When does an isometry on a Banach algebra preserve the multiplicative structure?

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The Banach-Stone theorem asserts that unital commutative C^*-algebras are isometric as Banach spaces if and only if they are isomorphic as Banach *algebras*.

Problem. *For which (commutative) Banach algebras does the Banach space structures ensure the Banach algebra structure?*

A theorem of Nagasawa (1959), or deLeeuw, Rudin and Wermer (1960) states that a surjective complex-linear isometry between uniform algebras is a weighted composition operator. Hence a uniform algebra satisfies the mentioned property in Problem. A standard proof of the theorem depends on the so-called extreme point argument. The Arens-Kelley theorem asserts that an extreme point of the closed unit ball of the dual space of a uniform algebra is the point evaluation at a Choquet boundary point followed by a scalar multiplication of the unit modulus. Thus the dual map of the given isometry gives the correspondence between the Choquet boundaries, which induces the composition part of the isometry. It is interesting that the first result on isometries of the Hardy spaces depend on this theorem. On the other hand, the dual space of the Wiener algebra $W(\mathbb{T}) = \{f \in C(\mathbb{T}) : \sum |\hat{f}(n)| < \infty\}$ is $\ell^\infty(\mathbb{Z})$, and an Arens-Kelley theorem does not hold for the Wiener algebra. For any bijection φ from the set of the positive integers onto the set of all integers, the map $T : W(\mathbb{T}) \to W_+(\mathbb{T})$ defined by

$$T(f)(e^{i\theta}) = \sum_{n=0}^{\infty} \hat{f}(\varphi(n)) e^{in\theta}, \quad f \in W(\mathbb{T})$$

is a surjective complex-linear isometry, where $W_+(\mathbb{T}) = \{f \in W(\mathbb{T}) : \hat{f}(n) = 0, \forall n < 0\}$ is a closed subalgebra of $W(\mathbb{T})$. On the other hand, $W(\mathbb{T})$ is not algebraically isomorphic as Banach algebra to $W_+(\mathbb{T})$ since the maximal ideal spaces of these two algebras are not homeomorphic to each other. This reminds us that the class of Banach algebras which satisfy the mentioned property in Problem is not so large. The answer to Problem is far from being completed.

I will give a survey talk concerning to Problem and the related subjects such as isometries on spaces of analytic functions.

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