Probability Theory

Solutions #11

11.2 Denote by X the number of times the digit 7 appeared. Then X is a binomial with parameters $(10000, 1/10)$ which means that $\frac{X-1000}{\sqrt{100001/109/10}}$ is approximately normal. This means that the probability in question is approximately $\Phi(-29/30) \approx .166$

11.4 Denote the number of heads by X. Then we need a k for which $\mathbf{P}(485 < X < k) \approx .5$. SInce X is binomial with parameters $(1000, .5)$, by the central limit theorem the previous probability is approximately $\Phi((k-500)/\sqrt{250}) - \Phi(-15/\sqrt{250})$. From this one can easily calculate that $k = 507$.

11.7 If n is the number of coin-flips and S_n is the number of heads then we need

$$\mathbf{P}(|S_n - n/2| < 0.01n) \geq 0.99$$

By the CLT this translates to $2\Phi(0.02\sqrt{n}) - 1 \geq 0.99$ which means $n \leq 16641$. (It depends how many signigficant digits you take ...)

11.10 If the unknown probability is p and the relative frequency of Democratic voters is p' in a sample of n then we need $\mathbf{P}(\left|\frac{p'-p}{p}\right| < 0.02) > 0.99$. np' is a binomial with parameters (n, p) thus by the normal approximation:

$$\mathbf{P}(\left|\frac{p'-p}{p}\right| < 0.02) \approx 2\Phi(0.02\sqrt{np/q}) - 1.$$

From this we get $n \geq 16641p/q$, and since $0.4 < p < 0.6$, with the choice of $n > 16641.6/.4 = 24962$ will suffice.
Of course, as we discussed in class, it size of the sample does not depend on the size of the population.

11.12 Denote the number of votes party B gets from the voters of the state by X. If they import k sure voters, then they will win if $X + k > (8.000.000 + k)/2$, i.e. if $X > 4.000.000 - k/2$. Thus we need a k with $\mathbf{P}(X > 4.000.000 - k/2) > 0.9$. Since X is binomial with parameters $(8.000.000, .5)$ we may approximate this probability using the CLT: $\Phi(-\frac{k}{2\sqrt{2.000.000}})$, which gives $k > 3622$.