Probability Theory

Solutions #5

5.4 It is clear that $A_{i,j}$ and $A_{k,l}$ are independent if $|\{i,j,k,l\}| = 4$ (i.e. all indices are different). We only have to check the case when there is one common index, e.g. the independence of $A_{1,2}$ and $A_{1,3}$. This follows from $\mathbf{P}(A_{1,2}) = \mathbf{P}(A_{1,3}) = 1/6$ and $\mathbf{P}(A_{1,2} \cap A_{1,3}) = \mathbf{P}(\text{first three throws are the same}) = 1/36$.

However, the events are not completely independent: if $A_{1,2}$ and $A_{1,3}$ occur then so will $A_{2,3}$.

5.7 The probability of getting the right answer for a given question is $p + r/2$. The probability of getting 20 or 19 good answers (binomial distribution!):
$(p + r/2)^{20} + 20(p + r/2)^{19}(q + r/2)$.

5.9 The probability of given configuration where there are x blocked routes, is $p^x(1-p)^{5-x}$. The probability in question may be computed by collecting all those configurations when there is a path from A to D, or by suitable conditioning. The answer is
$1 - 2p^2 - 2p^3 + 5p^4 - 2p^5$ which gives $1/2$ in the case when $p = 1/2$. (Can you prove this without the actual formula?)

5.10 By symmetry, we can fix the position of A to a given point. Then our sample space is $[0, 2\pi] \times [0, 2\pi]$ with uniform measure on it. The event in question is

$$\{(x,y) \in [0,2\pi]^2 : \min(x,y) < \pi, \max(x,y) - \min(x,y) < \pi, 2\pi - \max(x,y) < \pi\}$$

Finding its area and dividing by $4\pi^2$ we get that its probability is $1/4$

5.12 The probability of getting a given dart between 10 and 20 cm of the center is $\frac{20^2-10^2}{30^2} = 1/3$ (this is the ratio of the areas). Thus the probability of not getting a dart there in 3 tries is $(1 - 1/3)^3 = 8/27$.