sum-free, giving a lower bound of order $n^{1/3}$. We are going to prove the almost matching upper bound $n^{1/3+o(1)}$, improving earlier results of Alon-Freiman and Sárközy.

Our approach combines analytic and combinatorial techniques, making use of a recent result of Szemerédi and Vu on Erdős-Folkman conjecture concerning complete sequences (a sequence of natural numbers is complete if its partial sums contain all sufficiently large natural number).

Joint work with H. Nguyen (OSU).

Probability Theory

Márton Balázs: Technical University Budapest

Asymmetric exclusion: a way to anomalous scaling

Abstract: It has been known for a long time that t independent, similar (and not too wild) random quantities sum up to have square root of t fluctuations. If the summands have slight dependence on each other, like in a stochastic process that is a bit more complicated than independent summands, still square root of t fluctuations stay. Any model that differs from this in the scaling of fluctuations is exceptional and very interesting. I will show such an example, and will sketch an argument that leads to $t^{1/3}$ scaling of fluctuations instead of $t^{1/2}$.

József Beck: Rutgers University

What is geometric entropy, and does it really increase

Abstract: We study the time evolution of a large point billiard system in a box (=unit cube). Our goal is to answer the following questions. Starting from an arbitrary but fixed initial configuration (say, all points are in the left half, or even "Big Bang" where at the beginning all points are the same), and moving with typical velocities, how long does it take to reach "spatial equilibrium"? How to define "spatial equilibrium"? Is there a precise concept of "entropy function" that describes the increasing disorder of the typical time evolution of the system? Is the "entropy function" really increasing?

Itai Benjamini: Weizmann Institute

Random walk on planar graphs

Abstract: We will comment on several aspects of the geometry of planar graphs and the behaviour of random walk on them.

Krzysztof Burdzy: University of Washington

Forward Brownian Motion

Abstract: I will present processes which have the distribution of standard Brownian motion (in the forward direction of time) starting from random points on the trajectory which accumulate at $-\infty$. These processes do not have to have the distribution of standard Brownian motion in the backward direction of time, no matter which random time we take as the origin. I will discuss the maximum and minimum rates of growth for these processes in the backward direction. I will also address the question of which extra assumptions make one of these processes a two-sided Brownian motion.

Joint work with Michael Scheutzow.

Jean-François Le Gall: Université Paris-Sud and IUF

The range of tree-indexed random walk

Abstract: A famous paper of Dvoretzky and Erdős in 1951 studied the range of simple random walk on the d-dimensional lattice. If R_n stands for the number of distinct sites visited by random walk up to time n, Dvoretzky and Erdős proved that, in dimension $d \geq 3$, $n^{-1}R_n$ converges to a positive constant, whereas in the case $d = 2$, $n^{-1}(\log n)R_n$ converges to π. We address a similar problem for tree-indexed random walk. Let T_n be a random tree uniformly distributed over all plane trees with n edges, and let N_n be the number of distinct sites visited by simple random walk indexed by T_n on the d-dimensional lattice. We prove that, if $d \geq 5$, $n^{-1}N_n$ converges to a positive constant, whereas if $d = 4$, $n^{-1}(\log n)N_n$ converges to $\pi^2/2$. In low dimensions $d \leq 3$, we prove that $n^{-d/4}N_n$ converges in distribution to the range of the d-dimensional Brownian snake driven by a normalized Brownian excursion. Similar results hold for more general random walks and more general branching structures. This is a joint work with Lin Shen.

Tamás Móri: Eötvös Loránd University

A random graph model with duplication

Abstract: We examine an evolving random graph model, where the basic step is the duplication of a vertex. That is, a new vertex is created and it is connected to the neighbours of a randomly chosen vertex. Such kind of random graph models are frequently used in biological sciences, e.g., for modelling interactions of proteines, where it often happens that different proteines have similar roles and therefore they take part in the same kind of interactions, resulting in many common neighbours in the graph.

Evolving random graph models have been examined from many points of view. A popular direction of analysis is describing the degree distribution of a randomly chosen vertex, and then the asymptotic study of the proportion of vertices with large degree. If the proportion of vertices of degree d tends to some constand c_d almost surely as the size of the graph tends to infinity, and c_d is a polynomially decaying function of d, then the random graph model is said to have the so called scale free property. Many real world networks are pointed out to have their degree distribution not far from polynomially decaying sequences.

In the last decades several scale free random graph models were constructed and examined. However, in the case of duplication–deletion models, the existence of an asymptotic degree distribution has usually been proved in a weaker sense: with the convergence of expectations instead of almost sure convergence. Sometimes the latter wasn't even true.

In the talk a linearly growing duplication–deletion model is introduced and analysed by using martingale techniques. The method is based on a coupling argument. We modify the model in such a way that the new graph sequence has a very special structure, which makes it possible to easily determine the asymptotic degree distribution. Then we transfer the result to the original model. In addition, this kind of argument also reveals the structure of the graph.

Finally, we deal with the decay of the asymptotic degree distribution. It is between the exponential decay which is characteristic of uniform recursive graphs, and the polynomial one of scale free models.

Joint work with Ágnes Backhausz.

Gábor Pete: Technical University Budapest

Exceptional times in dynamical percolation, and the Incipient Infinite Cluster

Abstract: In dynamical percolation on the triangular lattice, every site is switching between open and closed according to an independent Poisson clock, keeping critical percolation as the stationary measure. It is known that there are exceptional times when the origin is connected to infinity, and the Hausdorff dimension of this random set of times is 31/36. In joint work with Alan Hammond and Oded Schramm, we show that, at a "typical" exceptional time, the cluster of the origin has the law of Kesten's Incipient Infinite Cluster, but, at the first exceptional time, the cluster is different. I will sketch a proof of this, together with several open problems.

Zhan Shi: Université Paris VI

Local time of random walks on trees

Abstract: I am going to make some discussions on the local time of biased random walks on supercritical Galton–Watson trees. Join work with Y. Hu.

Domokos Szász: Technical University Budapest

Erdős-type theorems for billiard models

Abstract: Beside being gems of probability theory, random walks - a central subject from the rich world of Erdős' interests - also serve as stochastic models of Brownian motion. I discuss results for Lorentz processes: billiard-like mechanical models of Brownian motion. In them the dynamics is deterministic rather than stochastic. These results extend classical random walk results of Erdős obtained jointly with Kac (in 1947), with Dvoretzky (in 1951) and with Taylor (in 1960).

Bálint Tóth: Technical University Budapest and University of Bristol

Erdős-Rényi Random Graphs + Forest Fires = Self-Organized Criticality

Abstract: We modify the usual Erdős-Rényi random graph evolution by letting connected clusters 'burn down' (i.e. fall apart to disconnected single sites) due to a Poisson flow of lightnings. In a range of the intensity of rate of lightnings the system sticks to a permanent critical state. This phenomenon is called *self-organized criticality* in the physics literature. We investigate the global and local behaviour of the system, deriving an infinite system of nonlinear ODE-s which describe the time evolution of cluster size densities and the Markovian evolution of one typical cluster.

The talk will be based on joint work with Balázs Ráth and (work in progress) with Ed Crane and Nic Freeman.

Bálint Virág: University of Toronto

Independent sets in sparse graphs

Abstract: A set of vertices S of a graph G is called independent if there are no edges between them. How large can S be? This question inspired exciting combinatorial work by Erdős, and it is a very active research topic in computer science and statistical physics.

I will review recent developments and the connection to spectral theory and random graphs.

Ramsey Theory

Jacob Fox: MIT

A relative Szemerédi theorem

Abstract: A famous conjecture of Erdős and Turán from 1936 states that every set of integers of positive relative density contains arbitrarily long arithmetic progressions. Szemerédi proved the Erdős-Turán conjecture in 1975.

One of the main motivations of the Erdős-Turán conjecture was in trying to prove that there are arbitrarily long arithmetic progressions of primes. This was finally proved a few years ago in the celebrated work of Green and Tao. The proof has two parts. The first part is a relative Szemerédi theorem which says that any subset of a pseudorandom set of integers of positive relative density contains long arithmetic progressions, where a set is pseudorandom if it satisfies two conditions, the linear forms condition and the correlation condition. The second part is in finding a pseudorandom set in which the primes have positive relative density.

In this talk, I will discuss recent joint work with David Conlon and Yufei Zhao in which we give a simple proof for a strengthening of the relative Szemerédi theorem, showing that a weak linear forms condition is sufficient for the theorem to hold. By removing the correlation condition, our strengthened version can be applied to give a relative Szemerédi theorem for k-term arithmetic progressions in pseudorandom subsets of $\mathbb{Z}_N$ of density N^{-c_k}. It also simplifies the deduction of the Green-Tao theorem by removing the need for certain number theoretic estimates in the second part of their proof.

The key component in our proof is an extension of the regularity method to sparse pseudorandom hypergraphs, which we believe to be interesting in its own right. From this we derive a relativized hypergraph removal lemma. This is a strengthening of an earlier theorem used by Tao in his proof that the Gaussian primes contain arbitrarily shaped constellations and, by standard arguments, allows us to deduce the relative Szemerédi theorem.

Ronald Graham: University of California, San Diego

Paul Erdős and Egyptian Fractions

Abstract: One of Paul Erdős' earliest mathematical interests was the study of so-called *Egyptian fractions*, that is, finite sums of distinct fractions having numerator 1. In this talk we survey various results in this subject, many of which were motivated by Erdős' problems and conjectures on such sums.

Yoshiharu Kohayakawa: University of Sao Paulo

The regularity method and Ramsey theory

Abstract: The regularity method plays a crucial role in Ramsey theory. This talk will discuss some recent results in the area, including a bound for the Ramsey number of almost all planar graphs obtained by Böttcher, Taraz and Würfl. The proof of this result makes use, among others, of a blow-up lemma for arrangeable graphs, obtained by those authors together with the speaker.

Tom Sanders: University of Oxford

Roth's theorem on arithmetic progressions

Abstract: We shall talk about Roth's theorem on arithmetic progressions which states that if A is a sufficiently large subset of the integers $\{1, \ldots, N\}$ then A contains an arithmetic progression with at least three distinct elements. The talk will concentrate on work from 2010 on how large 'sufficiently large' is.

József Solymosi: University of British Columbia

On the sum-product problem

Abstract: More than 30 years ago Erdős and Szemerédi conjectured that for any finite set of integers, either the sum-set or the product set should be large. The sum-set of a set A consists of the pairwise sums of the elements in A, that is $A + A = \{a + b | a, b \in A\}$. The product-set, AA, is defined in the analogous way. The Erdős-Szemerédi conjecture is that for any $\varepsilon > 0$

there is a threshold n_0 such that if $|A| \geq n_0$ then $|A+A|+|AA| \geq |A|^{2-\varepsilon}$. The conjecture is still open. Similar questions can be asked in arbitrary rings. There is a large family of exciting and important results which all belong to the so called *sum-product phenomenon*. I will talk about the original problem and some of the extensions.

Probabilistic Method

David Conlon: University of Oxford

On the KŁR conjecture in random graphs

Abstract: The KŁR conjecture of Kohayakawa, Łuczak, and Rödl is a statement that allows one to prove that asymptotically almost surely all subgraphs of the random graph $G_{n,p}$, for sufficiently large $p := p(n)$, satisfy an embedding lemma which complements the sparse regularity lemma of Kohayakawa and Rödl. We prove a variant of this conjecture which is sufficient for most known applications of this conjecture to random graphs. In particular, our result implies a number of recent probabilistic versions, due to Conlon, Gowers, and Schacht, of classical extremal combinatorial theorems. We also discuss several further applications. Joint work with Tim Gowers, Wojciech Samotij and Mathias Schacht.

Mihyun Kang: Technische Universität Graz

Recent developments in phase transitions and critical phenomena: 54 years since the seminal work of Erdős and Rényi

Abstract: The phase transition is a phenomenon observed in mathematics and natural sciences in many different contexts. It deals with a sudden change in the properties of a large structure caused by altering a critical parameter. The phase transition in random discrete structures (e.g. random graphs, random satisfiability problems, Ising/Potts model, percolation) has captured the attention of many scientists in recent years. This talk will discuss the development of phase transitions and critical phenomena in various random graph models, since Erdős and Rényi first discussed the phase transition in random graphs in 1959.

Oliver Riordan: University of Oxford

The evolution of Achlioptas processes

Abstract: In the *Erdős–Rényi random graph process*, starting from an empty graph, in each step a new random edge is added to the evolving graph. One of its most interesting features, both mathematically and in terms of applications, is the 'percolation phase transition': as the ratio of the number of edges to vertices increases past a certain critical point, the global structure changes radically, from only small components to a single macroscopic ('giant') component plus small ones.

An *Achlioptas process* is any one of a family of variants of the classical Erdős–Rényi process: starting from an empty graph, in each step *two* potential edges are chosen uniformly at random, and using some rule *one* of them is selected and added to the evolving graph. One might expect these processes to behave not too differently from the Erdős–Rényi one. However, simulations of Achlioptas, D'Souza and Spencer suggested that for certain rules (in particular the 'product rule' suggested by Bollobás) the percolation phase transition has a radically different form: more or less as soon as the macroscopic component appears, it is already extremely large. This phenomenon is known as 'explosive percolation'. I will briefly explain this striking and unusual phenomenon, and discuss results for general classes of Achlioptas processes obtained in joint work with Lutz Warnke.

Joel Spencer: Courant Institute

Six Standard Deviations Still Suffice

Abstract: A quarter century or so ago the speaker resolved a conjecture of Erdős, showing that any n sets on n vertices may be two colored so that all sets have discrepancy at most $K\sqrt{n}$, K (originally 6) an absolute constant. We discuss recent works of Bansal and of Lovett and Meka that reprove this result, giving an algorithm (long conjectured by the speaker *not* to exist!) finding the coloring. We emphasize the approach of Lovett and Meka that uses floating colors in $[-1, +1]$ which move around in a restricted Brownian motion.

Angelika Steger: ETH Zürich

How to learn quickly

Abstract: Throughout a day we are confronted with new information, often present for only a second or less. Nevertheless, hours later we are still able to recall most of it. How can this be? – In this talk we show that an appropriate modeling of the brain allows to use techniques and results from random graph theory that can explain these phenomena to a large extent.

Nick Wormald: University of Waterloo

A new model for analysis of the random graph d-process

Abstract: A graph d-process starts with an empty graph on n vertices, and adds one edge at each time step, chosen uniformly at random from those pairs which are not yet edges and whose both vertices have current degree less than d. Erdős posed the question of finding the distribution of the degree sequence of the vertices in the final graph. Once upon a time, Ruciński and I showed using a martingale argument that asymptotically almost surely there is at most one vertex of degree less than d. We now have an entirely new approach to analysing this process which allows us to obtain much more accurate answers to a number of questions, such as: when does the last vertex of degree 0 disappear?

This joint work with Andrzej Ruciński.

Set Theory

Gregory Cherlin: Rutgers University

Universal Graphs with Forbidden Subgraphs

Abstract:
Problem (1)**.** *For which finite graphs C is there a countable universal C-free graph?*

Problem ($1'$)**.** *For which finite graphs C is there a countable universal C-free graph with oligomorphic automorphism group? (I.e., there are finitely many orbits in the action on n-tuples, for each n.)*

Problem $1'$ is a necessary step toward Problem 1, and a much clearer problem, when one works out the issues involved. There is a clear combinatorial criterion for problem $1'$, and where we have satisfactory partial results on Problem 1, they come by analyzing marginal cases of Problem $1'$.

In addition, when there is an oligomorphic automorphism group then we may look at it as a topological group with interesting topological dynamics, following [KPT 2005]. Nešetřil will address an instance of this in his talk.

We expect a complete solution to both Problems to emerge eventually.

On the other hand one really should allow not just a single forbidden subgraph C, but a finite set of forbidden graphs $\mathcal{C}$. Then I do not expect an explicit solution to either problem, but it is reasonable to ask whether one can provide an algorithmic solution to the general case.

Some concrete results are as follows.

Theorem. *Let $\mathcal{C}$ (or C, if $\mathcal{C} = \{C\}$), be one of the following. Then there is a countable universal $\mathcal{C}$-free graph.*

- *$\mathcal{C} = \emptyset$ [Rado 1964] (oligomorphic)*
- *C complete [Henson 1971] (oligomorophic)*
- *C a path or nearpath [KMP1988, ChT 2007] (oligomorphic: paths only)*
- *$\mathcal{C}$ is homomorphism-closed [ChShSh 1999] (oligomorphic)*
- *C is a bow tie (two triangles joined at a vertex) [Komjáth 1999] (oligomorphic)*

Theorem. *Conversely, in the following cases, if there is a universal $\mathcal{C}$-free graph then it is on the list above.*

- *C is 2-connected [FK 1997]*
- *C is a tree [ChShe 2005]*
- *$\mathcal{C}$ is a set of cycles [ChShi 1996]*

I will make some cheerfully optimistic conjectures and take note of the terrors of noncomputability that lurk in the shadows. In particular, [FK 1997] suggests that C shoud be made of solid blocks.

If time allows, I might comment on the connection with isometric universality, cf. [Moss 1989, Moss 1991].

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Matthew Foreman: UC Irvine

Random graph techniques and banking system failures

Abstract: As the events of 2008 in the United States illustrate, banking systems are susceptible to catastrophic crashes. Surprisingly, much of the vulnerability can be attributed to the network structure of the system (as opposed to the actions of single banks). In this talk we discuss adapting random graph techniques from epidemiology to the study of financial contagion.

Sy-David Friedman: University of Vienna

Cardinals and Ordinal Definability

Abstract: An important development in large cardinal theory is the construction of inner models M all of whose sets are definable from ordinals and which serve as good approximations to the entire universe V. The former means that M is contained in HOD, the universe of hereditarily ordinal-definable sets, and the latter can be interpreted in a number of ways. One such interpretation is that the cardinal structure of M is "close" to that of V in the sense that α^+ of M equals α^+ of V for many cardinals α. This is for example the case if V does not contain $0^\#$ and M equals L or if V does not contain an inner model with a Woodin cardinal and M is the core model K for a Woodin cardinal.

The result presented in this talk states that we cannot hope to approximate the cardinals of V by those of (inner models of) HOD in general:

Theorem. *Assuming the consistency of a supercompact, it is consistent that for every cardinal α, α^+ of V is greater than α^+ of HOD.*

For the proof one chooses a suitable inner model $V[G_0]$ of an extension $V[G]$ of V via supercompact Radin forcing such that $V[G_0]$ and V have the

same cardinals but $V[G]$ is a generic extension of $V[G_0]$ via a homogeneous forcing.

This is joint work with James Cummings and Mohammad Golshani.

István Juhász: Rényi Institute

Resolvability of topological spaces

Abstract: A topological space is said to be *κ-resolvable* if it contains κ many pairwise disjoint dense sets. The space X is *maximally resolvable* if it is $\Delta(X)$-resolvable, where $\Delta(X)$ is the minimum cardinality of a non-empty open set in X.

Classical results show that "nice" spaces, like compact, metric, or linearly ordered ones, are maximally resolvable. In this talk I intend to point out that the resolvability properties of some natural generalizations of these spaces are much harder to deal with.

For instance, the existence of a monotonically normal space that is not maximally resolvable is equiconsistent with the existence of a measurable cardinal. (Monotonically normal spaces form a common generalization of metric and linearly ordered spaces.) This is a joint result with M. Magidor.

I also present some recent results, joint with L. Soukup and Z. Szentmiklóssy, concerning a problem of Malychin about the resolvability of regular Lindelöf spaces in which all non-empty open sets are uncountable.

Piotr Koszmider: Polish Academy of Sciences

Uncountable combinatorics of Boolean algebras in Banach spaces

Abstract: We describe some recent progress in the theory of Banach spaces and operators on them achieved by reducing functional analytic problems to combinatorial problems concerning Boolean algebras. The latter problems are being attacked using the standard tools like stepping-up, forcing or book-keeping principles.

Jean Larson: University of Florida

Counting G-tops

Abstract: A G-top is a pair $(T, <)$ where T is a set of binary sequences closed under initial sequences, $<$ is a linear order on T, and the pair $(T, <)$ satisfies some additional constraints. G-tops are critical types for a Ramsey theorem of Džamonja, Larson and Mitchell.

The t in G-top stands for tree and note that for any G-top $(T, <)$, T is a set theoretic tree under end-extension. The o stands for order. We use the absolute value notation for the length $|s|$ of a sequence s, and identify a binary sequence s of length m with the corresponding function from $\{0, 1, \ldots, m-1\}$ to $\{0, 1\}$. The G in G-top stands for the graph $G(T) = (V(T), E(T))$ where $V(T)$ is the set of tree-tops of T and $E(T)$ is the set of pairs $\{s, t\}$ for which $|s| < |t|$ and $t(|s|) = 1$. The *meet* of s and t from T is $s \wedge t$, the longest initial segment common to both s and t. With this notation in hand, $(T, <)$ is a *G-top* if it satisfies the following conditions:

(a) (permutation) If $V(T)$ is enumerated in lexicographically increasing order as $v_0, v_2, \ldots, v_{2n}$, then $(v_0, v_0 \wedge v_2, v_2, \ldots, v_{2n-2}, v_{2n-2} \wedge v_{2n}, v_{2n})$ is a down-up permutation of $\{0, 1, \ldots, 2n\}$;

(b) The order $<$ respects length order, i.e. $|s| < |t|$ implies $s < t$; and

(c) The order $<$ satisfies a technical condition called the *vip protocol.*

We will discuss ways to count the set of G-tops that give rise to permutations on $\{0, 1, \ldots, 2n\}$, and ask about possible generating functions related to counting G-tops.

Norbert Sauer: University of Calgary

Edge labelled graphs and metric spaces

Abstract: We will discuss connections between the following:

Given an edge labelled graph G with labels in $\mathcal{R} \subseteq \Re$. What are the conditions on G and $\mathcal{R}$ so that there exists a metric space M on $V(\mathrm{G})$ with $\mathrm{dist}(\mathrm{M}) \subseteq \mathcal{R}$ whose distances extend the labelling?

Which homogeneous metric spaces are approximately indivisible?

(A metric space M being approximately indivisible if for every $\epsilon > 0$ and partition (A, B) of M there exists a copy M^* of M in M whose distance from A or from B is less than ϵ.)

Which homogeneous metric spaces are "oscillation stable"?

For which subsets $\mathcal{R} \subseteq \Re$ does there exist a homogeneous, universal, separable, complete metric space M with dist(M) = $\mathcal{R}$?

Lajos Soukup: Rényi Institute

On properties of families of sets

Abstract: We begin our journey in the friendly land of finite and countable sets, prolific with the nicest results using are beloved axioms of ZFC. Brave as we are, we continue to the realm of uncountable; needless to say, we are about to face the dark art of forcing and as many dreadful independence results as questions there are. However, our quest shall end in the heavenly surroundings of Shelah's pcf theory; a new world opens up before our eyes as we see that the right questions do have answers solely in ZFC.

Theory of Computing

Miklós Ajtai: IBM Almaden Research Center

Lower Bounds for RAMs and Quantifier Elimination

Abstract: For each natural number d we consider a finite structure $\mathbf{M}_d$ whose universe is the set of all $0,1$-sequence of length $n = 2^d$, each representing a natural number in the set $\{0, 1, \ldots, 2^n - 1\}$ in binary form. The operations included in the structure are the four constants $0, 1, 2^n - 1, n$, multiplication and addition modulo 2^n, the unary function $\min\{2^x, 2^n - 1\}$, the binary functions $\lfloor x/y \rfloor$ (with $\lfloor x/0 \rfloor = 0$), $\max(x, y)$, $\min(x, y)$, and the boolean vector operations $\wedge, \vee, \neg$ defined on $0, 1$ sequences of length n, by performing the operations on all components simultaneously. These are essentially the arithmetic operations that can be performed on a RAM, with wordlength n, by a single instruction. We show that there exists an $\varepsilon > 0$ and a term (that is, an algebraic expression) $F(x, y)$ built up from the mentioned operations, with the only free variables x, y, such that if $G_d(y)$, $d = 0, 1, 2, \ldots$, is a sequence of terms, and for all $d = 0, 1, 2, \ldots$, $\mathbf{M}_d \models \forall x, [G_d(x) \leftrightarrow \exists y, F(x, y) = 0]$, then for infinitely many integers d, the depth of the term G_d, that is, the maximal number of nestings of the operations in it, is at least $\varepsilon(\log d)^{\frac{1}{2}} = \epsilon(\log\log n)^{\frac{1}{2}}$. The theorem implies that there is a search problem for RAMs with word length n with the property that a solution can be verified in constant time by a program which does not depend on n, but the question whether there exists a solution at all cannot be decided even in time $\varepsilon_n(\log\log n)^{\frac{1}{2}}$ for infinitely many integers n, where $\varepsilon_n = \varepsilon'(\log\log\log n)^{-1}$ and $\varepsilon' > 0$ is a sufficiently small constant.

László Babai: University of Chicago

Testing Isomorphism of Steiner 2-designs and Strongly Regular Graphs

Abstract: We report recent progress on the Graph Isomorphism problem. In a 1980 paper, Erdős, Selkow, and the speaker have shown that the vertex-refinement heuristic yields a canonical form for almost all graphs, exploiting the irregularity of most graphs. The new results reported in this talk show that the same heuristic combined with individualization (assignment of unique colors to a moderate number of vertices) yields strong bounds on the complexity of testing isomorphism of highly regular struc-

tures such as those in the title, improving results by the speaker (1980) and Spielman (1996).

As a by-product, we obtain an optimal $\exp(O(\log^2 n))$ asymptotic upper bound on the number of automorphisms of Steiner 2-designs (block designs where each pair of points is contained in $\lambda = 1$ block), improving previous moderately exponential bounds (B - Pyber (1994), Spielman (1996)). Such a bound remains an open question for block designs with $\lambda = 2$.

The probabilistic method is central to the arguments, including an application of the second moment method to an addressing scheme that produces a hierarchy of rapidly growing families of pairwise independent, uniformly distributed points of a Steiner 2-design.

Joint work with John Wilmes and partly simultaneous and partly joint work with Xi Chen, Xiaorui Sun, and Shang-Hua Teng.

Boaz Barak: Microsoft Research New England

On Algebraic vs Combinatorial Computational Problems

Abstract: The interplay between combinatorics and algebra permeates much of theoretical computer science, as well as Paul Erdős's work. In particular some computational problems, such as satisfiability of CNF formulas (SAT) seem more ”combinatorial” in nature, while others such as integer factoring seem more ”algebraic”.

In this high level and somewhat informal talk, I will discuss how, despite the fact that we don't yet have a precise definition of these notions, there do seem to be inherent differences between combinatorial and algebraic problems. At least heuristically, understanding the complexity of combinatorial problems seems significantly easier, and such problems often display a *dichotomy* behavior, where the problem can be solved by a fairly simple algorithm in one regime of parameters, and seems exponentially hard in the other. In contrast, algebraic problems often have non-trivial algorithms (including quantum ones).

One area in which this distinction seems important is cryptography, where there are many *private key* encryption schemes based on combinatorial problems, but the most well studied *public key* schemes are based on algebraic problems, raising some questions on their resistance to unforeseen (or foreseen) classical or quantum attacks. I will talk about efforts to base public key encryption on combinatorial problems, as well as the fascinating phenomenon that some computational problems might satisfy

a *threshold* phenomenon, where in one parameter regime the problem is combinatorial, and in another regime the problem is algebraic. As we'll see, the computational complexity of combinatorial problems seems also inherently connected with the quintessential Erdős technique of the *probabilistic method*— with its inherently non-constructive and non-local nature, it gives rise to a way to *prove* that an object has a certain property without giving an *efficiently verifiable* certificate of that fact.

I will assume no background in cryptography. Parts of the talk are based on joint works with Benny Applebaum, Guy Kindler, David Steurer, and Avi Wigderson.

Moses Charikar: Princeton University

Local Global Tradeoffs in Metric Embeddings

Abstract: Suppose that every k points in a metric space X are embeddable into ℓ_1 with distortion D. What is the least distortion with which we can embed the entire space X into ℓ_1? In other words, what do *local* properties (embeddability of subsets) of the space tell us about *global* properties (embeddability of the entire space)?

This is a natural question about metric spaces in the same spirit as graph theoretic questions investigated by Erdős. It is motivated by numerous applications of low distortion metric embeddings in computer science and mathematics. Such local-global tradeoff questions arise naturally in the study of strong families of mathematical programming relaxations obtained from lift-and-project hierarchies. Solutions to such relaxations have distributions on integer solutions consistent with any small subset of variables. Despite this local consistency, can the global solution be far from a distribution on integer solutions?

In this talk, I will discuss upper and lower bounds for this local-global tradeoff question for metric embeddings. This leads to constructions of metrics where small subsets are isometrically embeddable into ℓ_2, yet the entire metric is far from ℓ_1 embeddable. These are used in constructing integrality gap examples for lift-and-project relaxations of various combinatorial optimization problems.

This is joint work with Konstantin Makarychev and Yury Makarychev.

Pavel Pudlák: Czech Mathematical Institute

The proof complexity of the finite Ramsey theorem

Abstract: There are several computational problems about the Finite Ramsey Theorem. In particular 1. how difficult is it to compute the Ramsey number $R(n)$ and 2. how difficult is it to construct graphs that witness lower bounds on $R(n)$? It seems very difficult to prove bounds on the computational complexity of these problems. It is, however, possible to prove some bounds on the *proof complexity* of the Finite Ramsey Theorem. Namely, it is possible to prove lower bonds on 1. the complexity of the proof of the theorem and 2. the complexity of proving that a given graph is a witness of a good lower bound of $R(n)$. The results about the proof complexity of the Finite Ramsey Theorem can be viewed as pieces of evidence confirming the conjecture that the above computational problems are hard.

Chris Umans: Caltech

On sunflowers and matrix multiplication

Abstract: I'll describe two conjectures related to matrix multiplication. The first is the "strong Uniquely Solvable Puzzle" conjecture of Cohn, Kleinberg, Szegedy and Umans; the second is the "no 3 disjoint equivoluminous subsets" conjecture of Coppersmith and Winograd. Each conjecture implies an exponent 2 matrix multiplication algorithm.

Then I'll describe several variants of the classical sunflower conjecture of Erdős and Rado, and show that two of these imply negative answers to the CKSU and CW conjectures. Along the way we'll see how two seemingly quite different classical sunflower conjectures can be viewed uniformly as conjectures about sunflowers in $\mathbb{Z}_D^n$, and we'll formulate a "multicolored" strengthening of the well-known (ordinary) sunflower conjecture in $\mathbb{Z}_3^n$.

Joint work with Noga Alon and Amir Shpilka.

List of accepted posters

Posters displayed on Thursday and Friday

- Ron Aharoni, Daniel Kotlar and Ran Ziv: *Rainbow sets in the intersection of two matroids*
- Saeid Alikhani: *Some new results on domination roots of a graph*
- Spencer Backman: *Hereditary chip-firing models and spanning trees*
- German Badia and Carmen Sanguesa: *Log concavity for Bernstein-type operators using stochastic orders*
- Michael A. Bennett, István Pink and Zsolt Rábai: *On the number of solutions of binomial Thue inequalities*
- Hans-Peter Blatt and Regina Fieger: *Erdős-Turán-Type Discrepancy for the Zeros of Rational Functions*
- Alicia Cachafeiro, Elias Berriochoa, Jaime Diaz and Eduardo Martínez: *Asymptotic constants for the error of Hermite-Fejér interpolation on the circle*
- Huilan Chang, Hung-Lin Fu and Chie-Huai Shih: *Nonadaptive algorithms for threshold group testing with inhibitors*
- Min Chen and Andre Raspaud: *(3,1)-choosability of planar graphs without adjacent short cycles*
- Ewa Ciechanowicz: *Maximum modulus points of meromorphic functions and Valiron's defect*
- Annalisa Conversano and Anand Pillay: *Connected components of groups and o-minimal expansions of real closed fields*
- Jesus De Loera and Yvonne Kemper: *Polyhedral Embeddings of Cayley Graphs*
- Jesus De Loera, Jon Lee, Susan Margulies and Jacob Miller: *Systems of Polynomials for Detecting Orientable Matroids*
- Louis Debiasio and Theo Molla: *Semi-degree threshold for anti-directed Hamiltonian cycles*
- Natasha Dobrinen and Stevo Todorčević: *Ramsey-classification theorems and their applications to the Tukey theory of ultrafilters*

- Ehsan Ebrahimzadeh, Linda Farczadi, Pu Gao, Abbas Mehrabian, Cristiane Sato, Nick Wormald and Jonathan Zung: *On the Longest Path and the Diameter in Random Apollonian Networks*
- Sergii Favorov: *Discrete unbounded sets in a finite dimensional space and beyond*
- Alan Filipin and Ljubica Bacic: $D(4)$*-pair* $\{k-2, k+2\}$ *and its extension*
- Galina Filipuk: *The Painleve equations and orthogonal polynomials*
- Jan Florek: *On Barnette's Conjecture and* H^{+-} *property*
- William Gasarch and Sam Zbarsky: *Applications of the Erdős-Rado Canonical Ramsey Theorem to Erdős-type problems*
- Roman Glebov, Daniel Kráľ and Jan Volec: *An application of flag algebras to a problem of Erdős and Sós*
- Boris Granovsky and Dudley Stark: *Asymptotic enumeration of decomposable combinatorial structures with multiple singularities*
- Adriana Hansberg: *Reviewing some Results on Fair Domination in Graphs*
- Mario Huicochea and Amanda Montejano: *Rainbow linear equations on three variables in* $\mathbb{Z}_p$
- Jesus Illan, Alicia Cachafeiro, Elias Berriochoa and Jose Manuel Rebollido: *Modified Gauss rules for approximate calculation of some strongly singular integrals*
- Ana Jurasić: *Diophantine* m*-tuples for quadratic polynomials*
- Roma Kacinskaite: *Universality of various zeta-functions*
- Gulcan Kekec: *On the algebraic independence of certain Mahler's U-numbers*
- Sergey Komech: *Boundary distortion in dynamical systems and in image analysis*
- Tünde Kovács, Gyöngyvér Péter and Nóra Varga: *On some polynomial values of repdigit numbers*
- Ewa Kubicka, Grzegorz Kubicki and Kathleen McKeon: *Chromatic Sums for Colorings Avoiding Monochromatic Subgraphs*

Posters displayed on Tuesday and Wednesday

- Henry Liu and Teresa Sousa: *Monochromatic K_r-Decompositions of Graphs*
- Alexander Makhnev: *Arc transitive distance-regular covers of cliques*
- Johann Makowsky and Elena V. Ravve: *On the Location of Roots of Graph Polynomials*
- Ivan Marchenko: *On the maximum modulus points, deviations of meromorphic functions and a problem of Erdős*
- Giuseppe Mastroianni and Incoronata Notarangelo: *Embedding theorems with an exponential weight on the real semiaxis*
- Fred R. McMorris, Henry Martyn Mulder, Beth Novick and Robert C. Powers: *An axiomatic approach to location functions on finite metric spaces*
- Romeo Mestrovic: *A note on two Erdős's proofs of the infinitude of primes*
- Lucia Morotti: *Reconstruction of Fourier Sparse Signals over Elementary Abelian Groups*
- Ferenc Móricz: *Generalizations of a theorem of Paul Erdős with application*
- Tomoki Nakamigawa: *A Ramsey-type Theorem for Multiple Disjoint Copies of Induced Subgraphs*
- Lothar Narins: *Extremal Hypergraphs for Ryser's Conjecture*
- Jonathan Noel: *Choosability of Graphs with Bounded Order: Ohba's Conjecture and Beyond*
- Sergey Norin and Liana Yepremyan: *Sparse halves in dense triangle-free graphs*
- Shoetsu Ogata: *Normality of 3-dimensional lattice polytopes*
- Florian Pausinger: *Van der Corput sequences and linear permutations*
- Alexey Pokrovskiy: *Partitioning edge-coloured complete graphs into monochromatic cycles*
- Monika Polak, Urszula Romańczuk, Vasyl Ustimenko and Aneta Wróblewska: *On the applications of Extremal Graph Theory to Coding Theory and Cryptography*

- Andrei Raigorodskii and Ekaterina Ponomarenko: *A new intersection theorem and its applications to bounding the chromatic numbers of spaces*
- Saeedeh Rashidi and Nasrin Soltankhah: *On the possible volume of three way trades*
- Juanjo Rué and Ana Zumalacárregui: *Threshold functions for systems of equations on random sets*
- Katarzyna Rybarczyk: *Coupling methods to establish threshold functions in random graphs*
- Dora Selesi and Tijana Levajkovic: *Chaos expansion methods of stochastic processes for Malliavin-type equations*
- Vladimir Shlyk: *Integer Partitions from the Polyhedral Point of View*
- Fiona Skerman: *Modularity in random regular graphs and on lattices*
- Sinisa Slijepcevic: *Positive exponential sums, difference sets and recurrence*
- Ivan Soldo: $D(z)$*-quadruples in the ring* $\mathbb{Z}[\sqrt{-2}]$*, for some exceptional cases of* z
- Dániel Tamás Soukup and Lajos Soukup: *Partitioning bases of topological spaces*
- Rafael Tesoro: *Sets of integers avoiding congruent subsets*
- Lola Thompson: *On the divisors of* $x^n - 1$
- Thilo Weinert: *Transfinite Ramsey Numbers*
- Quan-Hui Yang: *A generalization of Chen's theorem on Erdős-Turán conjecture*
- Dmitrii Zhelezov: *Product sets cannot contain long arithmetic progressions*
- Maksim Zhukovskii: *Extension of the Zero-one* k*-law*

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