Probability Theory

Solutions

1.2 (d) Both sides equal to $(A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C) \cup (A \cap B \cap C)$

(e) Both sides equal to $A \cap \overline{B}$.

(h) Both sides equal to $(A \cap \overline{B} \cap \overline{C}) \cup (A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C)$.

(l) $(A \circ B) \cup (B \circ C) = (A \cup B \cup C) \setminus (A \cap B \cap C) = (A \circ C) \cup (B \circ C) \supset A \circ C$

(u) $[A \cap (B \cup C)] \cup [B \cap (C \cup A)] \cup [C \cap (A \cup B)] =$
$((A \cap B) \cap (C \cap A)) \cup ((B \cap C) \cap (A \cap B)) \cup ((C \cap A) \cup (B \cap C))$.

1.3 We throw three dice at the same time. How many observable simple events do we have

(a) $\binom{6+3-1}{3} = 56$,

(b) $\binom{6+2-1}{2}6 = 126$

(c) $6^3 = 216$

(d) $\frac{1}{2}$, by the symmetry of the sample space.

1.5 Odd number of events occur from A_1, A_2, ... , A_n. Proof by induction: for $n = 1$ and $n = 2$ it holds. If it is true for n then it will hold for $n + 1$: one only has to look at $(A_1 \circ A_2 \circ \cdots \circ A_n) \circ A_{n+1}$ and use the induction hypothesis.

1.7 Sample space: sequences of H and T, where the last two symbols are equal and before that no two consecutive symbols are the same. For each $n \geq 2$ there are two such sequences with length n, each has a probability of 2^{-n}.

(a) the experiment ends before the sixth toss: $\sum_{n=2}^{5} 2 \times 2^{-n} = \frac{15}{16}$.

(b) an *even* number of tosses is required: $\sum_{n=1}^{\infty} 2 \times 2^{-2n} = \frac{2}{3}$

1.8 (a) $\mathbf{P}(A \cap B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cup B) \geq \mathbf{P}(A) + \mathbf{P}(B) - 1$.

(b) Use induction. For $n = 2$ we just proved it. If it holds for n:

$$\begin{aligned}
\mathbf{P}\left((A_1 \cap A_2 \cap \ldots A_n) \cap A_{n+1}\right) &\geq \mathbf{P}\left(A_1 \cap A_2 \cap \ldots A_n\right) + \mathbf{P}(A_{n+1}) - 1 \\
&\geq \mathbf{P}(A_1) + \mathbf{P}(A_2) + \ldots \mathbf{P}(A_n) - (n-1) + \mathbf{P}(A_{n+1}) - 1 \\
&= \mathbf{P}(A_1) + \mathbf{P}(A_2) + \ldots \mathbf{P}(A_n) + \mathbf{P}(A_{n+1}) - n
\end{aligned}$$

(First we used that the inequality holds for 2 events, the next line we used the induction hypothesis.)