Decomposition of multiple packings with subquadratic union complexity

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(joint work with János Pach)

Let \( k \) be a positive integer and \( \mathcal{X} \) be a \( k \)-fold packing of simply connected compact sets in the plane, that is, a family such that every point belongs to at most \( k \) sets. Suppose that there is a function \( f(n) = o(n^2) \) with the property that any \( n \) members of \( \mathcal{X} \) surround at most \( f(n) \) holes, which means that the complement of their union has at most \( f(n) \) bounded connected components. We use tools from extremal graph theory and the topological Helly theorem to prove that \( \mathcal{X} \) can be decomposed into at most \( p \) packings, where \( p \) is a constant depending only on \( k \) and \( f \).