Workshop on

Fourier Analysis Extremal Problems and Approximation

Rényi Institute, Budapest, 2005.09.19. - 09.25.

The Jackson inequality for the best mean-square approximation of functions by entire functions of given exponential type and other extremal problems for positive definite functions

Vitaly V. Arestov
(University of Ekaterinburg)

In the talk we shall discuss the following two related extremal problems for positive definite functions of one and many variables.

1) The Jackson inequality with the least possible constant for the best mean-square approximation of functions by entire functions of given exponential type.

2) The Delsarte problem for functions representable by series with respect to ultraspherical polynomials. The problem is related to extremal problems for spherical codes.

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The Delsarte problem and related extremal problems of approximation theory and combinatorial geometry

Alexander Babenko
(Russian Academy of Sciences, Ural Branch, Ekaterinburg)

The talk will be devoted to the Delsarte problem and its applications to the Jackson inequality on approximation of periodic functions in L^2 by trigonometric polynomials and to bounds of packings in certain metric spaces.

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On Turán's Problem for ℓ-1 Radial, Positive Definite Functions

Elena Berdysheva
(University of Hohenheim, Germany)

Turán's problem is to determine the greatest possible value of the integral $\int_{\mathbb{R}^d} f(x)\,dx/f(0)$ for positive definite functions $f(x)$, $x \in \mathbb{R}^d$, supported in a given convex centrally symmetric body $D \subset \mathbb{R}^d$. In this talk we consider the Turán problem for positive definite functions of the form $f(x) = \varphi(\|x\|_1)$, $x \in \mathbb{R}^d$, with φ supported in $[0, \pi]$. An essential part of the talk is devoted to the planar case ($d = 2$), in this case we could settle and solve the corresponding discrete problem. Some of our results are proved for an arbitrary dimension.

Joint work with H. Berens (University of Erlangen-Nuremberg, Germa-ny).

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Some approximation properties of Nörlund logarithmic means on Vilenkin groups

István Blahota
(Institute of Mathematics and Computer Science, College of Nyíregyháza)

The (Nörlund) logarithmic means of the Fourier series is:

$$\frac{1}{l_n}\sum_{k=1}^{n-1}\frac{S_k(f)}{n-k}, \text{where } l_n=\sum_{k=1}^{n-1}\frac{1}{k}.$$

In general, the Fejér means has better properties, than the logarithmic o nes. In this talk we compare them, and we show that in the case of some unbounded Vilenkin systems the situation differs.

On the one hand there exists an integrable function $f \in L^1(G_m)$ such that the Fejér means $\sigma_{M_n} f$ does not converges to f in the Lebesgue norm, on the other hand we can give a class of unbounded generating sequence m such that the norm convergence $t_{M_n} f \to f$ hold for all $f \in L^1(G_m)$.

References

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An ideal density for hyperbolic packings

Károly J. Böröczky
(Rényi Institute of Mathematics, Hungarian Academy of Sciences)

It has been a challenge for approximately 50 years to find a meaningful notion of density for packings of congruent convex bodies in the hyperbolic space. The by now classical examples due to K. Böröczky (senior) show that no naive approach can work. Lately real breakthrough has been achieved by L. Bowen and Ch. Radin. They managed to handle "homogeneous packings" using recent advances on the ergodic theory of semi simple groups of rank one.

In this talk, a new approach is introduced to define density in the hyperbolic space. The advantage of the new notion is that it handles any packing. More precisely not a single number is assigned to packings but a density function on the ideal points. The new notion possesses many important properties of density for Euclidean packings. Methods related to Fourier analysis show that the new notion of density coincides with the "traditional" ones in the case of periodic packings.

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The Taikov problem for algebraic polynomials on a multidimensional sphere

Marina Deykalova
(University of Ekaterinburg)

We shall discuss the Taikov problem about the least value of measure of the set of non-negativity of algebraic polynomial of fixed degree with zero mean value on a multidimensional Euclidean sphere.

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Maximal convergence spaces of some operators related the Fourier series of two-variable functions on the Walsh group

György Gát
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1991 *Mathematics Subject Classification* 42C10
Key words: Walsh group, one and two-variable integrable functions, Cesàro means, Sunouchi operator, diadyc differentiability, almost everywhere convergence, maximal convergence spaces.

Let x be an element of the unit interval $I := [0, 1)$. The $\mathbf{N} \ni n$th Walsh function is

$$\omega_n(x) := (-1)^{\sum_{k=0}^{\infty} n_k x_k}.$$

The Dirichlet and Fejér kernel functions: $D_n := \sum_{k=0}^{n-1} \omega_k, \quad K_n := \frac{1}{n}\sum_{k=0}^{n-1} D_k$.

The Fourier coefficients, the n-th partial sum of the Fourier series, the n-th $(C, 1)$ mean of $f \in L^1(I)$:

$$\hat{f}(n) := \int_I f(x)\omega_n(x)d\mu(x), \quad S_n f := \sum_{k=0}^{n-1} \hat{f}(k)\omega_k, \quad \sigma_n f := \frac{1}{n}\sum_{k=0}^{n-1} S_k.$$

Let f be a two-variable integrable function, i.e. $f \in L^1(I^2)$. The two-parameter $(C, 1)$ means of f is $\sigma_{n,k} f := \frac{1}{nk}\sum_{i=0}^{n-1}\sum_{j=0}^{k-1} S_{i,j} f$.

In 1992, Móricz, Schipp and Wade proved (*Cesàro summability of double Walsh-Fourier series*, Trans Amer. Math. Soc., **329 (1)** (1992), 31-140)

for functions in $L\log^+ L(I^2)$ the a.e. convergence of the double $(C, 1)$ means of the Walsh-Fourier series $\sigma_{n,k} f \to f$ $(\min(n, k) \to \infty)$.

In 2000, Gát proved (*On the divergence of the* $(C, 1)$ *means of double Walsh-Fourier series*, Proc. Am. Math. Soc. **128 (6)** (2000), 1711–1720) that the theorem of Móricz, Schipp and Wade above can not be improved.

Namely, let $\delta : [0, +\infty) \to [0, +\infty)$ be a measurable function, $\delta(\infty) = 0$. Then, $\exists\, f \in L^1(I^2)$ such that $f \in L \log^+ L\delta(L)$, and $\sigma_{n,k} f$ does not converge to f a.e. as $\min(n, k) \to \infty$.

In other words, the maximal convergence space for the two-paramater $(C, 1)$ means is $L\log^+ L(I^2)$. The aim of this talk is to give a résumé of the recent developments concerning this matter. Besides, some other operators such as the Sunouchi operator, the dyadic difference operator are investigated in this point of view.

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Some extremal problems for periodic positive definite functions

Valery Ivanov
(Tula State University)

The solution of discrete variant of Fejer problem about the largest value in zero of even nonnegative trigonometric polynomial with fixed average value will be suggested. As a consequence, the largest average values of continuous 1-periodic even functions with nonnegative Fourier coefficients, fixed value in zero and equal zero in the closed interval [h,1-h] (Turan problem) or nonpositive in this closed interval (Delsarte problem) will be found for all rational h.

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A survey on the uncertainty principle

Philippe Jaming
(Université d'Orléans)

The uncertainty principle is a general fact stating that a function and its Fourier transform cannot both be arbitrarily concentrated. There are many mathematical ways to measure concentration, leading to the classical uncertainty principles (Heisenberg, Hardy,...). In this talk, I will survey several aspects of the uncertainty principle and present some recent results, starting with my results joint with A. Bonami and B. Demange and then presenting B. Demange's improvements.

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The minimality problem of positive realizations of transfer functions

Máté Matolcsi
(Rényi Institute of Mathematics, Hungarian Academy of Sciences)

The existence and minimality problem of positive realizations of transfer functions will be discussed.

Given a rational transfer function $H(z) = \frac{p(z)}{q(z)}$, the problem of positive realization is to find a matrix A and vectros b, c with nonnegative entries such that $H(z) = c^T(zI - A)^{-1}b$ holds. The minimality problem is to find the smallest possible dimension for A, b, c.

A necessary condition for the existence of such realiztions is that the inverse Z-transform of $H(z)$ (the so-called impulse response sequence) is nonnegative. Some recent results concerning the minimality problem, including lower and upper bounds on the dimension of the realizations will be reviewed.

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Convergence properties of the Marczinkiewicz-Fejér means with respect to the Walsh-Kaczmarz system

Károly Nagy
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In 1948 A. A. Šneider [7] introduced the Walsh-Kaczmarz system and showed that the inequality $\limsup_{n\to\infty} \frac{D_n^\kappa(x)}{\log n} \geq C > 0$ holds a.e. In 1974 F. Schipp [5] and Wo-Sang Young [8] proved that the Walsh-Kaczmarz system is a convergence system. V. A. Skvorcov in 1981 [6] showed that the Fejér means converges uniformly to f for any continuous functions f. G. Gát (1998 [1, 2]) proved, for any integrable functions, that the Fejér means converges almost everywhere to the function.

We introduce the Marcinkiewicz-Fejér means and Marcinkiewicz-Fejér kernels [3] as:

$$\mathcal{M}_n^\alpha f := \frac{1}{n}\sum_{k=0}^{n} S_{k,k}^\alpha f, \quad \mathcal{K}_n^\alpha := \frac{1}{n}\sum_{k=0}^{n} D_{k,k}^\alpha.$$

The behavior of the Marcinkiewicz-Fejér kernels with respect to Walsh-Kaczmarz system was studied in [4], thus we can state some convergence propeties of Marcinkiewicz-Fejér means.

References

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Malmquist-Takenaka systems and equilibrium conditions

Margit Papp
(University of Pécs)

The Malmquist-Takenaka systems $(\Phi_n, n \in \mathbb{N}^*)$ form an orthonormal system on the unit circle $\mathbb{T}$. The restriction of the inite collection $(\Phi_n, n = 1, ..., N)$ to a subset $\mathbb{T}_N^a = \{e^{i\tau_1}, ..., e^{i\tau_N}\}$ of $\mathbb{T}$ is a disc rete orthonormal system with respect to the scalar product $[.,.]_N$. It is showed that $(e^{i\tau_1}, ..., e^{i\tau_N})$ is a stationary point for a potential function.

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$2 \neq 3$

Imre Ruzsa
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2 dimensions differ from 3 in the following aspect: if $A \subset \mathbb{Z}^2$ has positive density then there exists a set $B \subset \mathbb{Z}$ such that $\overline{d}(B) > 0$ and $B \times B \subset A$. The analogous result in $\mathbb{Z}^3$ is false. Joint work with V. Bergelson.

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Negative results concerning Fourier series on the complete product of $\mathcal{S}_3$

Rodolfo Toledo
(Institute of Mathematics and Computer Science, College of Nyíregyháza)

In this talk I deal with the studies about convergence in L^p-norm of the Fourier series based on representative product systems on the complete product of arbitrary finite groups. Suppose that each finite group has discrete topology and normalized Haar measure. Let G be the compact group formed by the complete direct product of the finite groups with the product of theirs topologies, operations and measures. The most simple example of this groups is the complete product of $\mathcal{S}_3$, i.e. the symmetric group on 3 elements. The orthonormal system with which we work, is the product system (see [2]) of the normalized coordinate functions φ_k^s of the finite groups, namely

$$\psi_n(x) := \prod_{k=0}^{\infty} \varphi_k^{n_k}(x_k) \qquad (x \in G),$$

For the complete product of $\mathcal{S}_3$, G. Gát and R. Toledo (see [1]) proved there exists a $1 < p < 2$ and a function f in $L^p(G)$ for which the n-th partial sum of its Fourier series $S_n f$ does not converge to f in L^p-norm. Now, we proved this statement for all $1 < p < \infty$ and $p \neq 2$.

In general, we proved

Theorem 0.1. *Let G be a bounded group and suppose all the finite groups appearing in the product of G have the same system φ at all of their occurrences. If the sequence Ψ is unbounded, then the operator S_n is not of type (p,p) for all $p \neq 2$.*

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Jackson inequality between the value of the best approximation of a periodic function and its modulus of continuity

Stanislav Vasilyev
(Russian Academy of Sciences, Ural Branch, Ekaterinburg)

The talk will be about exact constant in Jackson inequality with modulus of continuity generated by difference operator with constant coefficient and some adjacent problems.