RESEARCH STATEMENT

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My main research objective is to derive mathematically rigorous proofs about the *hydrodynamic behavior* of stochastic interacting particle systems.

The central problem of non-equilibrium statistical physics is the study of the dynamics of interacting particle systems. Generally, the size of these systems is enormous, thus tracking every single particle is hopeless, even if we know everything about the microscopic dynamics. The proper way to look at this problem is from the macroscopic point of view. We characterize the state of the system with the local densities of certain physically relevant conserved quantities (e.g. particle number, momentum, energy). These quantities change slowly on a microscopic time-scale, and after appropriate rescaling, their local densities become deterministic. The time-evolution of these functions, which is usually driven by coupled partial differential equations (pde's), gives us the needed description.

Hydrodynamic limit (hdl) is the device to obtain these systems of pde's from the microscopic dynamics. In the physics literature there are a number of well-known phenomenological derivations of hdl's for several systems. It is a challenging and important program of mathematical physics to give *mathematically rigorous* versions of these derivations. For completely deterministic systems, this is still an unsolved problem. However, in the last decades there have been considerable advances in the case of *stochastic interacting particle systems*, in particular much effort has been made in the analysis of lattice gas models with conserved quantities (e.g. simple exclusion, zero range, Ginzburg-Landau models).

My field of interest is the investigation of *hyperbolic systems*. In that case the natural scaling one has to apply is Eulerian (i.e., space and time are scaled the same way) and in the limit we get *hyperbolic conservation laws*. The theory of these equations is a much investigated, important field of pde theory. One of their most interesting features is the following: arbitrary smooth initial conditions yield *shocks* in finite time (apart from some very specially prepared cases), which means we do not have global classical solutions. The study of weak solutions is the next natural step, but then we lose uniqueness: we may have an infinite number of weak solutions with the same initial condition. It is possible to single out the relevant (so-called entropy-) solution, and for one-component hyperbolic equations the problem is essentially solved: the existence and uniqueness of entropy-solutions is established. For *multi-component systems* (i.e., when there are more than one conservation laws) the pde theory is still not completely resolved, which means that in these cases the hydrodynamic description is also quite difficult.

The results of my PhD thesis deal with the hydrodynamic behavior of *multi-component hyperbolic systems*. The hdl under Eulerian scaling is derived for a fairly general class of lattice models and the hydrodynamic behavior of the evolution of a small perturbation around an equilibrium point is also analyzed. One of the main ingredients used in the proofs is H.T. Yau's *relative entropy method*, which is a robust, essentially model-independent technique. Unfortunately, the method requires smooth solutions of the limiting pde, hence all previously mentioned results hold only in the smooth regime, up until the first appearance of shocks. It would be of great interest if these results (particularly the Eulerian scaling limit) could be *extended beyond the shocks*. The investigation of this problem is my primary goal.

For one-component systems which exhibit the so-called attractivity property there are deep and strong results about the hydrodynamic behavior, even after the loss of smoothness. The key idea, first used by F. Rezakhanlou, is the implementation of certain pde techniques (by Kruzkov and Oleinik) on a stochastic level. In the case of multi-component systems attractivity is not a natural feature and thus the tools developed for attractive systems can not be used. Recently, J. Fritz introduced a new method, which consists of transposing some pde tools developed for the study of multi-component hyperbolic systems (by Tartar, Murat and DiPerna) to stochastic systems. This technique (the method of *compensated compactness*) was successfully applied by J. Fritz and B. Tóth to a particular two-component system. This was the first (and is still the only) result where hyperbolic hydrodynamic Euler equations are derived for a multi-component system which are valid beyond the appearance of shock waves. It is hoped, that this method may be *extended to more general systems* (including non-attractive one-component models). For this general case, *stochastic versions of other relevant pde tools* (most importantly the Lax-Chuey-Conley-Smoller maximum principle) would be very useful. Exploring this question would be the first step in my research.