

Strong Morita equivalence for semigroups with local units

Morita equivalence is a widely used tool for rings with identity. (Two rings are said to be Morita equivalent if the categories of unitary modules over them are equivalent.) For monoids, this notion is not really useful: in most cases it reduces to isomorphism. As the theory of Morita equivalence could be developed for the more general case of rings with local units, and then for idempotent rings, the question arose whether this could lead to a more fruitful development for semigroups. Indeed, in the mid nineties, Talwar could carry over the basic theorems of Morita equivalence to semigroups with local units, showing also the relevance of this notion in the structure theory of semigroups. The theory got stuck at this point, however – for instance, hardly any Morita invariant properties were known.

Recently, there has been tremendous progress in this field. Lawson, Laan and Márki have exhibited seven different approaches to Morita equivalence, all equivalent for semigroups with local units. (One of Lawson's approaches makes fundamental use of a recent result of Pécsi.) Laan and Márki have also cleared up the relation of these approaches for factorisable semigroups (those in which every element can be decomposed into a product), which is the limit for Morita equivalence theory. In addition, they have proved Morita invariance of a number of properties for semigroups with various kinds of local units.

References

- [1] V. Laan and L. Márki, Strong Morita equivalence for semigroups with local units, *J. Pure Appl. Algebra* **215** (2011), 2538-2546.
- [2] V. Laan and L. Márki, Morita invariants for semigroups with local units, *Mh. Math.* **166** (2012), 441-451.
- [3] M. V. Lawson, Morita equivalence for semigroups of local units, *J. Pure Appl. Algebra* **215** (2011), 455-470.
- [4] B. Pécsi, On Morita contexts in bicategories, *Applied Categ. Struct.* **20** (2012), 415-432.