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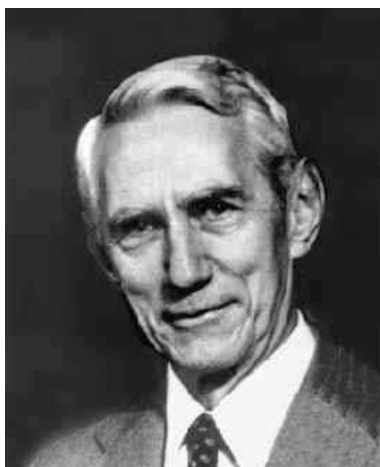
Theoretical and Mathematical Physics

Quantum
Information Theory
and Quantum
Statistics

 Springer



John von Neumann



Claude Shannon



Erwin Schrödinger

Preface

Quantum mechanics was one of the very important new theories of the 20th century. John von Neumann worked in Göttingen in the 1920's when Werner Heisenberg gave the first lectures on the subject. Quantum mechanics motivated the creation of new areas in mathematics, the theory of linear operators on Hilbert spaces was certainly such an area. John von Neumann made an effort towards the mathematical foundation and his book "*The mathematical foundation of quantum mechanics*" is still rather interesting to study. The book is a precise and self-contained description of the theory, some notations have been changed in the mean time in the literature.

Although quantum mechanics is mathematically a perfect theory, it is full of interesting methods and techniques, the interpretation is problematic for many people. An example of the strange attitudes is the following: "*Quantum mechanics is not a theory about reality, it is a prescription for making the best possible prediction about the future if we have certain information about the past*" (G. 't' Hooft, 1988). The interpretations of quantum theory are not considered in this book. The background of the problems might be the probabilistic feature of the theory. On the one hand, the result of a measurement is random with a well-defined distribution, on the other hand the random quantities do not have joint distribution in many cases. The latter feature justifies the so-called quantum probability theory.

Abstract information theory was proposed by the electric engineer Claude Shannon in the 1940's. It became clear that coding is very important to make the information transfer efficient. Although quantum mechanics was already established, the information considered was classical, roughly speaking this means the transfer of 0-1 sequences. Quantum information theory was born much later in the 1990's. In 1993 C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W. Wootters published the paper *Teleporting an unknown quantum state via dual classical and EPR channels* which describes a state teleportation protocol. The protocol is not complicated, it is somewhat surprising that it was not discovered much earlier. The reason can be that the interest in quantum computation motivated the study of the transmission of quantum states. Many things in quantum information theory is related to quantum computation and to its algorithms. Measurements on a quantum system provide classical information and due to the randomness classical statistics can be used to estimate the true state. In some examples, quantum information can appear, the state of a subsystem can be so.

The material of the book was lectured at the Budapest University of Technology and Economics and at the Central European University mostly for physics and mathematics majors, and for newcomers in the area. The book addresses graduate students in

mathematics, physics, theoretical and mathematical physicists with some interest in the rigorous approach. The book does not cover several important results in quantum information theory and quantum statistics. The emphasis is put on the real introductory explanation of certain important concepts. Numerous examples and exercises are also used to reach this goal. The presentation is mathematically completely rigorous but friendly, whenever this is possible. Since the subject is based on non-trivial applications of matrices, the appendix summarizes the relevant part of linear analysis. Standard undergraduate courses of quantum mechanics, probability theory, linear algebra and functional analysis are assumed. Although the emphasis is on quantum information theory, many things are explained from classical information theory as well. Some knowledge about classical information theory is convenient, but not necessary.

I thank to my students and colleagues, especially, Tsuyoshi Ando, Thomas Baier, Imre Csiszár, Katalin Hangos, Fumio Hiai, Gábor Kiss, Milán Mosonyi and József Pitrik for helping me to improve the manuscript.

Dénes Petz

Contents

1. Introduction	9
2. Prerequisites from Quantum Mechanics	11
2.1 Postulates of Quantum Mechanics	12
2.2 State Transformations	22
2.3 Notes	29
2.4 Exercises	29
3. Information and Its Measures	33
3.1 Shannon's Approach	33
3.2 Classical Source Coding	36
3.3 von Neumann Entropy	41
3.4 Quantum Relative Entropy	43
3.5 Rényi Entropy	51
3.6 Notes	54
3.7 Exercises	55
4. Entanglement	57
4.1 Bipartite Systems	57
4.2 Dense Coding and Teleportation	66
4.3 Entanglement Measures	70
4.4 Notes	72
4.5 Exercises	73
5. More About Information Quantities	75
5.1 Shannon's Mutual Information	75
5.2 Markov Chains	76
5.3 Entropy of Partied Systems	77
5.4 Strong Subadditivity of the von Neumann Entropy	79
5.5 The Holevo Quantity	81
5.6 The Entropy Exchange	81

5.7	Notes	82
5.8	Exercises	83
6.	Quantum Compression	85
6.1	Distances Between States	85
6.2	Reliable Compression	87
6.3	Universality	90
6.4	Notes	91
6.5	Exercises	92
7.	Channels and Their Capacity	93
7.1	Information Channels	93
7.2	The Shannon Capacity	94
7.3	Holevo Capacity	97
7.4	Classical-quantum Channels	105
7.5	Entanglement-assisted Capacity	106
7.6	Notes	106
7.7	Exercises	106
8.	Hypothesis Testing	109
8.1	The Quantum Stein Lemma	110
8.2	The Quantum Chernoff Bound	114
8.3	Notes	118
8.4	Exercises	118
9.	Coarse-grainings	119
9.1	Basic Examples	119
9.2	Conditional Expectations	121
9.3	Commuting Squares	128
9.4	Superadditivity	130
9.5	Sufficiency	130
9.6	Markov States	135
9.7	Notes	138
9.8	Exercises	138
10.	State Estimation	141
10.1	Estimation Schemas	141
10.2	Cramér-Rao Inequalities	147
10.3	Quantum Fisher Information	151
10.4	Contrast Functionals	158

10.5 Notes	159
10.6 Exercises	160
11. Appendix: Auxiliary Linear and Convex Analysis	161
11.1 Hilbert Spaces and Their Operators	161
11.2 Positive Operators and Matrices	163
11.3 Functional Calculus for Matrices	166
11.4 Distances	170
11.5 Majorization	172
11.6 Operator Monotone Functions	175
11.7 Positive Mappings	183
11.8 Matrix Algebras	189
11.9 Conjugate Convex Function	191
11.10 Some Trace Inequalities	193
11.11 Notes	193
11.12 Exercises	194
Index	199
Literature	203