On Why-Questions in Physics

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1 INTRODUCTION

In natural sciences, the most interesting and relevant questions are the so-called why-questions. What is a why-question? A why-question is nothing else than a question in the form "Why P?" (or "Why is P true?") where P is an arbitrary statement.

There are several different approaches to why-questions and explanations in the literature, see, e.g., [6], [7], [8], [18]. However, most of the literature deals with why-questions about particular events, such as "Why did Adam eat the apple?". Even the best known theory of explanation, Hempel's covering law model, is designed for explaining particular events. Here we only deal with purely theoretical why-questions about general phenomena of physics, for instance "Why can no observer move faster than light?" or "Why are Kepler's laws valid?".

Here we are not going to develop a whole new theory of why-questions in physics. We will just touch upon some ideas and examples relevant to our subject.

2 ANSWERING WHY-QUESTIONS

How to answer a why-question? For example, let NoFTL be the statement "No observer can move faster than light," which is one of the several astonishing predictions of relativity. As it is a statement that hard to believe, it is natural to ask why we think it is true. The standard answers to the question "Why NoFTL?" are

1. "NoFTL is true because the 4-dimensional Minkowski spacetime over $\mathbb{R}$ (the field of real numbers) is a good model of our physical world; and in this model NoFTL is valid."

2. "NoFTL is an axiom (of Special Relativity)."

Neither of these answers is satisfactory to a logician. The problem with the first is that it refers to one particular model and not to a list of axioms. The second does not really answer the question, it is a kind of "just because" answer.

How to give satisfactory answers to why-questions in physics? First of all to answer the question "Why P?", we need a formal language in which we can formulate P and the possible answers to it. Let us fix one such language.[1] The *possible answers* to the question "Why P?" are consistent theories which do not contain P as an axiom.[2] Here we do not require theories to be deductively closed systems, as we would like to separate the assumptions of the theory from its consequences. Hence we use the term theory as a

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[1] Every concept we introduce here is relative to a fixed language in which P can be formulated.

[2] It is important that we restrict our definition to possible answers to why-questions in physics because, for example, in mathematics the possible answers to why-questions are usually proofs and not theories. See, e.g., [15].

synonym for axiom system. Let us call a possible answer *acceptable answer* if it implies P.

To present some examples of acceptable answers to the question "Why NoFTL?", let us consider the following two-sorted first-order language:

$$\{ Q, +, \cdot, <; B, \mathsf{IOb}, \mathsf{Ph}; \mathsf{W} \},$$

where Q is the sort of quantities and B is the sort of bodies; $\cdot$ and $+$ are binary function symbols and $<$ is a binary relation symbol of sort Q; IOb (inertial observers) and Ph (light signals or photons) are unary relation symbols of sort B; and W (the world-view relation) is a 6-ary relation symbol of the type $B \times B \times Q \times Q \times Q \times Q$. Relations $\mathsf{IOb}(o)$ and $\mathsf{Ph}(b)$ are translated as "o is an inertial observer," and "b is a photon," respectively. We use the world-view relation W to speak about coordinatization by translating $\mathsf{W}(o, b, x, y, z, t)$ as "observer o coordinatizes body b at spacetime location $\langle x, y, z, t \rangle$," (that is, at space location $\langle x, y, z \rangle$ and at instant t).

In this language we can define the term $speed_o(b)$ as the speed of a body b according to observer o. Hence we can formulate that no observer can move faster than light. Moreover, we can prove the following:

$$\mathsf{SpecRel} \models \forall o, o', p \ \mathsf{IOb}(o) \wedge \mathsf{IOb}(o') \wedge \mathsf{Ph}(p) \implies speed_o(o') < speed_o(p),$$

where $\mathsf{SpecRel}$ is the consistent axiom system of the following axioms:

AxField: The *quantity part* $\langle Q; +, \cdot, < \rangle$ is an ordered field.

AxSelf: Every observer coordinatizes itself at a coordinate point if and only if its space component is the origin, that is, space location $\langle 0, 0, 0 \rangle$.

AxPh: The speed of light signals is 1 according to every inertial observer.

AxEv: Every inertial observer coordinatizes the very same set of events.

AxSymd: Inertial observers agree as for the spatial distance between events if these events are simultaneous for both of them.

For the formulation of these concepts and axioms, see, e.g., [5]. So SpecRel is an acceptable answer to the question "Why NoFTL?", see [5, Thm.1].

3   HOW TO COMPARE THE DIFFERENT ANSWERS

How to get better and better answers to a certain why-question? The basic idea is that "the less it assumes, the better an answer is." Let us try to make this idea more precise by introducing the following concepts:

1. Th_2 is *nonworse* than Th_1 as an answer to the question "Why P?" if all the formulas of Th_2 are consequences of Th_1, that is, Th_1 implies φ_2 for any formula $\varphi_2 \in Th_2$.

2. Th_2 is *piecewise nonworse* than Th_1 as an answer to the question "Why P?" if for any formula $\varphi_2 \in Th_2$ there is $\varphi_1 \in Th_1$ such that φ_1 implies φ_2.

It is easy to see the following:

1. Both concepts above are preorders on sets of formulas, that is, they are transitive and reflexive relations.

2. Th_2 is a piecewise nonworse answer than Th_1 if $Th_2 \subseteq Th_1$.

3. Th_2 is a nonworse answer than Th_1 if it is piecewise nonworse.

Let us say that Th_2 is *(piecewise) better* than Th_1 if it is (piecewise) nonworse and nonequivalent according to the equivalence relation defined by the corresponding preorder.

These definitions fit in with the following idea of Michael Friedman [6]: "Science increases our understanding of the world by reducing the total number of independent phenomena that we have to accept as ultimate or given."

To present another example, let us note that the following theorem can also be proved:

$$\mathsf{SpecRel}_0 \models \forall o, o', p \; \mathsf{IOb}(o) \wedge \mathsf{IOb}(o') \wedge \mathsf{Ph}(p) \implies speed_o(o') < speed_o(p),$$

where $\mathsf{SpecRel}_0$ consists of the following four axioms only: $\mathsf{AxField}$, AxSelf, AxPh and AxEv. Hence $\mathsf{SpecRel}_0$ is a piecewise better answer to the question "Why NoFTL?" than $\mathsf{SpecRel}$. Further answers to this question, which are not comparable to $\mathsf{SpecRel}_0$ according to the preorders above, can be found in [13, Thms. 3 and 5].

Let us note that, according to the definition above, a theory consisting of two (nonequivalent) axioms is a piecewise better answer than a theory consisting of their conjunction. That is a nice property as it makes the introduction of the following concept possible. Let us say that a possible answer to the question "Why P?" is *pointless* if there is a piecewise better answer which contains P as an axiom. Accordingly, the conjunction of Kepler's laws and Boyle's law is a pointless answer to the question "Why Kepler's laws are valid?". So it eliminates a problem that motivated Hempel to introduce his covering law model only for the explanation of particular events, see [7].

4 GENERAL ANSWERS

In the previous section, we dealt with answers to one particular question. However, in physics it is a general desire to search for unified theories, that is, theories answering more questions. So a good answer in physics is char-

acterized by assuming little, but implying a lot.[3]

The unification of theories is the point where we have to leave the convenience of fixed languages and search for suitable unification of the languages of the theories in question, too. For example, the juxtaposition of the axioms of relativity and quantum theories in their combined language will result in one theory but it will not solve the problem of their reconciliation. Though it will be consistent (as its parts are consistent) and will imply all the prediction of relativity and quantum theories, it will not be what we mean by a unified theory. To achieve a truly unified theory, we need a richer language in which we can formulate the interrelations between the concepts of these theories. However, unifying the languages in an appropriate way and formulating new axioms in this unified language to establish the interrelations between their concepts such that the unified theory has new experimentally testable predictions is a very difficult task. Unifying theories is the subject of amalgamating theories in algebraic logic, see, e.g., [10].

5   THE TRUTH OF AXIOMS OF PHYSICS

Most theories of why-questions require the explanations to be true, see, e.g., [7]. However, it is a fundamental requirement of a physical theory to be experimentally testable, hence refutable. So in physics we do not know whether an axiom is true or not, we just presume so. Therefore, if we require the explanations to be true, we will never ascertain whether our theory is really an explanation or not. Hence in the case of why-questions of physics, it is better not to require the axioms to be true.

For example, if we required the axioms of physics to be true, we could not

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[3]Of course there are other desired requirements of a physical theory, such as simplicity and comprehensibility of the axioms or experimental testability. However, it is not easy to define these concepts precisely if it is possible at all.

say that Newton's theory is an explanation of Kepler's laws because Newton's theory is refuted by some experimental facts.[4] That would be inconvenient as Newton's theory is the standard example in the literature for explanation of Kepler's laws. Hence it is better to treat the axioms as possible truths and treat the matter only in conditional, that is, an answer to the question "Why P?" means that "If the theory is true, then so is P." In this sense it is still meaningful to say that Newton's theory explains Kepler's laws. And according to the above definition Newton's theory is an acceptable answer to the question "Why Kepler's laws are valid?".

6   SOME FURTHER EXAMPLES

We can take the Twin Paradox (TwP) and ask "Why TwP?". In the language of SpecRel, we cannot formulate the full version of TwP only its inertial approximation called the Clock Paradox (ClP). Here we only concentrate on TwP but for similar investigation of ClP see [16], [17]. To formulate the full version of TwP we have to extend the language above for accelerated (that is, non-inertial) observers, which is done by adding one more unary relation for accelerated observers on the sort of bodies. In this language we can formulate TwP, see [11], [16]. Let us denote the formulated version of TwP as TwP.

The most natural axiom to assume about accelerated observers is the following:

AxCmv: At each moment of its world-line, every accelerated observer coordinatizes the nearby world for a short while in the same way as an inertial observer does.

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[4]For example, the perihelion advance of Mercury is a well known experimental fact that disproves Newton's theory.

For precise formulation of this axiom, see [11], [16]. Let AccRel be the axiom system consisting of AxCmv and all the five axioms of SpecRel.

Surprisingly, AccRel does not answer the question "Why TwP?". Moreover, the following can be proved:

$$\mathsf{AccRel} \cup \mathsf{Th}(\mathbb{R}) \not\models \mathsf{TwP},$$

where $\mathsf{Th}(\mathbb{R})$ is the whole first-order theory of real numbers, see [11]. At first sight this result suggests that the question "Why TwP?" cannot be answered within first-order logic, which would be depressing as there are weighty methodological reasons for staying within first-order logic, see, e.g., [3], [16]. However, there is a first-order axiom scheme (IND) in the above language such that AccRel together with this scheme answers the question "Why TwP?", see [11], [16].

The why-question "Why gravity slows time down?", can be answered by the theory AccRel ∪ IND and Einstein's equivalence principle, see [12], [16].

For further examples, let us consider the question of "Why does relativistic mass increase?" or "Why the sum of the rest masses of inelastically colliding inertial bodies is smaller than the rest mass of the originated inertial body?". Theories SpecRelDyn and SpecRelDyn$^+$ are possible answers to these questions, respectively, see [4], [14], [16].

7 CONCLUDING REMARKS

In the spirit of reverse mathematics, we have introduced a precise definition of acceptable answers to why-questions in physics and some ideas about how to compare these answers. We also presented several examples mainly from axiomatic relativity. Finally let us note that the work done by the research group of Logic and Relativity led by Andréka Hajnal and István Németi can

be considered as providing explanations to why-questions of relativity, see references.

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