ERRATA FOR OUR BOOK ‘CENTRAL SIMPLE ALGEBRAS AND GALOIS COHOMOLOGY’

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The following list rectifies errors we are currently aware of. They will be hopefully corrected in future printings.

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p. 20 Lemma 2.2.2 holds more generally for an arbitrary algebraic extension $K|k$. The reduction to the case of a finite extension is easy. In the proof of Theorem 2.2.1 it is this more general statement that is applied with $K = \overline{k}$.

p. 21, line –9: The multiplication rule should rather be $e_i e_j = \sum a_{ijl} e_l$.

p. 23, line –16: An idempotent is of course an element satisfying $e_i^2 = e_i$, as its name shows.

p. 28, line -13: There is a small mistake at the end of the proof of Lemma 2.3.8. The invariant subspace of G on $V \otimes_k K$ is generated over $K \otimes_k K$ by the element $\sum_{\sigma \in G} e_\sigma$ and not by e_1. To deduce from this the isomorphism $(V \otimes_k K)^G \otimes_k K \cong V \otimes_k K$ one may use the decompositions $K \otimes_k K \cong \bigoplus K e_\sigma$ and $V \otimes_k K \cong \bigoplus W e_\sigma$ to identify $K \otimes_k K$ with the ring of functions $G \to K$ and $V \otimes_k K$ with the $K \otimes_k K$-module of functions $G \to W$. The elements of $(V \otimes_k K)^G$ then correspond to the constant functions $G \to W$, and using the system of orthogonal idempotents e_σ of $K \otimes_k K$ (corresponding to 0-1 functions on G) we see that they generate $V \otimes_k K$ as a $K \otimes_k K$-module.

p. 38, lines 15–16: ‘replacing the matrix C above by some $D^{-1}CD$’ should read ‘replacing the matrix C_σ above by some $D^{-1}C_\sigma \sigma(D)$’.

p. 42, line 13: 2.8.2 is a proposition, not a theorem.

p. 56, line -4: $\mathbf{Z}[G^{i+1}]$ is indeed a free $\mathbf{Z}[G]$-module, but not of rank $i+1$ but with basis G^i. For the argument to follow only freeness matters.

p. 57: in 3.2.2 the last displayed formula defines δ_{i-1}^*, not δ_i^*.

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p. 78, Exercise 1: One should moreover require that $\phi^0_A : A^{G_2} \to A^{G_1}$ is the natural inclusion map.

p. 81, Proposition 4.1.3: The group $\mathrm{Gal}(M|K)$ should be replaced by $\mathrm{Gal}(M|k)$. Also, towards the end of the proof the two $\mathrm{Gal}(L|K)$'s should read $\mathrm{Gal}(L|k)$.

p. 88, line 6: in the source of the map replace the cohomology group $H^i((G/U_\alpha)/(H \cap U_\alpha), A^{H\cap U_\alpha})$ by $H^i(G/HU_\alpha, A^{HU_\alpha})$.

p. 95, line 3: instead of 'all Galois extensions' read 'all finite Galois extensions'.

p. 101, l. –1: 'In particular, $P_a(T)$ is irreducible over $\overline{k}$': this is of course nonsense. Replace the rest of the proof by the following text:

'Consider the k-algebra $K = k[T]/(P_a(T))$ and the homomorphism $\phi : K \to A$ induced by sending T to a. Tensoring ϕ by $\overline{k}$ we get the inclusion of the diagonal subalgebra of $M_n(\overline{k})$, so ϕ is injective. As A is a division algebra, K is then a subfield which is separable over k because $P_a(T)$ has distinct roots. Since $\deg P_a(T) = \deg_k(A) = \mathrm{ind}_k(A)$, we conclude by the previous proposition.'

p. 112, Exercise 3: One should moreover assume that K contains the m^2-th roots of unity. Otherwise the image of $\sqrt[m]{a}$ in $K^\times/K^{\times m}$ is not fixed by G.

p. 116, Proof of Theorem 5.1.3: The proof of implication (2) $\Rightarrow$ (3) may break down when k is finite. But in this case one may apply a well-known general lemma of Lang and Nishimura (S. Lang, *Amer. J. Math.* 76 (1954), 362–374; H. Nishimura, *Mem. Coll. Sci. Kyoto Ser. A*, 29 (1955), 189–192) that works over an arbitrary field.

Alternatively, one can remark that (3) always holds over a finite field anyway (by Theorem 5.2.1 and the triviality of the Brauer group of a finite field; this argument does not use Theorem 5.1.3).

p. 118, Theorem 5.2.2: The target of ρ is $\mathbf{P}_k^{N-1}$, not $\mathbf{P}_k^{n-1}$. Similarly, that of ρ_K is $\mathbf{P}_K^{N-1}$.

p. 121, bottom: The two sentences before the diagram should read: "Therefore the cocycle $a_\sigma : \sigma \mapsto \phi \circ \sigma(\phi^{-1})$ defining the class of X in $H^1(G, \mathrm{PGL}_n(K))$ takes its values in the subgroup $P_d(K) \subset \mathrm{PGL}_n(K)$, and the class of Y is in the image of the map

$$H^1(G, P_d(K)) \to H^1(G, \mathrm{PGL}_n(K))."$$

p. 123: in l. 12 it is better to write ab^{-1} instead of $a - b$ to avoid confusion.

Also, there are some misprints from l. –8 onwards. Instead of $M_{(r-1)d}(B)$ one should read $M_{r-1}(B)$ (3 times), and $M_r(B)$ should be replaced by B (twice).

p. 127, first paragraph and line 15: there is an unfortunate confusion here between co-induced modules and permutation modules. The correct argument for the vanishing of $H^1(G, \mathrm{Div}(X_K))$ is as follows. By exactly the same arguments as on pp. 152–153, we find that $H^1(G, \mathrm{Div}(X_K))$ is a direct sum of the modules $H^1(G_P, \mathbf{Z})$, where P runs over prime divisors on X and G_P is the stabilizer in G of a prime divisor of X_K above P. But the groups $H^1(G_P, \mathbf{Z}) = \mathrm{Hom}(G_P, \mathbf{Z})$ are trivial as $\mathbf{Z}$ is torsion free.

p. 130, after formula (6): A_σ is the matrix of $\lambda^{-1} \circ \sigma(\lambda)$, not of $\sigma(\lambda) \circ \lambda^{-1}$.

p. 133, line 4: In Exercise 5.1, the equation of Y_b should be

$$bx^n = N_{K/k}\Big(\sum_{i=0}^{n-1} (\sqrt[n]{a})^i \, y_i\Big).$$

p. 137, lines 5–9: what we are using is that on the *degree 2 terms* of the said special projective resolutions the map is given by sending σ to the image of $m\sigma$ in $\mathbf{Z}/n\mathbf{Z}$. (As C. Vial points out, this is not so in all degrees: we have the natural projection π in degrees 0 and 1, then π multiplied by m for 2 and 3, then π multiplied by m^2 for 4 and 5, etc.)

p. 144, Theorem 6.2.12: The formulation of the theorem is slightly misleading, because if the congruences have a common solution modulo some (t^N), they have a common solution modulo (t^M) for $0 < M < N$. We should rather say that the f_i have a common solution in $k[[t]]^n$ if and only if they have a common solution modulo (t^N) for all $N > 0$.

The role of N becomes clear if one formulates a stronger form of the theorem (which comes out of the proof): there exist positive integers c, ν depending only on S so that for N large enough each common solution modulo (t^N) can be approximated by a common solution in $k[[t]]^n$ congruent to the given one modulo $(t^{[N/c]-\nu})$.

The proof given in the text works in characteristic 0, but it is unfortunately incomplete in positive characteristic: since Proposition A.3.7 is false (see below), it may happen that on p. 145, l. –11 we have $W = V$, i.e. all points of V are singular. This case is handled by additional arguments in the original paper Greenberg [1].

p. 145, line 6: the zero is nontrivial. Afterwards, one should be a bit careful to avoid the trivial solution. This can be done by arranging that $a_j^{(N)}$ has constant coefficient 1 for some j and apply Theorem 6.2.12 to f with x_j replaced by $1 + tx_j$, or by applying the approximation form of the theorem stated above.

pp. 154–155: Theorems 6.4.4 and 6.4.5 should be stated only for $i \geq 2$ (otherwise the maps r_P are not defined).

p. 158, line 25: the argument is correct, but actually $H^1(G, \mathbf{Z}) = 0$, so the stated surjectivity is obvious. Therefore Lemma 6.5.3 is not used directly in the proof (but all the ingredients of its proof are).

p. 169, last paragraph: As G. Braun and E. Szabó observe, there is a much simpler way to show the non-triviality of the class $i_*(c(\overline{G}_n))$ (by the way, it should be $\iota_*(c(\overline{G}_n))$). Indeed, consider the extension

$$0 \to \mathbf{Q}/\mathbf{Z} \to E \to G \to 1$$

obtained as the pushout of the extension (13) by the map $\iota : \langle z \rangle \to \mathbf{Q}/\mathbf{Z}$. Were there a splitting $E \to \mathbf{Q}/\mathbf{Z}$, the composite $\langle z \rangle \to \overline{G}_n \to E \to \mathbf{Q}/\mathbf{Z}$ would be 0, because z is in the commutator subgroup of $\overline{G}_n$ and $\mathbf{Q}/\mathbf{Z}$ is commutative. But this composite map is none but ι, a contradiction.

p. 181, line 1: In Exercise 6.6.(c) the second formula should read

$$\mathrm{ind}_{k(t)}(B) = p \ \mathrm{ind}_K(A \otimes_k K).$$

p. 187, line 4: there is a sign error here. With our conventions (which are those of Milnor's original paper) we get the inverse of the tame symbol.

p. 224: As O. Gabber remarks, in the discussion of the Gersten complex in Milnor K-theory for excellent schemes one has to correct a mistake already present in Kato [4], and assume moreover that X is equipped with a dimension function δ. By definition such a δ associates an integer $\delta(x)$ with each point x in such a way that if y is a point of codimension 1 in the closure of x, then $\delta(y) = \delta(x) - 1$. One then defines X_i as the set of x with $\delta(x) = i$. The usual function sending a point x to the dimension of the closure of x does not necessarily satisfy the above property. For example, if V is a discrete valuation ring and $X = \mathrm{Spec}\, V[t]$, then X is irreducible of dimension 2 and the codimension 1 point $(1 - at)$ (for a nonzero non-invertible in V) is closed. However, for the applications in this book the usual dimension function works.

p. 275, Remark 9.3.7 (1): In the second line of this remark, replace 'extending K-linearly' by 'extending k-linearly'.

p. 304: As O. Gabber points out, Proposition A.3.7 is false. There is a simple counterexample: take k to be a perfect field of characteristic $p > 0$, and K the extension of $k((t))$ obtained by adjoining a p-th root u of t. Then K is purely inseparable, hence not separably generated over $k((t))$. In fact, a finitely generated extension of $k((t))$ is separably generated if and only if t is not a p-th power in it.

The mistake in the proof is on p. 305, l. 11: in the above counterexample the relation $F(u^p) = 0$ becomes $t - t = 0$, so the regrouping argument breaks down. However, when t is not a p-th power in K, the argument works.