Points of bounded height and geometry of Fano varieties

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Let V be a smooth geometrically integral projective variety over a number field with infinitely many rational points. It is then natural to choose an exponential height H on V and to study asymptotically the behavior of the rational points of bounded height on V.

The conjectures of Manin offer an interpretation of the number of points of bounded height in terms of the geometry of the variety, in which the anticanonical line bundle and the cone of effective divisors in the Néron-Severi group play an essential rôle.

In this setting, it is natural to consider the varieties such that the anticanonical line bundle belongs to the interior of the cone of effective divisor; Fano varieties and toric varieties are examples of such varieties. It then appears that the distribution of the points of bounded height on V with respect to the Zariski topology is directly related to the extremal rays in the cone of effective divisors.

The aim of these talks is to give a survey of the work done about the conjectures of Manin with a stress on the connection with the minimal model program.