CONTEMPORARY MATHEMATICS

Volume 76

The Structure of Finite Algebras

David Hobby and Ralph McKenzie

AMERICAN MATHEMATICAL SOCIETY

Providence • Rhode Island

INTRODUCTION

By a *finite algebra* we mean a finite set of elements together with a (possibly infinite) set of operations acting on this set of elements. This concept includes finite groups and rings and many other algebraic systems of interest in mathematics. Excluded are finite systems with infinitary operations, and those having "partial operations" (operations defined for some, but not all, n-tuples of elements). By a *locally finite variety* we mean a class of algebras of one type, closed under the formation of homomorphic images, subalgebras, and direct products, whose finitely generated algebras are finite. The class of groups satisfying $x^3 = 1$ is an example of a locally finite variety.

The main discovery presented in this book is that the lattice of congruences of a finite algebra determines very deeply the structure of that algebra. Our theory reveals a sharp division of locally finite varieties of algebras into six interesting new families, each of which is characterized by the behavior of congruences in the algebras. We use the theory to derive many new results that will be of interest not only to universal algebraists, but to other algebraists as well.

The utility of congruence lattices for revealing the structure of general algebras has been recognized since Garrett Birkhoff's pioneering work in the 1930's and 1940's. Our theory, nevertheless, is of very recent origin; and its germ can be found in the paper [27] of P.P. Pálfy and P. Pudlák. In 1981, McKenzie obtained two crucial results for the theory (rudimentary versions of Theorem 2.8 and Theorem 2.11) and applied them in [22]. Further impetus was given by results of Hobby in [18] (an early version of Theorem 5.5) and of Pálfy in [26] (Theorem 4.7). The theory then rapidly