

INVARIANT ALMOST COMPLEX STRUCTURES ON $S^{2n+1} \times S^{2p+1}$.

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ABSTRACT. Product of odd-dimensional spheres has 2-parametric family of complex structures $I_{a,c}$, constructed firstly in [1]. These structures are deplete all class of invariant almost complex structures on $S^{2n+1} \times S^{2p+1}$ as $U(n+1)/U(n) \times U(p+1)/U(p)$. All of them are tamed by the same 2-form $\omega_{t,\lambda,\lambda'}$, so we have the 5-parametric homogeneous Hermitian triple $(g_{a,c,t,\lambda,\lambda'}, I_{a,c}, \omega_{t,\lambda,\lambda'})$ on $S^{2n+1} \times S^{2p+1}$. The exact estimates of sectional curvature of $g_{a,c,t,\lambda,\lambda'}$ are found.

While all invariant almost complex structures on $U(n+1)/U(n) \times U(p+1)/U(p)$ are all in Calabi-Eckmann class, this matter is not such clear for the case $S^3 \times S^3$ as $SU(2) \times SU(2)$. The space of all leftinvariant almost complex structures is already 18-dimensional. We know that:

1. $S^3 \times S^3$ has 2-parametric family Calabi-Eckmann complex structures.
2. $S^3 \times S^3 = SU(2) \times SU(2)$ has complex structure as compact Lie group from Samelson construction [2].
3. 2-parametric family of complex structures satisfy to classical theorem about compact homogeneous complex manifolds [3].

It seems that class of leftinvariant complex structures on $SU(2) \times SU(2)$ is exactly equal to class of Calabi-Eckmann manifolds, and therefore we can say the same about Samelson complex structure on $SU(2) \times SU(2)$.

Fix Killing-Kartan metric on $SU(2) \times SU(2)$. Now we can select class \mathcal{Z} of a.c.s., orthogonal with respect to this metric. \mathcal{Z} is compact and functional of Nijenhuis tensor norm get its minimal (complex case) and maximal value. The maximal value is found. The family of structures, which give this maximal value is described.

In the talk I will also speak about some constructions on the space of leftinvariant a.c.s. on this Lie group and about one open question.

REFERENCES

- [1] Calabi E., Eckmann B. A class of compact complex manifolds which are not algebraic. Ann.Math. Vol. 58(1935), P. 494-500.
- [2] Samelson, H.: A class of complex-analytic manifolds, Portugaliae Math. 12, 129-132 (1953)
- [3] Grauert H., Remmert R. Uber kompakte homogene komplexe Mannigfaltigkeiten // Arch. Math. Vol.13(1962), S. 498-507.
- [4] Butruille, J.-B.: Classification des variétés approximativement kähleriennes homogènes. Ann. Global Anal. Geom. 27, 201-225 (2005)